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COMMON SCHOOL
ARITHMETIC;

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ELEMENTS OF THE SCIENCE,

WITH THEIR

PRACTICAL APPLICATIONS TO BUSINESS.

BY

JOHN H. FRENCH, LL.D.

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NEW YORK:
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PREFACE.

Business men generally agree in the statement that, after leaving school, they were obliged to learn, and in many cases to devise for themselves, methods of computation adapted to their use in business. The universality of this experience led the Author of this Series of Arithmetics to a careful and protracted investigation into the philosophy of the development of the mathematical powers of the mind, and to a critical examination into the present methods of teaching Arithmetic. From these investigations he has become convinced that, in order to make *practical arithmeticians*, a radical change in the plan of text-books upon the subject is necessary.

The acknowledged requisites of a good method of instruction are these:

1st. It must be adapted to the nature of both subject and learner;

2d. It must be an uninterrupted progress from the easy to the difficult;

3d. It must use facts known to the learner, in imparting to him a knowledge of the unknown subject;

4th. It must regard the natural order of development of the subject, and present it in that order;

5th. It must arrange the substance of the facts presented under each general division, into brief summaries, recapitulations, or general principles;

6th. It must thoroughly reach the understanding of the learner.

This book fully recognizes these requisites, in every Chapter, Section, and Case. The attention of teachers and parents is particularly invited to the following distinctive features of the work:

Order of Subjects.—A philosophical arrangement of subjects has been carefully observed. The four classes of numbers, Integers, Decimals, Compound Numbers, and Fractions, are presented in the order here stated. But Factors and Multiples precede Fractions, as a knowledge of the former subject is essential to a clear understanding of the latter. The Chapter on Converse Operations presents, in immediate connection, those operations in numbers which are the converse of each other. The succeeding Chapters present the subjects of Percentage, Ratio and Proportion, Evolution, Progressions, Mensuration, and Miscellaneous Problems.

Principles and Rules.—Each new process or method of computation is introduced inductively, and the Principles which follow are evident sequents of the Inductions. The learner is required to make practical applications of these Principles in solving a number of Problems, thus giving him a thorough comprehension of the principles upon which the practical calculations of after life are based. Rules are then presented, which are uniformly based upon the Principles previously established.

Illustrations.—The pictures are designed to aid the pupil in acquiring a clear understanding of the subjects and principles which they illustrate, and also to cultivate a taste for the beautiful in art. The Italic Figures, which were cut expressly for this Series, also add much to the beauty and attractiveness of the book.

Problems.—Much labor has been bestowed upon the preparation of the Problems, to make them the vehicle of practical knowledge. Nearly every occupation, trade, profession, and art has its own peculiar business terms, and its peculiar articles of commercial exchange, with their appropriate market values. The use of these business terms, and transactions in these articles of exchange, make up the every-day matters of the business world. Thus, the merchant, the manufacturer, the grocer, the druggist, the physician, the lawyer, the printer, the bookseller, etc., etc., has each his own statistics of business; and from these have been prepared problems that convey to the pupil a great amount of knowledge of the principles, customs, and details of business. Such problems are all the more interesting from their practical utility, while they are none the less illustrative of the Principles of Numbers. The Problems are all prepared from materials collected for this book, and present statements and business terms in a correct business way. In this way the arithmetic of the school-room may be made to meet, in a considerable degree, the practical wants of after life. Abstract problems possess little interest for pupils, and hence few of them are found in this book: but the number of practical problems drawn from the every-day transactions of business, greatly exceeds that in any similar work.

Useless Matter.—Reduction of State Currencies is nearly obsolete, Alligation Alternate is merely a curiosity of numbers, English Money is of little value to American youth, and many denominations in Compound Numbers often heard of in the school-room are unknown in business. Those who look for these matters in this book will look in vain.

New and Distinctive Features will be found in the Notation, Multiplication, and Division of Integers and Decimals; Compound Numbers; Factors and Multiples; Division of Fractions; Converse Operations; Table of Legal Rates of Interest, from official sources; Classification of all computations in Percentage under Five General Cases; Rules for Interest and Partial Payments; Average of Accounts; Proportion; Evolution; Progressions; Review and Miscellaneous Problems; and the deductions of Principles from Inductions, and the basing of Rules upon Principles.

In the preparation of this work for American Schools, the Author has had constantly in mind the present condition and the future requirements of American youth. The work differs, both in general plan and details, from other works upon the same subject. It is confidently believed that the adoption and introduction into schools of the distinctive features of the book will effect a change in methods of teaching, that shall result in making *good*, *practical Arithmeticians.*

MANUAL OF METHODS AND SUGGESTIONS

A Word with Teachers.—A hint, a suggestion, or an item of information not found in the body of a text-book, will often awaken thought, and start a train of inquiries in a class, that will greatly increase their interest in the study. Connected with the subject Arithmetic, and not found in text-books, are many items of interest and importance to pupils, to which their attention should, at the proper time, be directed. This MANUAL is intended to give you brief hints and suggestions, that will enable you the better to give instruction to your pupils in this important branch of study.

Page 9. Arithmetic, as a *Science*, logically investigates and philosophically classifies and arranges the principles and rules of the subject; as an *Art*, it applies the principles and rules for computations, to the practical affairs of life.

10. As the Roman Notation is not presented in this book, it may be well to spend time enough in giving oral instruction upon the subject, to make pupils familiar with the following facts and their application:

I. *The Roman Notation employs seven letters to represent numbers.*

II. *Each letter has a fixed value when used alone.* Thus, I.=1, V.=5, X.=10, L.=50, C.=100, D.=500, and M.=1,000.

III. *Repeating a letter repeats its value.* Thus, II.=2, XXX.=30, CC.=200.

IV. *When any letter stands after another expressing greater value, the number expressed is the sum of the values of the two letters.* Thus, VI.=6, XV.=15, CXVI.=116.

V. *When any letter stands before another expressing greater value, the number expressed is the difference of the values of the two letters.* Thus, IV.=4, IX.=9, XC.=90, CM.=900.

VI. *When a letter stands between two others, both of greater value, its value is taken from the sum of their values.* Thus, XIX.=19, CIV.=104, MXL.=1,040.

13. Beginners require abundant practice both in writing and reading numbers. Give them numerous exercises on each new period of figures.

15. Explain clearly that the *simple value* is the number of ones or units expressed by the figure, and the *local value* is the value given to these units by the place; *i. e.*, one is the value of the figure determined by its form, the other by the place it occupies.

Dull pupils may be aided in learning to write and read numbers, by allowing them at first to write a *Skeleton of Notation*, consisting of periods of ciphers; thus, 000,000,000,000; and under this to write the figures of any given number, in their proper places. They should also learn the *Family Name* of each period,—as ones, thousands, millions; and the *names of the places* in each family,—as ones, tens, hundreds.

19. Give original problems under each Section and Case in this Chapter, before assigning the problems from the book. This may be done in various ways;—the following has been found a good one: At the close of a recitation, or other convenient time, put problems on the blackboard, and let the class copy them upon their slates. At the commencement of the next recitation, call for the results (and solutions also, if you choose), to all the problems given out at the previous lesson.

25. More mistakes occur in addition than in any other process. Thorough drills in adding columns whose sums reach 100, will greatly lessen this defect. These 26 problems (41—67) afford this kind of drill; they should not be skipped.

30. Those teachers who prefer to use the "borrowing 10" method of subtraction, will find an explanation of that method in the Elementary Arithmetic of this Series.

32. An explanation of *Left-hand Subtraction* sometimes assists pupils in acquiring a clear understanding of the process commonly called "borrowing ten." The following course will enable you to make the explanation:

1st. Solve the problems in Case I., page 28, commencing at the left hand to subtract.

2d. Take a subtrahend whose right-hand figure only exceeds the corresponding figure in the minuend (as 582—347), commence at the left to subtract, calling the tens figure of the minuend 1 less, and adding the 1, as a ten, to the ones of the minuend.

3d. Proceed in a similar manner with problems in which other orders of units of the subtrahend exceed the like orders in the minuend, as shown in the following:

EXPLANATION.—Since 7 is more than 3, we have 4—1=3. Since 2 is more than 0, we have 12—7=5. Since 4 is not more than 6, we have 10—2=8. Since 8 is more than 2, we have 5—4=1. And 12—8=4.

SOLUTION.
53062
17248
35814

With practice, pupils will acquire great facility in left-hand subtraction.

34. The market grades or qualities of some kinds of goods are indicated by certain marks upon the cask or package. Thus, in sugars, we have **A** or "straight A," ◇A or "diamond A," Ⓐ or "circle A," **B**, and **C**; etc.

38. In explanations of solutions, use the true multiplicand for the multiplicand.

52. A clear understanding of this Principle will aid the pupil in the induction to division of decimals.

57. These problems should be solved by both long and short division.

59. In long division, require pupils to memorize the catch-words, *divide, multiply, subtract, bring down.* And indelibly impress upon their memories this fact:

For every figure of the dividend used after the first quotient figure is obtained, there must be a figure in the quotient.

61. Referring to **93**, Note 1, teach the pupil how to write a quotient containing a fraction; as $14\frac{27}{53}$.

69. Instruct pupils, when explaining solutions, to tell what is given, and what is required. Thus, "*In problem 4 are given a number and all its parts but one. To find this part, I subtract the sum of the given parts from the number.*"

In problem 9 explain the use of the " or repeating marks.

70. *Problem* 16. "As per annexed schedule" is a common business reference to a bill or memorandum on the same paper. Call the attention of pupils to the signification of the commercial terms found in the problems.

71. *Problem* 30. Cultivate neatness, order, and method in blackboard work. Thus, the solution of this problem might be placed upon the blackboard as here shown.

(*30*).
Bounty, *$949*
8 mo. @ *$13*= *$104*
8 " " *14*= *112*
13 " " *17*= *221*
7 " " *18*= *126*
36 mo. *$1512*
$1512+36=$42 per mo.

Explain the meaning of *Average*, and how it is found.

72. The Areas in this Table are from standard authorities. A little ingenuity will enable you to greatly increase the number of problems of the kind found on this page.

75. The only place in which *and* is properly used in reading numbers, is in a mixed number, after the integer. Thus, 5.7, 5 and 7 tenths; $4\frac{9}{16}$, 4 and 9 sixteenths.

78. Pupils should carefully compare Art. **129**, **130**, with Art. **25**, **26**, **27**.

80. Familiarize pupils with the reading of such numbers as 1400.010; 1000.410; .1400.

81. } **83.** } Instruct pupils to place the decimal point in the result, before adding or subtracting the ones. Require them to subtract without writing decimal ciphers in the minuend over decimal figures in the subtrahend.

57
5.91

94. Encourage pupils to solve problems in different ways. Thus, "In how many ways can this problem (23) be solved?" "Which way do you prefer, and why?"

97. *First Reference.*—The coins are shown in the cut, page 96, in perspective, and of course only the longest diameter is correct in measurement.

Second Reference.—Gold and silver coins are alloyed, to make them hard enough for use as a circulating medium, without depreciating perceptibly in value by wear.

Third Reference.—Pupils must make all their computations in decimals; and express parts of a cent, in final results, in fractions, when they are halves, fourths, or eighths.

113. "Fixed Standards" are weights and measures established by the General Government, or recognized and sanctioned by custom.

115. Abbreviations of denominations should always be written after the numbers, and be followed by periods. The signs $ and £ are written before the numbers.

118. By English Statute Law, a heaped bushel is 18½ inches in diameter, 8 inches deep, and heaped in a true cone to the height of 6 inches. This cone, 18½ inches in diameter and 6 inches high, is 1 peck.

125. } The denominations of square and cubic measures are used only in computa-
126. } tions, the measurements being taken in linear units.

131. Each lot of a Government Section of land is divided into 2 *Forties.* Hence, 1 section (640 A.)=4 quarter-sections (160 A. each)=8 80-acre lots (or 80's)=16 40's.

135.

1	solar	year	is		5 h.	48 min.	48 sec.	longer than	a	common	year.
4	"	years	are		23 "	15 "	12 "	"	4	"	years.
100	"	"	"	24 da.	5 "	20 "		"	100	"	"
400	"	"	"	96 "	21 "	20 "		"	400	"	"

Hence, if 97 days be added to every 400 years, the calendar will be only 2 h. 40 min. ahead of true time. These 97 days are distributed among 97 leap-years. (See **234.**)

137. Require pupils to point out, on all diagrams, the lines defined.

138. A geographic mile=$\frac{1}{60}$ of 69.16 Eng. mi.=1.15$\frac{4}{15}$ mi.=1 mi. 48.85 rd.

142. The commonly recognized units of the other Tables are, for Canada Money, Dollar; Sterling Money, Pound; Wood Measure, Cord; Surveyors' Linear Measure, Foot and Chain; Surveyors' Square Measure, Square Chain and Acre; Time, Day and Year; Circles, Degree, Right Angle, and Circumference.

144. As none of the tables or denominations of the Metric System have come into actual use, a presentation of the Tables is all that the present state of the subject demands.

150. } Problems like 17 are easily solved by left-hand subtraction.
152. } Commencing at the left, we have 17 mi.—14 mi. =3 mi. 3 rd. (4 rd.—1 rd.)—0 rd.=3 rd. 5½ yd.+2 yd. =7½ yd., and 7½ yd.—4 yd.=3½ yd.=3 yd. 1 ft. 6 in. 1 ft. 6 in.+1 ft.=2 ft. 6 in., and 2 ft. 6 in.—2 ft.=0 ft. 6 in.

$$\begin{array}{rrrrr} 17 \text{ mi.} & 4 \text{ rd.} & 2 \text{ yd.} & 1 \text{ ft.} & \\ 14 & 0 & 4 & 2 & \\ \hline 3 \text{ mi.} & 3 \text{ rd.} & 3 \text{ yd.} & 0 \text{ ft.} & 6 \text{ in.} \end{array}$$

167. If the multiplier or divisor is less than 1, the first two principles will be reversed.

178. The following method of finding the least common multiple is preferred by many. Take, for example, Problem 13, page 178.

We first arrange the numbers from least to greatest, and cancel or drop such as are factors of any of the others. We next divide through by any prime number that is a factor of any two, or by any number that is a factor of all the given numbers; and divide these results and the undivided numbers in the same manner; and so on, until the final quotients are prime to one another. The divisors and final quotients are the factors of the least common multiple.

$$\begin{array}{r|l} \cancel{9}, \cancel{12}, \cancel{15}, \cancel{18}, 35, 45, 60, 72 & 5 \\ \hline 7, \ 9, 12, 72 & 3 \\ \hline 7, \ 3, \ 4, 24 & 3 \\ \hline 7, \ 1, \ 4, \ 8 & 4 \\ \hline 7, \ 1, \ 1, \ 2 & \end{array}$$

$$7 \times 2 \times 5 \times 3 \times 3 \times 4 = 2520$$

183. Show wherein these General Principles are similar to the General Principles of Division, Art. **278.**

187. The fractional unit $\frac{1}{2}$ is 6 times as great as the fractional unit $\frac{1}{12}$. That is, *The less the denominator of a fraction, the greater is its fractional unit.*

193 Mixed numbers are readily subtracted by left-hand subtraction. For example, take Ex. 2, page 193. Since $\frac{20}{36}$ is more than $\frac{9}{36}$, we have 6—3=3, and $\frac{36+9}{36}-\frac{20}{36}=\frac{25}{36}$.

$$\begin{array}{r} 7\frac{1}{4}=7\frac{9}{36} \\ 3\frac{5}{9}=3\frac{20}{36} \\ \hline 3\frac{25}{36} \end{array}$$

206. Give several original problems like Problems 9, 10. Require pupils to write out a full explanation of a solution, and therefrom deduce the *Principle* stated in this Note.

211. Decimal figures which continually repeat, are called a *Repetend*. Its value is expressed by a fraction with the repeating figures for the numerator, and as many 9's for a denominator. Thus, $\frac{2}{3}=.666\ldots\ldots=\frac{6}{9}=\frac{2}{3}$; $\frac{3}{7}=.428571\ldots\ldots=\frac{428571}{999999}=\frac{3}{7}$.

221. C. is the abbreviation for the Latin *Centum*, signifying one hundred; and M. for the French *Mille*, signifying one thousand.

223. A logical explanation of any solution requiring more than one process or computation, or of the reasons upon which any principle or process is based, is an Analysis. This section applies particularly to the solution of problems which involve more than one process or computation.

224. Pupils should face south, and hold their books erect before them, while studying or explaining this astronomical cut.

252. The tax on any sum from \$1 to \$10,000 can be taken from a Table that gives only the tax on \$1, \$2, \$3..to....\$10 inclusive, if the table is carried to six decimal places. Thus, if the tax on \$1 is \$.023145, on \$10 it is \$.23145; on \$100, \$2.3145; on \$1,000, \$23.145; on \$10,000, \$231.45; and so of \$2, \$20, \$200, \$2,000, etc.

262. In New Hampshire and Connecticut teachers should require pupils to solve the problems on page 262, both by the U. S. Court Rule and the Rule for their own State. In the books used in Vermont will be found, in an appendix of 12 pages, the Vermont law for annual interest, with explanations and applications.

291. Explain that the third term is divided by the first, to find the value of a unit; and the result is multiplied by the second term, to find the value of the number of units; the same as in Analysis, Art, **370**. Also, require pupils to solve the problems by analysis, after solving them by proportion.

293. Pupils will learn to state problems very rapidly, if they are taught to first write the terms in two lines, as they occur, writing the second set of conditions under corresponding terms of the first. For example, in Problem 6, page 294, the pupil writes

10 h. 1365 bar. 13 da.
16 h. ? " 39 "

He has then only to arrange the couplets.

In solving problems, some teachers write the work in one of the forms here shown, in preference to the forms given on pages 290, 293.

(P. 292, prob. 24).

$$\begin{array}{r|ll} & \cancel{6}\ ft. & 2 \\ .5\ \cancel{1.5}\ \cancel{7.5} & \cancel{85} & 17 \\ \hline .5 & 34.0 & \\ \cline{2-3} & 6\,8\ ft. & \end{array}$$

(P. 292, prob. 24).

$$\begin{array}{rr|l} 2 & \cancel{6}\ ft. & \\ 17 & \cancel{85} & \cancel{7.5}\ \cancel{1.5}\ .5 \\ \hline & 34.0 & .5 \\ \cline{1-2} & 6\,8\ ft. & \end{array}$$

300. Other roots are indicated by placing over the radical sign the figure denoting the required root; as $\sqrt[4]{}$, $\sqrt[5]{}$, $\sqrt[6]{}$, etc.

310. Since $2^2 \times 2^2 = 2^4$, the $\sqrt[4]{}$ of a number $= \sqrt{}$ of $\sqrt{}$. And since $2^2 \times 2^2 \times 2^2 = (2^2)^3 = 2^6$, and $2^3 \times 2^3 = (2^3)^2 = 2^6$, the $\sqrt[6]{}$ of a number $= \sqrt{}$ of $\sqrt{}$ of $\sqrt{}$, or $\sqrt{}$ of $\sqrt[3]{}$, or $\sqrt[3]{}$ of $\sqrt{}$.

313. } **317.** } In Progressions, if any three of the five things are given, the other two may be found. The rules here given cover the ordinary applications of the subject.

321. Have a figure drawn to illustrate each definition and problem in this Chapter.

325. } **327.** } These Principles (V., p. 325, III., p. 327) may be made plain to the pupil by practical applications.

COMMON SCHOOL ARITHMETIC.

CHAPTER I.

INTEGERS

SECTION I. DEFINITIONS.

1. A *Unit* is a single thing, or one, of any kind.

2. A *Number* is a unit, or a collection of units.

NOTE.—Any number is either concrete or abstract.

3. A *Concrete Number* is a number applied to some object; as, four men, ten apples, fifty days.

4. An *Abstract Number* is a number not applied to any object; as, four, ten, fifty.

5. An *Integer* is a number the units of which are whole or undivided.

NOTE.—Integers are also called *Whole Numbers.*

6. *Unity* is the abstract unit 1.

7. *Arithmetic* is the science of numbers, and the art of computation. (See Manual, page 5.)

8. A *Solution* is a process of computation used to obtain a required result.

9. A *Problem* is a question requiring a solution.

10. An *Explanation* is a statement of the reasons for the manner of solving a problem.

11. A *Principle* is a general truth upon which a process of computation is founded.

12. An *Example* is a problem used to illustrate a principle, or to explain a method of computation.

13. An *Analysis* is a statement of the different steps in a solution.

14. A *Rule* is a brief direction for performing any computation.

NOTE.—These general definitions apply to all classes of numbers.

SECTION II.

NOTATION AND NUMERATION.

15. *Notation* in arithmetic is the art of expressing numbers by ten characters, called figures.

These figures are

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
called	*naught,*	*one,*	*two,*	*three,*	*four,*	*five,*	*six,*	*seven,*	*eight,*	*nine.*

The figure 0, also called *Cipher,* denotes nothing, or the absence of number; and the other figures represent the first nine integers, and are sometimes called *Digits.*

16. *Numeration* is the art of reading numbers expressed by figures. (See Manual, page 5.)

To express numbers greater than 9, two or more of the ten figures must be combined.

17. In writing numbers, every ten *ones* taken together are called a *ten.*

Ten is written				*10*
2 tens are called		*twenty,*	written	*20*
5 tens	"	*fifty,*	"	*50*
8 tens	"	*eighty,*	"	*80*
9 tens	"	*ninety,*	"	*90*

When two figures are written together to express a number, the left-hand figure expresses *tens,* and the right-hand figure *ones.* Thus,

Sixteen	consists of	1 ten and	6 ones,	written	*16*
Thirty-five	"	3 tens "	5 ones,	"	*35*
Seventy-two	"	7 tens "	2 ones,	"	*72*
Ninety	"	9 tens "	0 ones,	"	*90*

18. Every 10 tens taken together are called a *hundred.*

One hundred	is written	*100*
Two hundred	"	*200*
Seven hundred	"	*700*

When three figures are written together to express a number, the left-hand figure expresses *hundreds*, and the other two figures express *tens* and *ones*. Thus,

Four hundred twenty-seven consists of 4 hundreds 2 tens and 7 ones, and is written *427*

2 hundreds 5 tens and 6 ones, or two hundred fifty-six, is written *256*

7 hundreds 1 ten and 8 ones, or seven hundred eighteen, is written *718*

5 hundreds 3 tens and 9 ones, or five hundred thirty-nine, is written *539*

4 hundreds 6 tens and 0 ones, or four hundred sixty, is written *460*

1 hundred 0 tens and 5 ones, or one hundred five, is written *105*

EXERCISES.

1. Write in words, 10, 30, 70, 23, 99, 16, 11, 12.
2. Write in words, 100, 400, 700, 350, 280, 190.
3. Write in words, 596, 281, 694, 375, 333, 899.
4. Write 108, 904, 301, 707, 510, 811, 600, 150.

Express by figures the following numbers:

5. Fifty, ninety, forty-one, sixty-six.
6. Fourteen, one hundred, four hundred, six hundred.
7. Two hundred sixty, five hundred ninety.
8. Seven hundred ten, three hundred twenty-six.
9. Five hundred eighty-one, six hundred fifteen.
10. Two hundred four, five hundred three.
11. Seven hundred six, eight hundred one.
12. Six hundred fifty, seven hundred twelve.
13. Five hundred sixty-three, two hundred ninety.
14. One hundred nineteen, nine hundred ninety-nine.

19. In writing numbers, every 10 hundreds taken together are called a *thousand*, every 10 thousands taken together are called a *ten-thousand*, and every 10 ten-thousands are called a *hundred-thousand*.

When a figure stands at the left of hundreds in a number, it express thousands; when at the left of thousands, it expresses ten-thousands; and when at the left of ten-thousands, it expresses hundred-thousands.

One thousand	is written	*1,000*
Five thousand	"	*5,000*
Nine thousand	"	*9,000*
Ten thousand	"	*10,000*
2 ten-thousands, or twenty thousand,	"	*20,000*
Sixty thousand	"	*60,000*
One hundred thousand	"	*100,000*
Three hundred thousand	"	*300,000*
Eight hundred thousand	"	*800,000*

20. Every three figures in any number, counting from the right, are called a *Period*. Periods of figures are separated from each other by commas.

The first, or right-hand period, consists of ones, tens, and hundreds; and the second period of ones, tens, and hundreds of thousands.

of thousands.					
7	5	2,	1	9	4
hundreds	tens....	ones....	hundreds	tens....	ones....

Nine thousand one hundred	is written	*9,100*
Six thousand two hundred fifty	"	*6,250*
Two thousand seven hundred forty-two	"	*2,742*
Ten thousand four hundred	"	*10,400*
Fifty-six thousand six hundred seventy	"	*56,670*
Nineteen thousand one hundred thirty-six	"	*19,136*
Forty thousand seven hundred nine	"	*40,709*
Eighty-two thousand six	"	*82,006*
Three hundred seventy-six thousand	"	*376,000*
Five hundred ten thousand	"	*510,000*
Six hundred thousand five hundred	"	*600,500*
Three hundred fifty-two thousand seven hundred eighty-two	"	*352,782*

EXERCISES.

15. Read 5,000; 4,200; 1,360; 6,384; 3,569; 8,113.
16. Read 9,011; 5,608; 3,008; 1,040; 4,076.
17. Read 30,000; 57,000; 42,300; 65,850; 83,294.
18. Read 15,203; 47,056; 50,912; 90,052; 89,005.
19. Read 80,006; 25,030; 60,209; 40,475; 30,800; 55,763.

Write the following numbers:

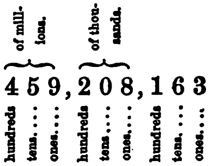
20. Two thousand; seven thousand five hundred.
21. Four thousand one hundred sixty.
22. Nine thousand six hundred fifty-three.
23. Three thousand eight hundred eleven.
24. Seven thousand forty-one.
25. One thousand one; two thousand fifty.
26. Five thousand four hundred nine.
27. Sixteen thousand five hundred.
28. Eighty-one thousand two hundred seventy.
29. Eleven thousand nine hundred eighty-five.
30. Read 275,000; 100,000; 860,000; 493,600; 815,350.

Write the following numbers:

31. Two hundred thousand.
32. Six hundred fifty thousand eight hundred.
33. One hundred nine thousand seven hundred twenty-six.
34. One hundred five thousand eighty. See Manual.

21. The third period of figures consists of *ones, tens*, and *hundreds of millions*.

In any full period the right-hand figure is ones, the middle figure tens, and the left-hand figure hundreds. Thus, in any number consisting of three full periods, there are ones, ones of thousands, and ones of millions; tens, tens of thousands, and tens of millions; and hundreds, hundreds of thousands, and hundreds of millions.



One million two hundred thirty-one thousand three hundred sixty-four is written		*1,231,364*
Twenty-five million	"	*25,000,000*
Nine hundred million	"	*900,000,000*
Four hundred six million	"	*406,000,000*

EXERCISES.

35. Read 4,000,000; 80,000,000; 73,000,000; 9,721,312.
36. Read 18,271,100; 300,000,000; 253,729,594; 604,000,000.

Write the following numbers:

37. Nine million; fourteen million.

38. Four hundred fifty-two million.
39. Nine hundred one million.
40. Three hundred million two hundred sixty-five thousand.
41. Five hundred nine million six hundred twelve thousand nine hundred eighty-five.

22. The first period is called the period of ones or units, the second the period of thousands, and the third the period of millions.

The fourth period is that of *billions*, the fifth that of *trillions*, and the sixth that of *quadrillions*.

of quadrillions			of trillions			of billions			of millions			of thousands			of units		
4	9	3,	3	6	7,	5	0	8,	2	1	0,	0	6	4,	1	1	9
hundreds	tens....	ones....	hundreds	tens....	ones....	hundreds	tens....	ones....	hundreds	tens....	ones....	hundreds	tens....	ones....	hundreds	tens....	ones....

EXERCISES.

42. Read 4,359,006,110; 19,000,000,000; 40,060,139,194.
43. Read 5,236,481,279; 10,500,600,000; 92,675,244,000.
44. Read 3,000,000,000,000,000; 396,728,136,294.
45. Read 17,252,005,030; 18,000,039; 410,000,060,000.

Write the following numbers:

46. Five billion two hundred million twenty-two thousand eight.
47. Forty-five billion one hundred fifteen million one hundred sixty-four thousand eighty-nine.
48. Fifty-two trillion.
49. One hundred nine quadrillion.
50. Nine billion three hundred six thousand.
51. Four hundred seventy-eight quadrillion two hundred thirty-four trillion eight billion five hundred sixteen million seven hundred thousand five hundred eight.
52. Six hundred nineteen million thirty.

23. The ones, tens, hundreds, thousands, etc., of any number are called *units of different orders;* ones being simple units, or units of the first order:

9th order.	8th order.	7th order.	6th order.	5th order.	4th order.	3d order.	2d order.	1st order.
5	9	3,	2	9	8,	7	5	6

tens, units of the second order; hundreds, units of the third order, and so on.

24. Every figure has an absolute or simple value, and a local value. Its *simple value* is the number of ones it expresses when taken alone. Its *local value* is the order of units it expresses in a number. Thus, 8 when taken alone expresses 8 things, 8 ones, or 8 simple units; but when taken with other figures it expresses different units, according to its place. In 80, it expresses 8 tens; in 800, 8 hundreds; in 8,000, 8 thousands, and so on. (See Manual, page 5.)

25. A unit of any order is ten times as great in value as a unit of the next lower order. Thus, a ten is 10 times a one; a hundred 10 times a ten; a thousand 10 times a hundred, and so on, as shown in the following

NOTATION AND NUMERATION TABLE.

10 ones	are 1 ten,
10 tens	" 1 hundred,
10 hundreds	" 1 thousand,
10 thousands	" 1 ten-thousand,
10 ten-thousands	" 1 hundred-thousand,
10 hundred-thousands	" 1 million,
and so on.	
1 ten	is 10 ones,
1 hundred	" 10 tens,
1 thousand	" 10 hundreds,
1 ten-thousand	" 10 thousands,
1 hundred-thousand	" 10 ten-thousands,
1 million	" 10 hundred-thousands,
and so on.	

26. *Principles of Notation.*

I. *The values of the different places in a number increase from right to left in a tenfold ratio.*

II. *The place which any figure occupies in a number determines the value expressed by it in that number.*

III. *The highest period of any number must stand at the left, and the succeeding periods in their order.*

IV. *Every full period must consist of three figures,—hundreds, tens, and ones; the place of any unit not named in the given number being filled by a cipher.*

V. *The three places of any period not named in a number must be filled by three ciphers.*

27. *Principles of Numeration.*

I. *Every integer consisting of more than three figures should be separated into periods.*

II. *Each period of an integer is read separately, as hundreds, tens, and ones; the name of the period being pronounced after the ones.*

III. *In reading any number, the names of places and periods filled with ciphers are omitted.*

EXERCISES.

53. Read 80; 290; 763; 409; 7,000; 2,009; 5,080.

54. Read 9,393; 6,500; 50,000; 83,400; 14,008; 10,086.

55. Read 512,694; 809,123; 559,026; 300,006; 110,090.

Write the following numbers:

56. Eighty; three hundred; nine hundred ten.

57. Fifty-five; seven hundred sixteen; four hundred one.

58. Eight thousand; fifty thousand; ninety-two thousand.

59. Six hundred twelve thousand one hundred sixty-five.

60. Fifteen thousand seventeen.

61. Four hundred thousand fifty-six.

62. Sixty million; seven hundred million.

63. One hundred eighty-two million three hundred fifty-five thousand four hundred eighty-eight.

64. Two hundred nine million eighteen thousand nine hundred ten.

65. Read 320,000,296; 200,165,000; 693,100,083; 501,080,276.

66. Read 433,279,187,695; 309,400,060,009.

67. Read 393,000,000,000,000,000; 117,371,545,903.

Write the following numbers:

68. Sixteen trillion three hundred ninety-six billion.

69. Two hundred forty-seven billion fifty-six thousand.

70. Seventy-one trillion two hundred forty-one.

71. Two hundred sixty-seven quintillion.

SECTION III.

ADDITION.

INDUCTION AND DEFINITIONS.

28. 1. MARIA had 3 peaches, and George gave her 4 more. How many peaches had she then?

2. Frank has 5 large rabbits and 6 small ones. How many rabbits has he?

3. How many apples are 6 apples, 4 apples, and 7 apples?

4. How many birds are 5 birds, 7 birds, 3 birds, and 6 birds?

5. Ella has 5 roses, Mary has 8, Olive has 4, Alice has 7, Louise has 9, and Flora has 6. How many roses have all the girls?

29. ***Addition*** is the process of uniting two or more numbers to form one number.

30. The ***Amount*** or ***Sum*** is the result obtained by Addition.

31. The ***Parts*** are the numbers which are united to form the sum.

32. The ***Sign of Addition,*** made thus +, is called *Plus;* and when written between numbers, it signifies that they are to be added.

33. The ***Sign of Equality,*** made thus =, when written between numbers or sets of numbers, signifies that they are equal to each other. Thus, $4 + 5 = 9$; $16 = 3 + 7 + 6$.

NOTE.—A number with the sign $ before it expresses dollars.

6. What is the sum of 5 cents, 9 cents, and 8 cents?

7. Add 9, and 5, and 3, and 4, and 7.

8. Add 6 books, 8 books, 5 books, 4 books, and 9 books.

9. 12 days + 3 days + 7 days + 1 day = how many days?

10. What is the amount of 5 pens, 11 pens, 8 pens, and 2 pens?

11. 15 pictures + 7 pictures + 8 pictures + 8 pictures + 9 pictures = how many pictures?

12. The parts are 12, 7, 4, 1, 5, and 8. What is the sum?

34. ADDITION TABLE.

0	0	1	2	3	4	5	6	7	8	9	5	0	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0	0		5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9		5	6	7	8	9	10	11	12	13	14
1	0	1	2	3	4	5	6	7	8	9	6	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1		6	6	6	6	6	6	6	6	6	6
	1	2	3	4	5	6	7	8	9	10		6	7	8	9	10	11	12	13	14	15
2	0	1	2	3	4	5	6	7	8	9	7	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2		7	7	7	7	7	7	7	7	7	7
	2	3	4	5	6	7	8	9	10	11		7	8	9	10	11	12	13	14	15	16
3	0	1	2	3	4	5	6	7	8	9	8	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3		8	8	8	8	8	8	8	8	8	8
	3	4	5	6	7	8	9	10	11	12		8	9	10	11	12	13	14	15	16	17
4	0	1	2	3	4	5	6	7	8	9	9	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4		9	9	9	9	9	9	9	9	9	9
	4	5	6	7	8	9	10	11	12	13		9	10	11	12	13	14	15	16	17	18

CASE I.

The sum of all the figures of any place not more than 9.

35. We can add apples to apples, dollars to dollars, pens to pens, or hours to hours; but we can not add apples to dollars, nor pens to hours. For 4 apples + 9 dollars = neither 13 apples nor 13 dollars.

Again, we can add ones to ones, tens to tens, or hundreds to hundreds; but we can not add ones to hundreds, nor tens to thousands. For 4 tens + 9 thousands = neither 13 tens nor 13 thousands.

36. Example. What is the sum of 4,216, 3,152, and 1,321?

Explanation.—Since we must add ones to ones, tens to tens, hundreds to hundreds, etc., it is most convenient to write the parts with like orders of units in the same column. We then add each column separately, writing the sum directly under the column added. The sum of the ones, 1 + 2 + 6, is 9; the sum of the tens,

SOLUTION.

$$\left.\begin{array}{r}4216\\3152\\1321\end{array}\right\}\text{ Parts.}$$
$$\begin{array}{r}\hline 8689\end{array}\text{ Sum.}$$

2 + 5 + 1, is 8; the sum of the hundreds, 3 + 1 + 2, is 6; and the sum of the thousands, 1 + 3 + 4, is 8. The result, 8,689, is the sum required. (See Manual.)

In this manner solve and explain the following

PROBLEMS.

Find the sum in each of the first five problems:

(1)	(2)	(3)	(4)	(5)
54	71	556	3,615	215,124
35	26	43	2,371	583,642

6. A man gave $22 for a coat, and $25 for an overcoat. How much did he pay for both? *$47.*

7. A gentleman paid $125 for a horse, and $162 for a carriage. How much did both cost him? *$287.*

8. A farmer has 14 cows, 11 oxen, and 23 young cattle. How many head of cattle has he? *48.*

9. One day a miller sold 321 barrels of flour, the next day 143 barrels, and the third day 235 barrels. How much flour did he sell in the three days? *699 barrels.*

10. What is the sum of $5,418, $51, and $430?

11. Add 13,300 miles, 2,051 miles, 1,435 miles, and 2,012 miles.

12. In the Congressional Library at Washington there are 50,700 volumes, and in the library of the Smithsonian Institute 25,000 volumes. How many volumes in both libraries? *75,700.*

13. Three men together purchase a vessel, A paying $11,725, B, $10,050, and C, $8,120. What is the cost of the vessel? *$29,895.*

14. One day a produce dealer bought from three men 720 bushels, 145 bushels, and 1,124 bushels of oats. How many oats did he buy? *1,989 bushels.*

15. An army containing 41,430 men received two reinforcements, the first of 13,225 men, and the second of 24,234 men. How many men were then in the army? *78,889.*

16. A dealer in real estate sold three city lots for $1,220 each, another lot for $2,125, and a farm for $12,210. For how much did they sell? $17,995.

CASE II.

The sum of all the figures of any place more than 9.

37. 5 + 8 + 3 = 16, and 16 = 1 ten and 6 ones.

5 hundreds + 8 hundreds + 3 hundreds = 16 hundreds, and 16 hundreds = 1 thousand and 6 hundreds.

7 tens + 9 tens + 8 tens = 24 tens, and 24 tens = 2 hundreds and 4 tens. Hence

When the sum of the units of any order exceeds 9, the tens of this sum are units of the next higher order.

38. Ex. What is the sum of 3,475, 2,694, and 1,363?

EXPLANATION. — We write the parts as in Case I., draw two horizontal lines underneath, as shown in the First Solution, and then add. The sum of the ones, 3 + 4 + 5, is 12, or 2 ones and 1 ten. We write the 2 ones below the lower line as the ones of the sum, and the 1 ten in tens' place, between the two lines, to be added with the column of the tens. The sum of the tens, 1 + 6 + 9 + 7, is 23, or 3 tens and 2 hundreds. We write the 3 tens as the tens of the sum, and the 2 hundreds in hundreds' place between the two lines. The sum of the hundreds, 2 + 3 + 6 + 4, is 15, or 5 hundreds and 1 thousand. We write the 5 hundreds as the hundreds of the sum, and the 1 thousand in thousands' place between the lines. The sum of the thousands, 1 + 1 + 2 + 3, is 7, and this we write as the thousands of the sum. The result, 7,532, is the sum required.

FIRST SOLUTION.

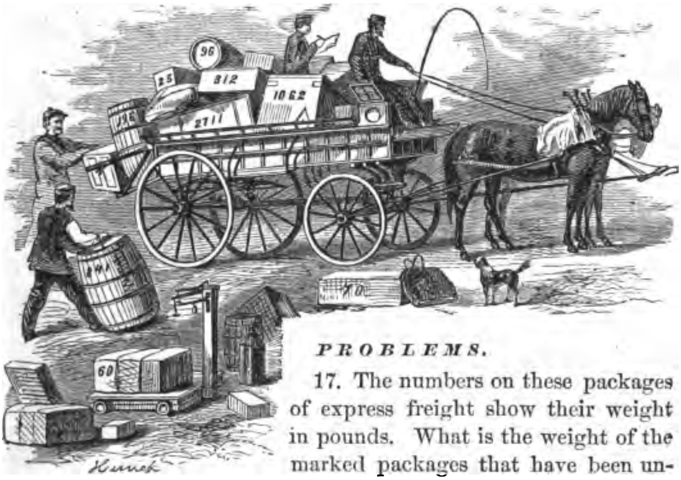
$$\begin{array}{r} 3475 \\ 2694 \\ 1363 \\ \hline 121 \\ \hline 7532 \end{array}$$

EXPLANATION.—In the Second Solution we write the parts as before, draw one horizontal line underneath, and then add. The sum of the ones is 12, or 2 ones and 1 ten. We write the 2 ones below the line for the ones of the sum, and add the 1 ten with the column of tens. The sum of all the tens is 23, or 3 tens

SECOND SOLUTION.

$$\begin{array}{r} 3475 \\ 2694 \\ 1363 \\ \hline 7532 \end{array}$$

and 2 hundreds. We write the 3 tens as the tens of the sum, and add the 2 hundreds with the column of hundreds. The sum of all the hundreds is 15, or 5 hundreds and 1 thousand. We write the 5 hundreds as the hundreds of the sum, and add the 1 thousand with the column of thousands. The sum of all the thousands is 7, and this we write as the thousands of the sum. The result, 7,532, is the sum required.



PROBLEMS.

17. The numbers on these packages of express freight show their weight in pounds. What is the weight of the marked packages that have been unloaded? *971 pounds.*

18. How many pounds do the cask, barrel, and half-barrel weigh? *1,133.*

19. What is the weight of all the marked boxes on the express wagon? *4,110 pounds.*

20. How much does all the marked freight on the wagon weigh?

21. What is the total weight of all the marked packages shown in the picture? *5,373 pounds.*

22. In the Old Testament are 39 books, and in the New Testament 27 books. How many books are in the Bible? *66.*

23. A cabinet-maker paid $125 for black-walnut lumber, and $90 for mahogany. How much did the lumber cost him? $215.

24. A merchant tailor bought three pieces of broadcloth, the first containing 27 yards, the second 45 yards, and the third 34 yards. How many yards did he buy?

25. One day a man traveled 241 miles by railroad, 57 by steamboat, and 14 by stage. How far did he travel? *312 miles.*

26. Henry is 16 years old, his father is 29 years older than he, and his grandfather 32 years older than his father. How old is his grandfather? *77 years.*

27. From the creation of the world to the Christian era were 4,004 years. How many years from the creation to the end of the present year?

28. A fruit dealer bought 56 barrels of russet apples, 74 barrels of pippins, 69 barrels of spitzenbergs, and 83 barrels of greenings. How many apples did he buy? *282 barrels.*

39. The facts deduced in Arts. **35** and **38** may be stated as

Principles of Addition.

I. *Only abstract numbers or like concrete numbers can be added.*

II. *Only like orders of units in different numbers can be added.*

III. *When the sum of the units of any order exceeds 9, the tens of this sum are units of the next higher order.*

40. Upon these principles is based the

Rule for Addition of Integers.

I. *Add like orders of units, and write the ones of the sum in the result.*

II. *Add the tens of the sum of any order with the next higher order.*

III. *Write the whole sum of the highest order of units given.*

NOTE.—Since the tens of the sum of any column must be added with the next left-hand column, it is in general more convenient to commence at *the right to add.*

PROBLEMS.

29. A nursery-man sold during the year 3,729 apple-trees, 1,415 pear trees, 974 peach trees, 567 plum-trees, 918 cherry-trees, and 1,584 ornamental trees. How many trees did he sell? *9,187.*

30. One year a farmer raised 649 bushels of oats, 422 bushels of corn, 178 bushels of wheat, and 96 bushels of barley. How much grain did he raise? *1,345 bushels.*

31. A merchant pays his book-keeper $1,250 a year, two clerks $825 each, and a boy $175. How much do their salaries amount to? *$3,075.*

32. An Erie canal-boat has on board 273 barrels of flour for Utica, 385 barrels for Albany, and 465 barrels for New York. How much flour has she on board? *1,123 barrels.*

33. A grocer, in purchasing his first stock, paid $466 for sugars, $387 for syrups, $196 for teas, and $1,760 for other goods. What was the cost of his stock? *$2,809.*

34. At an auction a woman bid off one carpet for $24, another for $36, some oil-cloth for $7, and some window-shades and fixtures for $12. What was the amount of her bill? *$79.*

35. At the New York Cattle Market the number of beeves on sale last Tuesday was as follows: number remaining over from Monday, 396; received by Erie Railroad, 1,516; by Hudson River Railroad, 1,044; by Harlem Railroad, 1,185; by Camden & Amboy Railroad, 296; by Hudson River boats, 210; by New Jersey Central Railroad, 329; and on foot, 311. How many beeves were on sale? *5,287.*

36. A man built a house, which cost, for brick and stone, $375; for lumber, $540; for other materials, $224; for excavation, $72; for mason work, $284; for carpenter work, $580; and for painting, glazing, and paper hanging, $225. How much did the house cost him? *$2,300.*

37. At one time the rolling stock of the New York Central Railroad was 211 locomotives, 196 first-class passenger cars, 41 second-class and emigrant cars, 61 baggage, mail, and express cars, 2,760 freight cars, and 350 gravel cars. What was the whole number of cars? 3,619.

38. I paid $325 for a lot, $1,426 for building a house upon it, $589 for building a barn and carriage house, $74 for fences, and $48 for grading the lot. For how much must I sell the property to gain $338? *$2,800.*

39. On Monday morning a merchant had $1,767 in the bank. That day he deposited $94; on Tuesday, $113; on Wednesday, $78; on Thursday, $141; on Friday, $52; and on Saturday, $279. How much had he on deposit at the end of the week? *$2,524.*

40. Find the sum of 15 million 9 thousand 17, 9 million 503, 675 thousand 899, and 245 million 326 thousand 8.

270 million 11 thousand 432.

41. In each of the two following sets of numbers, find the sum of all the numbers above *e*.

21,365	—*a*	42. From *a* to *d*.	2,194,756	—*a*
18,890	—*b*	43. From *b* to *e*.	40,373,254	—*b*
54,363	—*c*	44. Above *f*.	90,000,383	—*c*
27,541	—*d*	45. From *a* to *g*.	6,275,851	—*d*
53,027	—*e*	46. From *d* to *i*.	12,593	—*e*
34,198	—*f*	47. From *h* to *m*.	6,005	—*f*
44,254	—*g*	48. Below *n*.	373,582	—*g*
87,679	—*h*	49. Below *h*.	218	—*h*
73,250	—*i*	50. From *f* to *m*.	1,694,583	—*i*
19,000	—*j*	51. From *g* to *o*.	657,679	—*j*
48,468	—*k*	52. From *j* to *g*.	500,000,290	—*k*
91,516	—*l*	53. From *e* to *l*.	73,418	—*l*
60,009	—*m*	54. From *a* to *j*.	1,547	—*m*
38,482	—*n*	55. Above *i*.	4,293,500	—*n*
19,564	—*o*	56. Below *i*.	400,000	—*o*
65,587	—*p*	57. From *f* to *n*.	44	—*p*
28,385	—*q*	53. From *k* to *r*.	63,974	—*q*
78,126	—*r*	59. From *d* to *p*.	13,987,457	—*r*

60. What is the sum of the two answers of Problem 41?

61. The sum of the two answers of Problem 42?

62. The sum of the four answers of Problems 43 and 44?

63. Add the answers of Problems 45, 46, 47, and 48.

64. Add the answers of Problems 49, 50, 51.

65. Add the answers of Problems 52, 53, 54.

66. Add the answers of Problems 55, 56, 57, and 58.

67. What is the sum of the two answers of Problem 59?

See Manual.

68. How many rods of fence will it take to inclose a field that is 38 rods long on each side, and 29 rods wide on each end? *134.*

69. I have a farm 176 rods long and 115 rods wide. A fence extends around it; 3 inside fences extend from end to end; and 4 other fences from side to side. How many rods of fence on the farm? *1,570.*

70. Thomas Jefferson was born A.D. 1743, and lived to be 83 years old. In what year did he die?

71. A schooner cleared from Chicago for Buffalo, having on board 14,397 bushels of wheat, 5,810 bushels of corn, and 2,113 bushels of oats. How much grain had she in her cargo?

72. A grocer bought three hogsheads of sugar, containing 1,467 pounds, 1,324 pounds, and 1,296 pounds; also, two barrels containing 254 pounds and 237 pounds. How much sugar did he buy? *4,578 pounds.*

73. A man bought a farm, paying $2,375 down. After making three other payments of $1,148, $1,096, and $1,260, the amount unpaid was $5,896. What was the cost of the farm?

74. How many days in the first six months of the year, January, March, and May each having 31 days, April and June each 30 days, and February 28 days?

75. At the battle of Fort Donelson, the Union loss was 446 killed, 1,735 wounded, and 150 taken prisoners. The Confederate loss was 237 killed, 1,007 wounded, and 13,300 taken prisoners. What was the whole number killed? The whole number wounded? The whole number taken prisoners?

Killed, 683; wounded, 2,742; prisoners, 13,450.

76. What was the total loss to each army?

Union, 2,331; Confederate, 14,544.

77. The property in a certain school district is assessed as follows: to A, $3,875; B, $1,050; C, $13,250; D, $600; E, $5,875; F, $250; Glass Manufactory, $105,750; H, $3,000; I, $150; J, $860; K, $14,180; L, $375; M, $53,000; National Bank, $151,500; O, $13,760; P, $670; Q, $19,960; Railroad Company, $89,500; S, $960; T, $350; U, $26,675; V, $17,625; and W, $275. What is the assessed valuation of the district? $523,490.

SECTION IV.

SUBTRACTION.

INDUCTION AND DEFINITIONS.

41. 1. MYRA brought 8 roses to school, and gave 5 of them to her teacher. How many had she left?

2. In a garden there are 11 fruit-trees; 4 of them are plum-trees, and the others cherry-trees. How many are cherry-trees?

3. If you buy 12 Brazil-nuts, and eat 8 of them, how many will you have left?

4. Ellen had 14 books upon two shelves, and 7 of them were on the lower shelf. How many were on the upper shelf?

5. Robert is 13 years old, and Edward is 9. How much older is Robert than Edward?

6. One day a lawyer wrote 15 letters, writing 7 of them in the forenoon. How many did he write in the afternoon?

7. A laborer received $9 for his week's work, and spent $6. How much money had he left?

42. ***Subtraction*** is the process of taking one of two numbers from the other.

43. The ***Remainder*** or ***Difference*** is the result obtained by subtraction.

44. The ***Minuend*** is that one of two numbers from which the other is to be taken.

45. The ***Subtrahend*** is that one of two numbers which is to be taken from the other.

NOTE.—The subtrahend can never be a greater number than the minuend.

46. The ***Sign of Subtraction,*** made thus —, is called *Minus;* and when written between two numbers, it signifies that the number after it is to be subtracted from the number before it. Thus, 18 — 12 signifies that 12 is to be subtracted from 18.

8. 15 men — 9 men = how many men?

9. 17 pencils — 7 pencils = how many pencils?

10. From 14 hours subtract 5 hours.

11. From 18 pounds subtract 9 pounds. Which number is the minuend? Which is the subtrahend? Which, the remainder?

12. The minuend is 16, and the subtrahend 5. What is the remainder?

13. What is the difference between 13 and 10?

14. What is the remainder when 8 is subtracted from 17?

47. SUBTRACTION TABLE.

0	0	1	2	3	4	5	6	7	8	9	5	5	6	7	8	9	10	11	12	13	14
	0	0	0	0	0	0	0	0	0	0		5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10	6	6	7	8	9	10	11	12	13	14	15
	1	1	1	1	1	1	1	1	1	1		6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	10	11	7	7	8	9	10	11	12	13	14	15	16
	2	2	2	2	2	2	2	2	2	2		7	7	7	7	7	7	7	7	7	7
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9
3	3	4	5	6	7	8	9	10	11	12	8	8	9	10	11	12	13	14	15	16	17
	3	3	3	3	3	3	3	3	3	3		8	8	8	8	8	8	8	8	8	8
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9
4	4	5	6	7	8	9	10	11	12	13	9	9	10	11	12	13	14	15	16	17	18
	4	4	4	4	4	4	4	4	4	4		9	9	9	9	9	9	9	9	9	9
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9

CASE I.

No figure of the Subtrahend of greater value than the corresponding figure of the Minuend.

48. We can subtract apples from apples, dollars from dollars, pens from pens, or hours from hours; but we can not subtract apples from dollars, nor pens from hours. For 13 apples — 4 dollars = neither 9 apples nor 9 dollars.

Again, we can subtract ones from ones, tens from tens, or hundreds from hundreds; but we can not subtract ones from hundreds, nor tens from thousands. For 9 thousands — 4 tens = neither 5 tens nor 5 thousands.

49. Ex. From 5,267 subtract 2,215.

EXPLANATION.—Since we must subtract ones from ones, tens from tens, hundreds from hundreds, etc., it is most convenient to write the figures of the subtrahend under the figures of like orders in the minuend. We then subtract each figure of the subtrahend from the figure above it in the minuend, writing the result directly below in the remainder. 5 ones from 7 ones leave 2 ones; 1 ten from 6 tens leave 5 tens; 2 hundreds from 2 hundreds leave 0 hundreds; and 2 thousands from 5 thousands leave 3 thousands. The result, 3,052, is the difference or remainder required.

SOLUTION.

5267 *Minuend.*
2215 *Subtrahend.*
3052 *Remainder.*

In the same manner solve and explain the following

PROBLEMS.

	(1)	(2)	(3)	(4)	(5)
From	85	459	4978	13279 feet	$2384
Subtract	43	348	3264	3148 feet.	1073

6. From a chest of tea, which contained 76 pounds, a grocer sold 43 pounds. How much tea remained in the chest? *33 pounds.*

7. From a flock of 396 sheep a drover bought 194. How many sheep were left in the flock? *202.*

8. A man bought a house and lot for $2,375, paying $1,225 down. How much did he then owe on the place? *$1,150.*

9. A contractor received $7,875 for building a railroad bridge, and it cost him $5,450 to build it. How much was his profit?

10. In a city school there are 849 pupils, of whom 437 are girls. How many are boys? *412.*

11. A and B together bought a steamboat for $78,385, and A furnished $45,385 of the purchase-money. How much did B furnish? *$33,000.*

12. From 9 million 548 thousand 276, subtract 5 million 34 thousand 153. *Remainder, 4,514,123.*

CASE II.

Any figure of the Subtrahend of greater value than the corresponding figure of the Minuend.

50. If the minuend is 5, and the subtrahend is 2, the difference is 3.

$$\begin{array}{r}5\\\underline{2}\\3\end{array}\qquad\begin{array}{r}5+4=9\\2+4=\underline{6}\\3\end{array}\qquad\begin{array}{r}5+7=12\\2+7=\underline{\ 9}\\3\end{array}$$

If 4 be added to both minuend and subtrahend, the difference is 3, as before. Again, if 7 be added to both minuend and subtrahend, the difference is still 3. Hence,

The difference or remainder is not affected by adding the same number to both minuend and subtrahend.

51. Ex. 1. From 40,658, subtract 21,385.

SOLUTION.

$$\begin{array}{r}40658\\\underline{21385}\\19273\end{array}$$

EXPLANATION.—Writing the numbers as in Case I., we commence at the right to subtract. 5 ones from 8 ones leave 3 ones, which we write as the ones of the remainder. Since we can not subtract 8 (tens) from 5 (tens), and since the difference will not be affected by adding the same number to both minuend and subtrahend (**50**), we add 10 (tens) to the 5 of the minuend, and 1 (hundred = 10 tens) to the 3 of the subtrahend. We then subtract 8 from 15, and 4 from 6, writing the results 7 (tens) and 2 (hundreds), as the tens and hundreds of the remainder. Since we can not subtract 1 (thousand) from 0 (thousand), we add 10 (thousands) to the 0 of the minuend, and 1 (ten-thousand = 10 thousands) to the 2 of the subtrahend. We then subtract 1 from 10, and 3 from 4, and write the results, 9 (thousands) and 1 (ten-thousand), as the thousands and ten-thousands of the remainder. The result, 19,273, is the remainder required.

Ex. 2. From 923 subtract 48.

SOLUTION.

923
48
875

EXPLANATION.—Since we can not subtract 8 (ones) from 3 (ones), we add 10 (ones) to the 3 of the minuend, and 1 ten (= 10 ones) to the 4 of the subtrahend. Then 8 from 13 leaves 5. Since we can not subtract 5 tens (4 + 1) from 2 tens, we add 10 (tens) to the 2 of the minuend, and 1 (hundred = 10 tens) to the subtrahend. Then, 5 from 12 leaves 7, and 1 from 9 leaves 8. The result, 875, is the remainder required.

Ex. 3. From 1,000 subtract 257.

SOLUTION.

1000
257
743

EXPLANATION.—In solving this example, we first add 10 (ones) to the minuend, and 1 (ten) to the subtrahend, and subtracting 7 from 10, we obtain 3 ones. We next add 10 (tens) to the minuend, and 1 (hundred) to the subtrahend, and subtracting 6 from 10, we obtain 4 tens. We then add 10 (hundreds) to the minuend, and 1 (thousand) to the subtrahend, and subtracting 3 from 10 and 1 from 1, we obtain 7 hundreds and 0 thousands. The result, 743, is the remainder required. See Manual.

In the same manner solve and explain the following

PROBLEMS.

	(13)	(14)	(15)	(16)
From	93	416	1483 men	423150 miles
Subtract	28	198	645 men.	145316 miles.

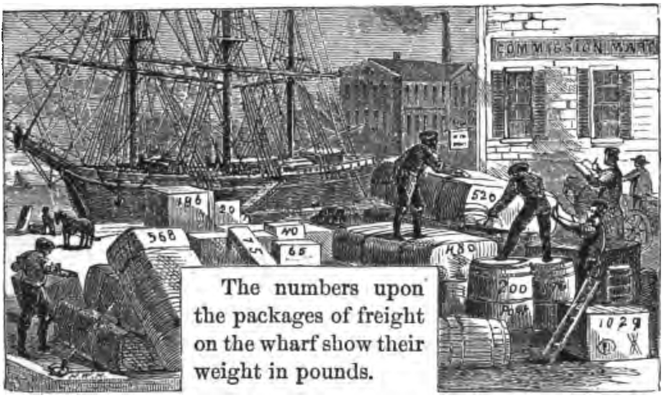
17. A man having $725 on deposit, draws out $268. How much has he left in the bank? *$457.*

18. A merchant sold a bill of goods for $173, and his profits were $37. How much did the goods cost him? *$136.*

19. A man's income is $1,675, and his expenses are $948. How much does he save? *$727.*

20. 1,378 tons — 985 tons = how many tons? *393.*

21. 1,045 bushels — 66 bushels = how many bushels? *979.*



The numbers upon the packages of freight on the wharf show their weight in pounds.

22. How much more does the heaviest bale of cotton weigh than the large box near the ship? *382 pounds.*

23. How much more do the 3 cotton bales weigh than the heaviest marked package?

24. What is the difference between the weight of the 2 barrels and the bale of cotton which the 3 men are handling?

25. Which weighs the most, the bale on which one of the men is standing, or the 4 lightest marked packages, and how much?

26. All the marked boxes are to be shipped on board the vessel lying at the wharf, and all the other marked freight has been landed from her. Will she take in more or less freight than she has discharged, and how much? *549 pounds less.*

27. A Boston provision dealer having 1,296 barrels of pork, ships 748 barrels to Liverpool. How many barrels has he then on hand?

28. In an election for Member of Congress, A received 12,031 votes, and B, 10,032 votes. Which candidate was elected, and by what majority? *A, by a majority of 1,999.*

29. A yoke of oxen, before being fattened, weighed 2,586 pounds, and after being fattened, 3,174 pounds. How much had they gained? *588 pounds.*

30. An ox weighed 1,326 pounds on foot, and 996 pounds when slaughtered. What was the difference between the live and dressed weights? *330 pounds.*

52. The facts deduced in Arts. 48, 50, may be stated as

Principles of Subtraction.

I. *Only abstract numbers or like concrete numbers can be subtracted the one from the other.*

II. *Only like orders of units can be subtracted the one from the other.*

III. *The difference or remainder is not affected by adding the same number to both minuend and subtrahend.*

53. Upon these principles is based the

Rule for Subtraction of Integers.

I. *Subtract units from units of like orders, writing each difference for the same order of units in the result.*

II. *When any order of units in the subtrahend is greater in value than the corresponding order in the minuend, add 10 units of the same order to the minuend, and 1 unit of the next higher order to the subtrahend.*

NOTES.—1. When no figure of the subtrahend exceeds in value the corresponding figure of the minuend, we may commence at the right or at the left to subtract.

2. When one or more figures of the subtrahend exceed in value the corresponding figures of the minuend, it is in general more convenient to commence at the right to subtract. See Manual.

PROBLEMS.

	(31)	(32)	(33)		(34)	
From	3250	10000	25600	gallons	342651	reams
Subtract	89	24	8008	gallons.	142652	reams.

35. A man's property is valued at $75,000, of which $48,766 is in real estate. How much is his personal property worth?

36. A grain buyer in Milwaukee receives an order for 12,500 bushels of "No. 1" wheat, and has only 7,645 bushels in store. How much must he purchase to fill the order? *4,855 bushels.*

37. A religious society, after raising $17,675 by subscription, contracted for the erection of a church for $22,500. How much remains yet to be raised? *$4,825.*

38. Christopher Columbus was born A.D. 1437, discovered America A.D. 1492, and died A.D. 1506. How old was he when he discovered America? How old when he died? *55 years; 69 years.*

39. 26,957,229 bushels of salt were used in the United States in 1860, and of this amount 14,094,227 bushels were imported. How many bushels were of home manufacture? *12,863,002.*

40. The distance from Albany to Buffalo is 297 miles, and from Albany to Rochester, 229 miles. How far is it from Rochester to Buffalo? *68 miles.*

41. At a saw-mill 100,000 feet of pine lumber were sawed in one month, and 47,250 feet of it were sold. How much remained at the mill? *52,750 feet.*

42. One year the receipts of the Third Avenue Railroad of New York city were $564,839, and the expenses were $307,188. How much were the profits? *$257,651.*

Lake Superior has an elevation of 623 feet above tide; Lake Huron, of 591 feet; Lake Erie, of 565 feet; Lake Ontario, of 232 feet; Great Salt Lake, of 4,200 feet; and Lake Titicaca, of 12,785 feet.

43. How much higher is Lake Superior than Lake Erie?

44. How much higher is Lake Superior than Lake Huron?

45. Great Salt Lake is how much higher than Lake Ontario?

46. How much fall is there in Niagara River?

47. Lake Superior is how much higher than Lake Ontario?

48. How much higher is Lake Titicaca than Lake Erie?

49. How much higher is Lake Huron than Lake Ontario?

50. How much fall is there between Lake Huron and Lake Erie?

51. Lake Titicaca is how much higher than Great Salt Lake?

52. From a farm of 417 acres, the owner sold 132 acres to one man, and 96 acres to another. How much land had he left?

53. A bank teller received a salary last year of $1,250. His personal expenses were $753, and he bought a village lot for $213, and paid out $149 for improvements upon it. How much money had he at the end of the year? *$135.*

54. A drover having 319 head of cattle, sold 98 head to one butcher and 127 head to another. How many cattle had he left?

55. An estate worth $35,474 is encumbered to the amount of $17,625. How much is it worth above the incumbrance?

56. A farmer having 113 sheep, sold 57 of them, and afterward bought 83 more. How many had he then? *139.*

57. A man at his death left an estate worth $48,765. He owed $13,596, and bequeathed $12,750 to his widow, $5,875 to charitable institutions, and the balance to his only son. How much did his son receive? *$16,544.*

58. A regiment was mustered into the service with 976 men, and afterward received 274 recruits. During service its losses were 38 killed in battle, 94 wounded, 54 taken prisoners, 69 discharged for sickness, 13 died from sickness, and 47 deserted. Of how many men did the regiment then consist? *935.*

59. A lumber dealer sold a quantity of plank for $746, making a profit of $148. How much did the lumber cost him? *$598.*

60. A hardware merchant, who owes a grocer $113 on account, sells him a cook stove for $32, a parlor stove for $28, and some pipe for $7, and pays the balance in cash. How much money does the grocer receive?

61. A clergyman had his life insured for $3,500. At the time of his death $376 of his salary was unpaid; he owned a house and lot worth $3,275, but upon it there was a mortgage for $1,390; and his other debts amounted to $294. How much did he leave his family? *$5,467.*

62. A block of stores, valued at $37,675, and goods worth $69,325, were destroyed by fire. The buildings were insured for $31,875, and the goods for $49,290. What was the loss on the buildings?

63. What was the loss on the goods?

64. How much did the loss on the goods exceed the loss on the buildings? *$14,235.*

65. June 1, a grocer bought 1,754 pounds of ◈ sugar, 1,249 pounds Ⓐ, 2,154 pounds B, 1,864 pounds C, 2,752 pounds W. I., and 1,954 pounds N. O. August 1, he had on hand 967 pounds ◈, 856 pounds Ⓐ, 1,182 pounds B, 1,692 pounds C, 2,158 pounds W. I., and 369 pounds N. O. How much sugar of each brand had he sold in the month? See Manual.

SECTION V.

MULTIPLICATION.

INDUCTION AND DEFINITIONS.

54. 1. JAMES found 4 hens' nests in the barn, and in each nest were 5 eggs. How many eggs did he find?

2. If a cooper can make 7 barrels in a day, how many barrels can he make in 5 days?

3. How many blades in 9 4-bladed knives?

4. A lady bought 7 spools of thread, at 7 cents a spool. How much did it cost her?

5. If 9 pounds of flour will last a family one week, how many pounds will last them 5 weeks?

6. How many dollars can a man earn in 6 days, if he earns $3 a day?

55. *Multiplication* is a short process of finding the sum of as many times one of two numbers as there are ones in the other.

56. The *Product* is the result obtained by Multiplication.

57. The *Factors* are the numbers used to obtain the product.

58. The *Multiplicand* is that factor which is to be taken any certain number of times.

59. The *Multiplier* is that factor which shows how many times the multiplicand is to be taken.

60. *Continued Multiplication* is the process of finding the product of more than two factors.

61. The *Sign of Multiplication,* made thus ×, when placed between two numbers, signifies that they are to be multiplied together. It is read "times," or "multiplied by." Thus, 5 × 8 is read "5 times 8," or "5 multiplied by 8."

7. 4 × 6 slates are how many slates?

8. What is the product of 6 × 7 oranges?

9. What is the product of 7 × 9?

10. The factors are 5 and 8. What is the product?

11. The multiplicand is 9, and the multiplier 8. What is the product?

12. What is the product of 3 × 3 × 7?

13. 4 × 2 × 6 pen-holders = how many pen-holders?

62. MULTIPLICATION TABLE.

1	0	1	2	3	4	5	6	7	8	9	6	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1		6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9		0	6	12	18	24	30	36	42	48	54
2	0	1	2	3	4	5	6	7	8	9	7	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2		7	7	7	7	7	7	7	7	7	7
	0	2	4	6	8	10	12	14	16	18		0	7	14	21	28	35	42	49	56	63
3	0	1	2	3	4	5	6	7	8	9	8	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3		8	8	8	8	8	8	8	8	8	8
	0	3	6	9	12	15	18	21	24	27		0	8	16	24	32	40	48	56	64	72
4	0	1	2	3	4	5	6	7	8	9	9	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4		9	9	9	9	9	9	9	9	9	9
	0	4	8	12	16	20	24	28	32	36		0	9	18	27	36	45	54	63	72	81
5	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9
	5	5	5	5	5	5	5	5	5	5		10	10	10	10	10	10	10	10	10	10
	0	5	10	15	20	25	30	35	40	45		0	10	20	30	40	50	60	70	80	90

CASE I.

The Multiplier One Figure.

63. Ex. How many are 673 + 673 + 673 + 673, or 4 times 673?

Explanation.—In the Solution by Addition we find the sum of 4 673's to be 2,692. But since 3 ones + 3 ones + 3 ones + 3 ones = 4 times 3 ones, and 7 tens + 7 tens + 7 tens + 7 tens = 4 times 7 tens, and 6 hundreds + 6 hundreds + 6 hundreds + 6 hundreds = 4 times 6 hun-

SOLUTION BY ADDITION

673
673
673
673
2692

dreds, we write the 673 but once, and write a 4 under its right-hand figure, to show how many times it is to be taken, as shown in the following Solution by Multiplication:

In this Solution we multiply each figure of the multiplicand by the multiplier. Thus, 4 times 3 ones are 12 ones, or 2 ones and 1 ten. We write the 2 ones as the ones of the product, and reserve the 1 ten in the mind to be added to the product of the tens. 4 times 7 tens are 28 tens, and 28 tens + 1 ten = 29 tens, or 9 tens and 2 hundreds. We write the 9 tens as the tens of the product, and reserve the 2 hundreds in the mind to be added to the product of the hundreds. 4 times 6 hundreds are 24 hundreds, and 24 hundreds + 2 hundreds = 26 hundreds, or 6 hundreds and 2 thousands. We write the 6 and 2 as the hundreds and thousands of the product. The result, 2,692, is the sum or product required.

SOLUTION
BY MULTIPLICATION.

673	*Multiplicand.*
4	*Multiplier.*
2692	*Product.*

64. From this explanation we learn that

The multiplicand is that factor which would be used as one of the parts in solving a problem or example by Addition.

65. We may add abstract numbers or like concrete numbers. (See **40**.) Hence,

The multiplicand may be either an abstract or a concrete number.

66. The multiplier is used simply to show how many times the multiplicand is taken. Hence,

The multiplier is always an abstract number.

67. In Addition the sum is of the same kind as the parts added. Hence,

The product is always of the same kind as the multiplicand.

PROBLEMS.

	(1)	(2)	(3)	(4)	(5)	
Multiply	43	132	491	6243	13562	pounds.
by	4	6	5	7	8	

6. How much will 3 cows cost, at $52 apiece?

7. How much will a mechanic earn in 6 months, if he earns $35 a month? *$210.*

8. A grocer bought 7 barrels of sugar, each containing 245 pounds. How much sugar did he buy? *1,715 pounds.*

9. How much will 9 tons of hay cost, at $14 a ton? *$126.*

10. How far will a railroad train run in 7 hours, at the rate of 39 miles an hour? *273 miles.*

11. A canal-boat captain bought 4 horses, paying $156 apiece for them. How much did they cost him? *$624.*

12. A manufacturer pays his hands $2,356 a month. How much do their wages amount to in 6 months? *$14,136.*

13. In a certain army corps there were 8 regiments of 966 men each. What was the number of men in the corps? *7,728.*

14. At $5 a week, how much will a year's board cost, there being 52 weeks in a year? *$260.*

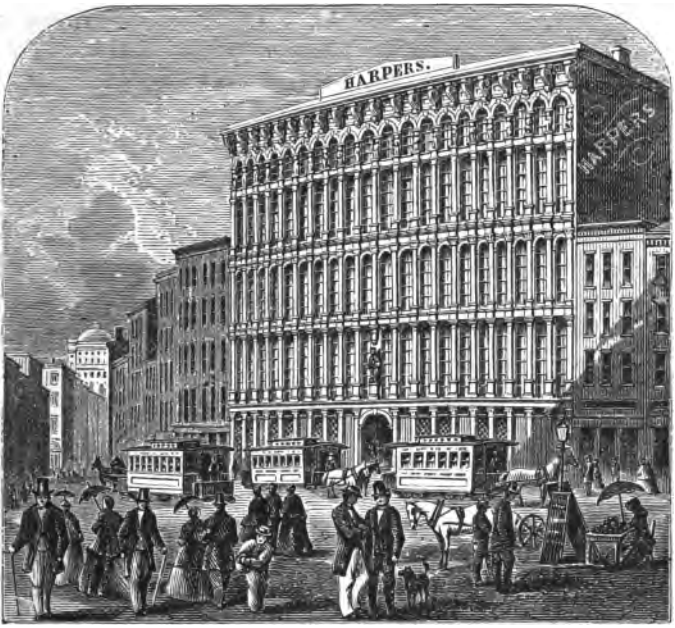
15. If the price of thrashing wheat is 4 cents a bushel, how much must be paid for thrashing 17,944 bushels? *71,776 cents.*

16. A miller sells 74 barrels of flour, at $9 a barrel. How much does the sale amount to? *$666.*

68. In explaining the solution of Problem 16, we would say, "74 barrels sell for 74 times as much as 1 barrel, or 74 times $9." Hence, $9, being a concrete number, is the *true multiplicand.* But since 74 times 9 is the same as 9 times 74, in solving the problem we may take 74 for the multiplicand, and use 9 as the multiplier. That is,

I. *In the solution of problems, either factor may be used as the multiplicand.* See Manual.

II. *In the explanation of the solution of problems containing concrete numbers, the concrete number is the true multiplicand.*



17. How many windows in the front of all the stories of this building except the first, there being 20 windows to each story?

18. How many iron pillars or columns in the front of the three upper stories, there being 21 columns to each story?

19. How many windows in 8 street cars, there being 18 windows in each car?

20. If 24 passengers ride in a car at one trip, and the fare is 6 cents, how much fare will the conductor collect?

21. A woman who keeps a fruit stand sold 178 oranges one day, at 4 cents apiece. How much did she receive for them?

22. How much did it cost to paint the two signs on this building, at $3 a letter?

23. How much would it cost all the persons in the foreground of this picture to go to Harlem and back on a horse-car, the fare being 7 cents each way? *210 cents.*

24. The Third Avenue Railroad, in New York City, is 7 miles long. How many miles does a car run in making 11 round trips?

25. A wood dealer sold 6,591 cords of wood, at $7 a cord. How much were his receipts? *$46,137.*

26. In 1 mile there are 1,760 yards, or 5,280 feet. How many yards in 8 miles? *14,080.*

27. How many feet in 8 miles? *42,240.*

28. Ten brick-layers finish the walls of a building in 9 days, laying 9,475 bricks each day. How many bricks are in the walls?

29. At $9 apiece, how much will it cost for the transportation of 33,875 soldiers from Washington to Chicago? *$304,875.*

CASE II.

The Multiplier any number of tens, hundreds, thousands, and so on.

69. Ex. Multiply 3,176 by 10.

EXPLANATION.—We write the numbers as shown in the Solution, and multiply each figure of the multiplicand by the multiplier, as in Case I. Comparing the multiplicand and product, we find that the figures of the product are the same as those of the multiplicand, with a cipher on the right. Hence,

SOLUTION.

$$\begin{array}{r} 3176 \\ \underline{10} \\ 31760 \end{array}$$

70. *Annexing a cipher to any number multiplies it by 10.*

71. *Annexing a second cipher multiplies by 10 again; that is, annexing two ciphers to any number multiplies it by 10 times 10, or 100.*

72. *Annexing three ciphers to a number multiplies it by 10 times 100, or 1,000.*

73. *Annexing four ciphers multiplies by 10,000; annexing five ciphers, by 100,000; and annexing six ciphers, by 1,000,000.*

74. Ex. Multiply 291 by 60.

EXPLANATION.—60, or 6 tens, = 6 times 10 or 10 times 6. Hence, 60 times 291 is 10 times as much as 6 times 291. We may therefore multiply 291 by 6, and to the product thus obtained annex a cipher.

SOLUTION.

$$\begin{array}{r} 291 \\ 60 \\ \hline 17460 \end{array}$$

75. *To multiply by 600, we multiply by 6, and annex two ciphers to the product; to multiply by 6,000, we multiply by 6, and annex three ciphers.*

76. *We multiply by any number of tens, hundreds, thousands, or units of higher orders in the same manner.*

PROBLEMS.

30. How many oats in 10 wagon loads of 65 bushels each?

31. A barrel of flour contains 196 pounds. How many pounds in 100 barrels? *19,600.*

32. The United States Government purchased 10,000 rifles, at $24 each. How much did they cost? *$240,000.*

33. Multiply 23,947 by 1,000. *Product, 23,947,000.*

34. Multiply 175,941 by 100,000. *Product, 17,594,100,000.*

35. A pork packer sells to a provision dealer 125 barrels of pork, at $20 a barrel. What is the amount of his bill? *$2,500.*

36. At $50 an acre, how much will a farm of 176 acres cost?

37. A gentleman bought a city lot, having a front of 22 feet, at $90 a foot. How much did it cost him? *$1,980.*

38. A drover sold 500 head of cattle, at $67 a head. How much did he receive? *$33,500.*

39. A manufacturer sells 594 sewing-machines, at $60 each. How much does he receive for them? *$35,640.*

40. In a certain county, 237 drafted men purchased their exemption by paying $300 each. How much did their exemption cost?

41. The President's Cabinet consists of 7 members, who receive a salary of $8,000 each. What do their united salaries amount to?

42. 400 × 495 = how many? *198,000.*

43. What is the product of 32,721 × 50,000? *1,636,050,000.*

CASE III.

The Multiplier more than One Figure.

77. Ex. Multiply 3,528 by 472.

FIRST SOLUTION.

Multiplying by 2.	*Multiplying by 70.*	*Multiplying by 400.*
3528	3528	3528
2	70	400
7056	246960	1411200

EXPLANATION.—Since 472 consists of 2 ones, 7 tens, and 4 hundreds, or 2, 70, and 400; and since we can not multiply 3,528 by the whole 472 at once, we multiply it first by 2, then by 70, and then by 400, and afterward add the results, or *Partial Products.* The result thus obtained, 1,665,216, is the sum of 2 × 3,528, 70 × 3,528, and 400 × 3,528, or 472 × 3,528.

Adding Partial Products.

$$\begin{array}{rl} \left.\begin{array}{r} 7056 \\ 246960 \\ 1411200 \end{array}\right\} & \text{Partial Products.} \\ \cline{1-1} 1665216 & \end{array}$$

In the First Solution, each of the four steps stands by itself; in the Second Solution they are placed together.

SECOND SOLUTION.

$$\begin{array}{rl} 3528 & \\ 472 & \\ \cline{1-1} \left.\begin{array}{r} 7056 \\ 246960 \\ 1411200 \end{array}\right\} & \text{Partial Products.} \\ \cline{1-1} 1665216 & \end{array}$$

In the Second Solution, the ciphers on the right of the second and third partial products serve merely to fill the places of ones and tens; and since the sum of any number of 0's is 0, they may be omitted without affecting the total product, as shown in the Third Solution.

THIRD SOLUTION.

$$\begin{array}{rl} 3528 & \\ 472 & \\ \cline{1-1} \left.\begin{array}{r} 7056 \\ 24696 \\ 14112 \end{array}\right\} & \text{Partial Products} \\ \cline{1-1} 1665216 & \end{array}$$

In this Solution the second partial product is found by multiplying by 7, instead of 70; and the third partial product by multiplying by 4, instead of 400. But we must always

Write the first figure of each partial product directly under the figure of the multiplier used to obtain it.

NOTE.—In the explanation of Solutions, the value of each partial product should be named. This is done by reading it as though the ciphers were written.

PROBLEMS.

(44)	(45)	(46)	(47)	(48)
34	73	281	2976	127492
23	45	54	81	647

49. A prairie farmer planted 176 acres of corn, which yielded 73 bushels to the acre. How many bushels in the crop? *12,848.*

50. At a certain recruiting station, in 1862, 43 men enlisted each day for 36 days. How many men enlisted? *1,548.*

51. An overland emigrant train traveled 23 miles a day for 17 days. How far did they travel? *391 miles.*

52. The skipper of a fishing smack received, as his share of the season's catch, 273 barrels of mackerel, which he sold at $11 a barrel. How much were his receipts? *$3,003.*

53. In a pump manufactory, 125 of the workmen receive $39 a month each. How much do their wages for a month amount to?

54. How much do their wages amount to in 1 year, or 12 months? *$58,500.*

55. A livery-stable keeper who has 23 horses, finds that it costs him $83 a year for the keeping of each horse. What is the cost of keeping all of them? *$1,909.*

56. A steamer sailed from New York for Liverpool with 376 first-class passengers. How much did their fares amount to, at $135 apiece? *$50,760.*

57. A railroad company contracted for 43 locomotives, at $18,725 apiece. What was the amount of the contract? *$805,175.*

58. In a certain paper-mill 184 reams of paper are made daily. How many reams are made in a year of 313 working days?

59. Multiply 1,372 by 861. *Product, 1,181,292.*

60. What is the product of 4,293 multiplied by 2,726?
11,702,718.

61. 417,293 × 581 = how many? *242,447,233.*

CASE IV.

One or more ciphers between other figures of the Multiplier.

78. Ex. Multiply 2,566 by 3,007.

Explanation.—In the Second Solution we have multiplied by 7 ones and 3 thousands (see **76**), omitting to multiply by 0 tens and 0 hundreds, because 0 times 2,566 is 0, as shown in the First Solution. Hence we may always

```
FIRST SOLUTION.      SECOND SOLUTION.
      2566                  2566
      3007                  3007
   -------               -------
     17962                 17962
     0000               7698
    0000                -------
   7698                 7715962
   -------
   7715962
```

Omit to multiply by ciphers that stand between other figures in the multiplier.

PROBLEMS.

	(62)	(63)	(64)	(65)
Multiply	426	1728	4765	29872
by	203	506	807	5008

66. At $105 each for horses, how much will it cost to mount a cavalry regiment of 1,043 men? *$109,515.*

67. In one season a manufacturer sold 307 reapers, at $135 apiece. How much did he receive for them? *$41,445.*

68. A furrier bought 108 buffalo-robes, at $17 apiece? How much did they cost him? *$1,836.*

69. A canal 203 miles long was kept in repair one season at an expense of $383 a mile. What was the expense for the whole canal? *$77,749.*

70. At a cotton manufactory 1,396 yards of cloth are made each day. How many yards are made in 307 days? *428,572.*

71. In a certain cotton factory are 203 looms, which turn out 69 yards of cloth per day, each. How many yards of goods are made daily?

72. At the same rate, what is the total product of the factory in a year of 308 working days? *4,314,156 yards.*

CASE V.

One or more ciphers on the right of either or both factors.

79. Ex. 1. Multiply 89 by 1,600.

To multiply by 1,600, we first multiply by 16, and then to the product annex two ciphers, as in Case II. (See **75.**)

SOLUTION.

$$\begin{array}{r} 89 \\ 1600 \\ \hline 534 \\ 89 \\ \hline 142400 \end{array}$$

Ex. 2. Multiply 54,000 by 37.

Explanation.—We first multiply 54 by 37, and obtain 1998. Since the 54 is thousands, this product must be thousands (see **67**); and we therefore write three ciphers in units' period; that is, annex three ciphers to the product of 37 × 54.

SOLUTION.

$$\begin{array}{r} 54000 \\ 37 \\ \hline 378 \\ 162 \\ \hline 1998000 \end{array}$$

Ex. 3. Multiply 73,000 by 2,600.

Explanation.—We first multiply 73 by 26, obtaining 1,898. But since the 73 is thousands, we annex three ciphers to this product for those at the right of the multiplicand. The result thus obtained is 26 × 73,000; and to make it 2,600 × 73,000, we annex two ciphers more. Hence, when there are ciphers on the right of one or both factors,

SOLUTION.

$$\begin{array}{r} 73000 \\ 2600 \\ \hline 438 \\ 146 \\ \hline 189800000 \end{array}$$

To the product of the other figures annex as many ciphers as there are ciphers on the right of both factors.

PROBLEMS.

73. In one barrel of beef there are 200 pounds. How many pounds in 37 barrels? *7,400.*

74. A carriage maker sold 30 top carriages, at $285 apiece. How much did they come to? *$8,550.*

75. A Missouri farmer had 130 acres of wheat, and the yield was 27 bushels to the acre. How much wheat did he raise?

76. In printing an edition of 5,000 copies of a book, 16 sheets of paper were used for each book. How much paper was used?

77. A Georgia planter had 94 acres of cotton that yielded 460 pounds to the acre. How much cotton did he raise?

78. At an ax factory 280 axes were made each day for 156 days. How many axes were made? *43,680.*

79. What will be the cost of 25,000 army blankets, at $5 apiece?

80. In one ream of paper there are 480 sheets. How many sheets in 260 reams? *124,800.*

81. A Pennsylvania oil well flowed 110 days, at the rate of 340 barrels of oil a day. How much oil did it yield? *37,400 barrels.*

82. How much will be the cost of building a line of telegraph 680 miles long, at $1,250 a mile? *$850,000.*

83. What is the product of 760,000 and 53,000? *40,280,000,000.*

84. The multiplicand is 694,000, and the multiplier 56,700. What is the product? *39,349,800,000.*

80. The facts deduced in Arts. **64** to **70** may be stated as

Principles of Multiplication.

In the Explanation of Solutions:

I. *The true multiplicand is that factor which would be used in solving the problem by Addition.*

II. *The multiplicand may be either an abstract or a concrete number.*

III. *The multiplier must always be an abstract number.*

IV. *The product is always of the same kind as the true multiplicand.*

In the Solution of Problems:

V. *In the solution of problems, either factor may be used as the multiplicand.*

VI. *Annexing a cipher to any number multiplies it by 10.*

VII. *The sum of all the partial products arising from multiplying one of two numbers by the ones, tens, hundreds, etc., of the other, is the product of the two given numbers.*

81. Upon these principles is based the

Rule for Multiplication of Integers.

I. The multiplier one figure.

Commence with the ones, and multiply successively each figure of the multiplicand by the multiplier. In the product write the ones of each result, and add the tens to the next result.

II. The multiplier more than one figure.

Multiply by each figure of the multiplier except ciphers, place the right-hand figure of each product under that figure of the multiplier used to obtain it, and add the partial products.

III. Ciphers on the right of either or both factors.

To the product of all the other figures annex as many ciphers as there are ciphers on the right of both factors.

PROBLEMS.

85. Every mile of a road 4 rods wide contains 8 acres. How many acres are there in 168 miles of such a road? *1,344.*

86. An army train was 9 hours in passing a given point, 84 wagons passing each hour. How many wagons were in the train?

87. How many poles will be required for a telegraph 328 miles long, if 16 poles are required for one mile? *5,248.*

88. A phonographic reporter, in taking down a speech, wrote 68 words a minute for 137 minutes. How many words were in the speech? *9,316.*

89. How much will it cost to build a railroad 134 miles long, at $65,475 a mile? *$8,773,650.*

90. Last season a cheese factory used the milk of 470 cows, and made 290 pounds of cheese to the cow. How much cheese was made? *136,300 pounds.*

91. In one square mile there are 640 acres, and in the State of Iowa there are 50,914 square miles. How many acres in the State? *32,584,960.*

92. The New American Cyclopædia contains 13,805 pages, of 3,036 ems each. How many ems in the work? *41,911,980.*

93. The factors are three hundred ninety-seven thousand five hundred, and nine thousand eight hundred. What is the product?
Three billion eight hundred ninety-five million five hundred thousand.

94. How many miles will a railroad conductor travel in a year, if he goes over a road 108 miles long once every day? *39,420.*

95. It is estimated that Mississippi River deposits 3,702,758,400 cubic feet of solid matter in the Gulf of Mexico every year. How many cubic feet have been deposited in 502 years? *1,858,784,716,800.*

96. How many yards of sheeting are there in 91 bales, each bale containing 23 pieces of 45 yards each? *94,185.*

97. At $13 a month, how much will the pay of the privates of a certain regiment, 896 in number, amount to in 12 months? *$139,776.*

98. A shoe dealer bought 27 cases of French calf boots, each case containing 12 pairs, at $5 a pair. What was the amount of the purchase? *$1,620.*

99. In a woolen factory there are 48 looms. How many yards of cloth will be made in the factory in 208 days, if 27 yards are woven upon each loom daily? *269,568.*

100. Just before an expected battle, 64 rounds of cartridges were given to each of 70,000 men. How many rounds were distributed to all of them? *4,480,000.*

CASE VI.

Powers.

82. A *Power* is the product of two or more equal factors; as 49, which is the product of 7 × 7.

83. A *Square* is the product of two equal factors; as 25, which equals 5 × 5.

84. A *Cube* is the product of three equal factors; as 64, which equals 4 × 4 × 4.

85. The *Fourth Power* is the product of four equal factors; the *Fifth Power* the product of five equal factors; the *Sixth Power* the product of six equal factors, and so on.

NOTES.—1. The process of finding the square of a number is called squaring it; and the process of finding its cube is called cubing it.

2. The process of finding any power of a number is called raising it to that power.

86. An *Index* is that one of two numbers which denotes the power to which the other number is to be raised. It is written at the right, and a little above the other number. Thus, in the expression 8^3, 3 is an index, and it denotes that 8 is to be cubed. So, also, 21^2 indicates the square of 21, and 59^4 indicates the fourth power of 59.

87. Ex. Raise 15 to the fourth power.

EXPLANATION. — We first find the square of 15, by multiplying it by itself. We then find the cube by multiplying the square by 15. We then multiply the cube by 15, and the result is the fourth power; because it is the product of 15 × 15 × 15 × 15.

In squaring a number there are 2 equal factors and 1 multiplication; in cubing it, 3 equal factors and 2 multiplications; and in raising it to the fourth power, 4 equal factors and 3 multiplications.

SOLUTION.

$$\begin{array}{rl} 15 & \\ \underline{15} & \\ 75 & \\ \underline{15} & \\ 225 & \textit{Square.} \\ \underline{15} & \\ 1125 & \\ \underline{225} & \\ 3375 & \textit{Cube.} \\ \underline{15} & \\ 16875 & \\ \underline{3375} & \\ 50625 & \textit{Fourth power.} \end{array}$$

In finding any power of a number, the number of multiplications is one less than the number of factors.

PROBLEMS.

101. What is the square of 9? *81.*

102. What is the cube of 5? *125.*

103. Square 423. Cube 47. *178,929; 103,823.*

104. Raise 12 to the fourth power. *20,736.*

105. What is the cube of 52? The cube of 901?

106. What is the square of 2,016? *4,064,256.*

107. Raise 218 to the sixth power. *107,334,407,093,824.*

108. Raise 63 to the seventh power.

109. Raise 14 to the eighth power.

110. Raise the following numbers to the powers denoted by their indices: 139^2, 97^3, 26^4, 18^5, 11^6.

SECTION VI.

DIVISION.

INDUCTION AND DEFINITIONS.

88. 1. Emma exchanged a 50-cent fractional-currency note for 5-cent pieces. How many 5-cent pieces did she receive?

2. How many oranges can I buy for 28 cents, if I pay 4 cents apiece for them?

3. A silversmith sold 30 teaspoons, in sets of 6 spoons each. How many sets of spoons did he sell?

4. A farmer put 15 bushels of oats into bags, putting 3 bushels in each bag. How many bags did he use?

5. One day Henry saw a pic-nic party of 36 persons passing by, in 4 carriages. How many persons were there for each carriage?

6. Two boys received 20 cents for carrying a lady's trunk to the depot, and they shared the money equally between them. How many cents had each boy?

7. In my garden are 21 fruit-trees, standing in 3 equal rows. How many trees in 1 row?

8. A grocer paid $54 for 9 barrels of cranberries. What was the price per barrel?

In solving each of the first four problems, we find how many times one of two numbers is contained in the other; and in solving each of the other problems, we separate one of two numbers into as many equal parts as there are ones in the other.

89. *Division* is the process of finding how many times one of two numbers is contained in the other; or of finding one of the equal parts into which a number may be divided.

90. The *Quotient* is the result obtained by Division.

91. The *Dividend* is the number to be divided.

92. The *Divisor* is the number by which the dividend is to be divided.

NOTES.—1. A *Partial Dividend* is that part of the dividend used to obtain one figure of the quotient, when the whole dividend is too large to obtain the entire quotient at one operation.

2. Division is *Exact* when all the dividend is divided and the quotient is an integer.

3. A *Remainder* is that part of the dividend left undivided, either when the division is only partially completed, or when exact division is impossible.

93. The *Sign of Division*, made thus ÷, when placed between two numbers, signifies that the number before it is to be divided by the number after it. It is read, "divided by." Thus, 175 ÷ 25 is read "175 divided by 25."

NOTES.—1. Division is also expressed by writing the dividend above, and the divisor below a horizontal line. Thus, $\frac{175}{25}$ is read, "175 divided by 25."

2. In writing numbers for solution, the divisor may be written either at the right of the dividend, thus, 175 ⌊25, or at the left of it, thus, 25) 175.

9. How many times are 6 cents contained in 54 cents?

10. What is the quotient of 40 divided by 8?

11. 56 ÷ 7 = how many? $\frac{33}{11}$ = how many?

12. The dividend is 24, and the divisor is 4. What is the quotient?

13. If as many of 17 apples be divided among 5 children as will give them whole apples, how many whole apples will each child have; and how many apples will be the remainder?

14. Divide 42 figs among 6 girls, and tell me the dividend, the divisor, and the quotient. Why is there no remainder?

94. DIVISION TABLE.

0	1	2	3	4	5	6	7	8	9	⌊1	0	6	12	18	24	30	36	42	48	54	⌊6
0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	
0	2	4	6	8	10	12	14	16	18	⌊2	0	7	14	21	28	35	42	49	56	63	⌊7
0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	
0	3	6	9	12	15	18	21	24	27	⌊3	0	8	16	24	32	40	48	56	64	72	⌊8
0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	
0	4	8	12	16	20	24	28	32	36	⌊4	0	9	18	27	36	45	54	63	72	81	⌊9
0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	
0	5	10	15	20	25	30	35	40	45	⌊5	0	10	20	30	40	50	60	70	80	90	⌊10
0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	

95. 2 is contained in 6, 3 times; in 6 tens or 60, 3 tens or 30 times; in 6 hundreds or 600, 3 hundreds or 300 times. 25 is contained in 75, 3 times; in 75 tens or 750, 3 tens or 30 times; in 75 hundreds or 7,500, 3 hundreds or 300 times.

In other words, if we divide ones, the quotient must be ones; if we divide tens, the quotient must be tens; if we divide hundreds, the quotient must be hundreds; if we divide thousands, the quotient must be thousands, and so on. Hence, in division of integers,

Any quotient figure must be of the same name or order of units as the right-hand figure of the partial dividend used to obtain it. See Manual.

CASE I.

The Divisor One Figure.

FIRST METHOD.

96. Ex. 1. Divide 936 by 3.

SOLUTION.

$$\begin{array}{lr|l} \textit{Dividend.} & 936 & 3 \ \textit{Divisor.} \\ & \underline{9} & \overline{312} \quad \textit{Quotient.} \\ & 3 & \\ & \underline{3} & \\ & 6 & \\ & \underline{6} & \end{array}$$

EXPLANATION. — We place the divisor at the right of the dividend, separating them by a line, and draw a line under the divisor, to separate it from the quotient. Then commencing at the left hand, we divide each figure of the dividend by the divisor, thus: 3 is contained in 9, 3 times; and as the 9 is hundreds, the 3 is hundreds (**95**), and we write it as the first figure of the quotient. We have now used the 9 hundreds of the dividend, and subtracting it from the dividend, we have no hundreds left. The next part of the dividend to be used is the 3 tens, which we bring down for a partial dividend. 3 is contained in 3, 1 time; and as the 3 is tens, the 1 is a ten (**95**), and we write it as the second figure of the quotient. We have now used the 3 tens of the dividend, and subtracting it from the dividend, we have no tens left. The next part of the dividend to be used is the 6 ones, which we bring down for another partial dividend. 3 is contained in 6, 2 times; and as the 6 is ones, the 2 is ones (**95**), and we write it as the third figure of the quotient. We have now used all the figures of the dividend, and the result, 312, is the quotient required.

Ex. 2. Divide 17,668 by 7.

SOLUTION.

$$\begin{array}{l|l}
17668 & 7 \\ \cline{2-2}
14 & 2524 \\ \cline{1-1}
36 & \\
35 & \\ \cline{1-1}
16 & \\
14 & \\ \cline{1-1}
28 & \\
28 & \\ \cline{1-1}
\end{array}$$

Explanation.—7 is not contained in 1 any number of times; we must therefore take 17 for the first partial dividend. Since 2 times 7 are 14 and 3 times 7 are 21, and 17 is more than 14, but less than 21, 7 is contained in 17, 2 times. As 17 is thousands, the 2 is thousands (**95**), and we write it as the first or thousands' figure of the quotient. We have now used 2 (thousands) times 7, or 14 (thousands) of the dividend; and to find how many thousands remain undivided, we subtract the 14 (thousands) from the 17 (thousands), and obtain a remainder of 3 (thousands). We next bring down the 6 (hundreds) of the dividend, and uniting it with the 3 (thousands), we have 36 (hundreds) for a second partial dividend. Since 5 times 7 are 35, and 6 times 7 are 42, and 36 is more than 5 × 7 and less than 6 × 7, 7 is contained in 36, 5 times. As 36 is hundreds, the 5 is hundreds (**95**), and we write it as the second or hundreds' figure of the quotient. We have now used 5 (hundreds) times 7, or 35 (hundreds) of the partial dividend; and to find how many hundreds remain undivided, we subtract the 35 (hundreds) from the 36 (hundreds), and obtain a remainder of 1 (hundred). We next bring down the 6 (tens) of the dividend, and uniting it with the 1 (hundred), we have 16 (tens) for another partial dividend.

Since 16 is more than 2 times 7 and less than 3 times 7, 7 is contained in 16, 2 times. As 16 is tens, the 2 is tens (**95**), and we write it as the third or tens' figure of the quotient. We have now used 2 (tens) times 7, or 14 (tens) of the last partial dividend; and to find how many tens remain undivided, we subtract the 14 (tens) from the 16 (tens), and obtain a remainder of 2 (tens). We now bring down the 8 (ones) of the dividend, and uniting it with the 2 (tens), we have 28 for a final partial dividend. 7 is contained in 28, 4 times; and as 28 is ones, the 4 is ones (**95**), and we write it as the last or ones' figure of the quotient. We have now used 4 times 7, or 28; and subtracting this from the last partial dividend, we have no remainder. All the figures of the dividend have been used; and the result, 2,524, is the quotient required.

97. The quotient in Division is sometimes an abstract, and sometimes a concrete number. It is therefore necessary, before proceeding to the solution of problems, that we determine when the quotient is abstract, and when concrete. To do this, we will take the following examples:

(1)
15 cents ⌊ *3 cents*
5

(2)
15 ⌊ *3*
5

(3)
15 cents ⌊ *3*
5 cents

(4)
15 ⌊ *3 cents*
Impossible

Writing in the places of numbers, words indicating the kinds of numbers used, we have:

(1)
Concrete ⌊ *Concrete*
Abstract

(2)
Abstract ⌊ *Abstract*
Abstract

(3)
Concrete ⌊ *Abstract*
Concrete

(4)
Abstract ⌊ *Concrete*
Impossible

98. These illustrations fully establish the following facts:

I. *The quotient will be an abstract number, when both dividend and divisor are abstract, or both concrete numbers.* (Ex. 1, 2.)

II. *The quotient will be a concrete number, when the dividend is a concrete, and the divisor an abstract number.* (Ex. 3.)

III. *Either the divisor or the quotient must always be an abstract number.* (Ex. 1, 2, 3.)

IV. *An abstract number can not be divided by a concrete number.* (Ex. 4.)

PROBLEMS.

(1)	(2)	(3)	(4)
648 ⌊2	$484 ⌊$4	273 tons ⌊7	7392 ⌊6

5. A stage company paid $396 for 3 horses. How much did they cost apiece? *$132.*

6. How many suits of clothes can be made from 1,248 yards of broadcloth, allowing 4 yards for each suit? *312.*

7. At a mortgage sale, 3 city lots were sold for $1,596. How much was that for one lot?

8. A railroad company bought 1,456 cords of wood, which they transported on platform cars, each carrying 8 cords. How many car loads were there? *182.*

9. An Ohio farmer raised a crop of 1,965 bushels of wheat, which he exchanged with a miller for flour, receiving 1 barrel of flour for every 5 bushels of wheat. How much flour did he receive for his wheat crop? *393 barrels.*

10. If 6 masons lay 15,894 bricks in a day, how many bricks can 1 mason lay? *2,649.*

11. If the yearly expenses of a family of 7 persons are $2,065, what are the expenses of 1 person?

12. An army-wagon train, 9 miles long, contains 1,944 wagons. How many wagons is that to the mile? *216.*

SECOND METHOD.

99. Ex. Divide 25,216 by 8.

SOLUTION.

Dividend. 25216 | 8 *Divisor.*

3152 *Quotient.*

EXPLANATION.—We write the dividend and divisor as in the First Method, but below the dividend we draw a horizontal line, under which to write the quotient. 8 is contained in 25 (thousands), 3 (thousands) times. This quotient figure we write directly below the last figure (5) of the part of the dividend used to obtain it. We multiply the divisor (8) by the 3 (thousands), and subtract the product from the 25, performing both computations mentally. We now mentally unite the remainder 1 (thousand) with the next figure of the dividend, 2 (hundreds), and divide the result, 12 (hundreds), by the divisor. 8 is contained in 12 (hundreds) 1 (hundred) time. We write the 1 as the second figure of the quotient, and then multiply 8 by it, and subtract the product from the partial dividend, 12 (hundreds), performing the computations mentally, as before. We next mentally unite the remainder, 4 (hundreds), with the 1 (ten) of the dividend, and divide the result, 41 (tens), by the divisor. 8 is contained in 41 (tens) 5 (tens) times. We write the 5 as the third figure of the quotient, and then multiply 8 by it, and subtract the product from the partial dividend, 41 (tens), performing the computations mentally, as before. We mentally unite the remainder, 1 (ten), with the 6 (ones) of the dividend, and divide the result (16) by the divisor. 8 is contained in 16 (ones) 2 (ones) times, and we write 2 as the fourth figure of the quotient. We have now used all the figures of the dividend; and the result, 3,152, is the quotient required.

In the Second Method, the same computations are performed as in the First Method; but the results of the subtractions and multiplications are not written, and hence fewer figures are used.

100. *Long Division* is that method of dividing, in which all the products and partial dividends are written.

101. *Short Division* is that method of dividing, in which only the dividend, divisor, and quotient are written.

See Manual.

PROBLEMS.

13. A city corporation paid $9,376 for 2 steam fire-engines. What was the cost of each? *$4,688.*

14. A man whose wages were $3 a day, earned $891 in a year. How many days did he work? *297.*

15. A steamboat which was owned by 4 men in equal shares, was sold for $39,724. How much was each man's share of the receipts? *$9,931.*

16. A farmer has 1,458 bushels of wheat, which he intends to carry to market in 2-bushel bags. How many bagfuls will he have? *729.*

17. If 3 horses eat 2,025 pounds of hay in a month, how much will 1 horse eat?

18. If I feed a horse 7 half-bushel measures of oats in a week, how many weeks will 483 half-bushels last him?

19. How long will 3 horses be in eating the same quantity of oats? *23 weeks.*

20. In a barrel in the granary are 96 quarts of corn, from which Clara feeds her ducks and chickens. How many quarts of corn are there for each one of the fowls?

21. How many weeks will the corn last, allowing 6 quarts a week for the poultry? *16.*

22. A man leased a farm for $865, at the rate of $5 an acre. How many acres were in the farm? 173.

23. A forwarder shipped 24,744 bushels of grain in 6 equal cargoes. How many bushels were in each cargo? *4,124.*

24. A builder paid $5,145 for bricks, at $7 a thousand. How many thousand did he buy? *735.*

25. It cost $6,504 to build a plank-road 8 miles long. What was the cost per mile? *$813.*

26. How long will it take a ship to make a voyage of 889 miles, if she sails 7 miles an hour? *127 hours.*

27. A boatman carried 8,532 barrels of flour from Oswego to New York in 9 down trips. How many barrels did he take each down trip?

28. A party of 8 men spent $1,672 on a journey to California, and they shared the expense equally. How much did each man pay?

CASE II.

The Divisor more than One Figure.

102. Ex. Divide 13,091 by 53.

EXPLANATION.—When the divisor consists of more than one figure, the division is commonly most easily performed by Long Division, or the First Method explained in Case I., as shown in the Solution.

SOLUTION.

```
13091 | 53
106   |----
----- | 247
 249
 212
 ---
  371
  371
  ---
```

103. It is sometimes difficult to tell, without trial, how many times the divisor is contained in a partial dividend.

For example, divide 25,474 by 47.

FIRST TRIAL.

```
25474 | 47
188   |----
---   | 4
 66
```

We can not readily tell how many times 47 is contained in 254, but we will suppose that it is contained 4 times. Writing 4 as the first quotient figure, we multiply and subtract, and obtain a remainder of 66. Since this remainder, which is a part of the partial dividend, is greater than the divisor, 47 is contained in 254 more than

4 times. We next try 5 as the quotient figure; and the remainder, 19, is less than 47. Hence, 47 is contained in 254, 5 times. We will now suppose that 47 is contained in 197, the next partial dividend, 5 times. Writing 5 as the second figure in the quotient, we multiply, and obtain a product of 235. Since this product is more than 197, 47 is not contained in 197 as many as 5 times. Hence,

SECOND TRIAL.

$$\begin{array}{l|l} 25474 & 47 \\ \underline{235} & \overline{55} \\ 197 & \\ \underline{235} & \end{array}$$

I. *When any remainder is greater than the divisor, the quotient figure is too small; and*

II. *When any product is greater than the partial dividend, the quotient figure is too great.* See Manual.

PROBLEMS.

29. A drover paid $312 for 13 beeves. How much did they cost him per head? *$24.*

30. A man on a journey traveled 608 miles in 19 days. At what rate per day did he travel? *32 miles.*

31. If 21 acres of land produce 945 bushels of barley, what is the yield per acre? *45 bushels.*

32. A farmer paid $6,804 for a farm of 108 acres. What was the price per acre?

33. A turnpike-road 46 miles long was kept in repair a year at an expense of $1,242. What was the cost per mile? *$27.*

34. In a certain school the aggregate or total attendance for a term of 65 days was 7,410. What was the average daily attendance? *114.*

35. How much must a man earn in each of the 313 working days of a year, to earn $1,252 in a year?

36. A dairy-man packed 10,304 pounds of butter in 56-pound tubs. How many tubs did he fill? *184.*

37. Last season it cost a milkman $576 to winter 32 cows. What was the cost per cow? *$18.*

38. If 1 sheet of paper is required for 24 pages of a book, how many sheets will be required for a book of 432 pages?

39. How many cars, each carrying 49 passengers, will be required to carry 392 passengers?

40. The entire cost of constructing a railroad 84 miles long was $4,065,264. What was the cost per mile? *$48,396.*

41. A person whose property is worth $9,375, spends yearly $625 more than his income. In how many years will he spend all his property? *15.*

42. In how many hours will an express train run the entire length of a railroad 544 miles long, running 32 miles an hour?

43. The stock of a bank, consisting of 875 shares, is worth $117,250. What is the value of each share? *$134.*

44. A grain buyer shipped 401,400 bushels of wheat from Milwaukee to Buffalo in 24 equal cargoes. How many bushels were there in each cargo? *16,725.*

104. Ex. Divide 48,507 by 69.

EXPLANATION.—In the First Solution we see that the second remainder, 20, is the same as the second partial dividend.

In the Second Solution we omit to multiply by the quotient figure, 0, and form the third partial dividend by annexing 7, the next figure of the dividend, to the 20, the second partial dividend. Hence,

Whenever the partial dividend is less than the divisor, we place 0 in the quotient, and bring down the next figure of the dividend for a new partial dividend.

FIRST SOLUTION.

48507 | 69
483 | 703
20
00
207
207

SECOND SOLUTION.

48507 | 69
483 | 703
207
207

PROBLEMS.

45. How much does a man earn each month, whose salary is $1,260 a year? *$105.*

46. In a factory 275 yards of cloth are made daily. How many days will be required to make 57,475 yards?

47. The total cost of constructing a telegraph line 359 miles long was $360,795. What was the cost per mile? *$1,005.*

48. The yield of a Pennsylvania oil well for 1 month was 30,186 gallons, which was put into casks, each holding 43 gallons. How many casks were used?

49. What is the quotient of 120,772,144 ÷ 592? *204,007.*

50. The dividend is 524,177,472, and the divisor 5,824. What is the quotient? *90,003.*

51. Divide 1 billion 962 million 198 thousand 504 by 28 thousand 9. *70 thousand 56.*

CASE III.

Remainders after Dividing last Partial Dividend.

105. Ex. After dividing 1,315 acres of land into as many farms as possible of 92 acres each, how much land will be left?

Explanation.—The divisor, 92, is contained in the last partial dividend, 395, 4 times, with a remainder of 27. As there is no figure of the dividend that has not been used, we have no figure to write with the 27 to form a new partial dividend, and the work is completed, leaving a remainder of 27. Hence, after dividing 1,315 acres of land into 14 farms of 92 acres each, there are 27 acres remaining undivided. See Manual.

SOLUTION.

$$\begin{array}{r|l} 1315 & 92 \\ \underline{92} & \overline{14} \\ 395 & \\ \underline{368} & \\ 27 & \text{Remainder.} \end{array}$$

PROBLEMS.

52. How many payments of $350 each must I make, to pay for a house and lot worth $3,400? *9, and one payment of $250.*

53. A farmer made 962 gallons of cider, which he put into casks holding 41 gallons each. How many full casks had he? *23, and 19 gallons over.*

54. The dividend is 80,963, and the divisor 376. What is the remainder? *123.*

55. A forwarder ships 15,500 barrels of flour to New York, by canal. If 856 barrels make a boat load, how many full boat loads has he to ship? *18, and 92 barrels remainder.*

56. If the directors of a railroad company appropriate $32,000 for the purchase of passenger cars, and the cars cost $1,875 each, how many cars can be bought with the appropriation?

17, and leave a surplus of $125.

57. What will be the remainder, after dividing 62,676 by 573?

58. How many bales of 396 pounds each can be made from 84,000 pounds of cotton? *Remainder, 48 pounds.*

59. 1,405,169 ÷ 2,376 = how many? *Remainder, 953.*

60. The dividend is 5 million 92 thousand 209, and the divisor 10 thousand 23. What is the remainder?

61. Divide 56,432,782 by 27,541. *Remainder, 1,273.*

CASE IV.

The Divisor any number of Tens, Hundreds, Thousands, and so on.

106. Ex. 1. Divide, 67,200 by 100.

FIRST SOLUTION.

$$\begin{array}{r|l} 67200 & \underline{100} \\ \underline{600} & 672 \\ 720 & \\ \underline{700} & \\ 200 & \\ \underline{200} & \end{array}$$

EXPLANATION.—By the First Solution it will be seen that, after removing as many figures from the right hand of the dividend as there are ciphers in the divisor, the remaining figures of the dividend are the same as the quotient. Therefore,

In the Second Solution we have brought down, for the quotient, all the figures of the dividend except as many of its right-hand figures as there are ciphers in the divisor.

SECOND SOLUTION.

$$\begin{array}{r|l} \underline{67200} & 100 \\ 672 & \end{array}$$

Ex. 2. Divide 49,392 by 1,000.

FIRST SOLUTION.

$$\begin{array}{r|l} 49392 & \underline{1000} \\ \underline{4000} & 49 \\ 9392 & \\ \underline{9000} & \\ 392 & \end{array}$$

EXPLANATION.—By the First Solution we see that, if we omit the three right-hand figures of the dividend, the other figures are the same as the quotient, and that the three right-hand figures thus omitted are the same as the remainder. Therefore, In the Second Solution we have brought down, for the quotient, all the figures of the dividend except three, (*i. e.*, as many of the right-hand figures as there are ciphers in the divisor), and for the remainder we have written the three right-hand figures. Hence,

SECOND SOLUTION.

$$\begin{array}{l|l} \underline{49392} & \underline{1000} \\ 49\ {}^{392} & \end{array}$$

I. *Removing the units' figure from a number, divides the number by 10.*

II. *Removing the units and tens, or the two right-hand figures from a number, divides it by 100.*

III. *Removing the three right-hand figures from a number, divides it by 1,000.*

IV. *Removing the four right-hand figures, divides by 10,000; removing five figures, by 100,000; removing six figures, by 1,000,000; and so on.*

V. *The figures removed are the remainder, and the other figures are the quotient.*

107. When the divisor contains more than one ten, hundred, thousand, etc., the manner of obtaining the final remainder is more difficult. For illustrating the method, we will take the following examples:

(1) (2) (3) (4)

$$\begin{array}{l|l} \underline{75} & \underline{15} \\ 5 & \end{array} \qquad \begin{array}{l|l} \underline{75} & \underline{3} \\ \underline{25} & \\ 5 & \end{array} \begin{array}{|l|l} \underline{15} & \underline{3} \\ 5 & \end{array} \qquad \begin{array}{l|l} \underline{75} & \underline{5} \\ \underline{15} & \\ 5 & \end{array} \begin{array}{|l|l} \underline{15} & \underline{5} \\ 3 & \end{array} \qquad \begin{array}{l|l} \underline{7500} & \underline{100} \\ \underline{75} & \\ 5 & \end{array} \begin{array}{|l|l} \underline{1500} & \underline{100} \\ 15 & \end{array}$$

By carefully examining these four examples, we see in Ex. 1 that the quotient of 75 divided by 15 is 5; in Ex. 2, 3, that the quotient is not changed by dividing both divisor and dividend by 3 or 5; and in Ex. 4, that the divisor, 1,500, and the dividend, 7,500, may both be divided by 100, and the results used as divisor and dividend, without affecting the final quotient. That is,

Both divisor and dividend may be divided by the same number, without affecting the value of the final quotient.

108. Ex. Divide 15,284 by 3,400.

FIRST SOLUTION.

$$\begin{array}{r|l} 15284 & 3400 \\ 13600 & \overline{\ \ 4} \\ \cline{1-1} 1684 & \end{array}$$

SECOND SOLUTION.

$$\begin{array}{r|l} 152|84 & 34|00 \\ 136\ \ \ \ & \overline{\ \ 4} \\ \cline{1-1} 1684 & \end{array}$$

EXPLANATION.—In the First Solution we divide as taught in Case III., and obtain a quotient of 4, and a remainder of 1,684.

In the Second Solution we first divide both divisor and dividend by 100, (which we do by cutting off, by a vertical line, the two right-hand figures from each term), obtaining 34 for a new divisor, 152 for a new dividend, and a remainder of 84. Dividing 152 by 34, we obtain a final quotient of 4, and a second remainder of 16. Since this remainder was obtained by subtracting 136 hundreds from 152 hundreds (see First Solution), it must be 16 hundreds; while the first remainder, 84, is the tens and ones of the dividend. We therefore unite these two remainders, by annexing the 84 to the 16 (hundreds), and we have 1,684 for the final remainder, the same as in the First Solution. Hence,

In dividing by any number of tens, hundreds, thousands, and so on,

The final remainder consists of the figures which were cut off from the dividend, annexed to the figures of the last remainder.

PROBLEMS.

62. A dealer sold 10 sewing-machines for $650. How much were they apiece?

63. A cental of grain is 100 pounds. How many centals in 47,300 pounds? *473.*

64. The capital or stock of a certain mining company is $225,000, and it is divided into $1,000 shares. How many shares of stock are there?

65. At $2,000 each, how many steam-tugs can be bought for $6,000?

66. A builder paid $5,760 for boards, at $30 per thousand. How many thousand feet did he buy? *192.*

67. A carriage maker received $1,200 for light carriages, at $200 apiece. How many carriages did he sell?

68. What is the quotient of 393,400,000 divided by 100,000?

69. A government agent paid $56,400 for horses for the army, at $150 apiece. How many did he buy? *376.*

70. A farmer having $83, wishes to purchase yearling calves, at $10 a head. How many can he buy? *8, and have $3 left.*

71. How many freight cars will be required to transport 58,293 barrels of flour, if 100 barrels make one car load?
582 full cars, and 1 car carrying 93 barrels.

72. A company purchase a hotel in New Orleans for $165,675, and the payments are to be $25,000 annually. How many payments must they make? *6 of $25,000 each, and 1 of $15,675.*

73. Divide 193,285 by 36,000. *Remainder, 13,285.*

74. What is the quotient of 17,630,000 divided by 24,000?

109. The facts deduced in Arts. **95**, **98**, **106**, **107**, may now be stated as

General Principles of Division.

I. *A concrete number can be divided by either a concrete or an abstract number.*

II. *An abstract number can be divided by an abstract number only.*

III. *The quotient is an abstract number, when the divisor and dividend are both abstract or both concrete numbers.*

IV. *The quotient is a concrete number, when the divisor is an abstract, and the dividend a concrete number.*

V. *Any quotient figure is of the same order of units as the last figure of the dividend used to obtain it.*

VI. *The removal of the right-hand figure from a number divides that number by 10.*

VII. *Both divisor and dividend may be divided by the same number, without affecting the value of the final quotient.*

VIII. *The right-hand figure of any remainder is of the same order of units as the last figure of the dividend used.*

110. Upon these principles is based the

Rule for Division of Integers.

I. For Long Division.

1. *Place the divisor at the right of the dividend, separate them by a line, and draw a line under the divisor to separate it from the quotient.*

2. *Find how many times the divisor is contained in the first partial dividend, and write the result for the first figure of the quotient.*

3. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend for a new partial dividend.*

4. *In the same manner, continue to divide, multiply, subtract, and bring down, until all the figures of the dividend have been used.*

II. For Short Division.

1. *Write the divisor and dividend as in Long Division, and draw a line under the dividend to separate it from the quotient.*

2. *Find how many times the divisor is contained in the first partial dividend, as in Long Division, and write the result under the last figure of the dividend used, for the first figure of the quotient.*

3. *Multiply, subtract, and form a new partial dividend, as in Long Division, performing the operations mentally.*

4. *Divide this partial dividend, and write the result as the second figure of the quotient.*

5. *Proceed in the same manner until all the figures of the dividend have been used.*

III. For dividing by any number of tens, hundreds, thousands, and so on.

1. *Cut off the ciphers by a line, and also an equal number of figures from the right of the dividend.*

2. *Divide the remaining figures of the dividend by the remaining figures of the divisor.*

3. *For the true remainder, annex to the last remainder the figures cut off from the dividend.*

PROBLEMS.

75. An army contractor paid $276,560 for beef, at $16 a barrel. How much beef did he buy? *17,285 barrels.*

76. In a cotton-factory are 54 looms, which were bought at a total cost of $9,720. What was the cost of each loom?

77. A canal 97 miles long was constructed at a cost of $14,131,930. What was the cost per mile? *$145,690.*

78. Peter having an ear of corn in which were 864 kernels, planted it in hills of 6 kernels each. How many hills did he plant?

79. He planted the corn in 8 equal rows. How many hills were there in each row? *18.*

80. The fare of 219 passengers by steam-ship from New York to Havre, was $36,135. How much was the fare of each passenger?

81. At $60 a head, how many cows can be bought for $1,650? *27, with $30 left.*

82. A city builder received $780,000 for building brown-stone front houses, at an average price of $10,000 each. How many houses did he build?

83. A manufacturer sold reapers at $130 each, and received $6,240. How many reapers did he sell? *48.*

84. How many barrels, each holding 200 pounds, will be required for packing 47,875 pounds of pork? *239, with 75 pounds of pork left.*

85. A wholesale grocer bought 3,440 pounds of tea, in 80-pound chests. How many chests did he buy? *43.*

86. How many canal boats can a transportation company buy with $34,000, at $1,000 each?

87. In the schools of a certain city 28,497 pupils are taught by 483 teachers. What is the average number of pupils to a teacher?

88. A paper maker having 361,920 sheets of foolscap, put it up for market in quires of 24 sheets each. How many quires were there?

89. He sold the paper by the ream of 20 quires. How many reams did he sell? *754.*

90. The United States Government paid $103,600 for 740 army wagons. How much was that for each wagon? *$140.*

91. What is the quotient of 447 billion 670 million 621 thousand 104 divided by 4 million 930 thousand 76? *90 thousand 804.*

92. Divide 660,886,723 by 982. *Remainder, 723.*

93. The dividend is 468,002,659, and the divisor 9,497. What is the remainder? *9,493.*

94. A planter raised 86,301 pounds of cotton on 223 acres of land. How many pounds was the yield per acre? *387.*

95. The New York and New Haven Railroad track is 401,280 feet long. How many miles from the New York to the New Haven Railroad depot, there being 5,280 feet in a mile? *76.*

96. In a certain county $2,039,688 were paid in bounties to 2,394 volunteers. What bounty was paid to each soldier? *$852.*

97. How long will 564,000 rations last an army brigade of 5,875 men? *96 days.*

98. A produce buyer purchased 417 bushels of wheat, 873 bushels of oats, and 314 bushels of barley. The bins in his storehouse will hold 72 bushels each. How many bins will each kind of grain fill?

Wheat, 5 bins, and 57 bushels over.
Oats, 12 " " 9 " "
Barley, 4 " " 26 " "

99. If all of the grain was of one kind, how many bins would it fill? *20 bushels, remainder.*

100. A man buys a farm of 113 acres, at $54 an acre. He pays $1,392 down, and agrees to pay the balance in 6 equal yearly payments. How much of the debt must he pay each year? *$785.*

SECTION VII.

REVIEW PROBLEMS IN INTEGERS.

1. THE parts of a number are 73, 427, 856, and 32,519. What is the number? See Manual.

2. The minuend is 59,408, and the subtrahend 14,642. What is the remainder?

3. The sum of two numbers is 1 million 56 thousand, and one of the numbers is 304 thousand 9. What is the other number?

4. A reward of $7,650 was shared among 4 detectives, the first of whom received $2,225, the second $1,750, and the third $1,875. How much did the fourth receive?

5. The multiplicand is 185,046, and the multiplier 4,309. What is the product?

6. What is that number, the factors of which are 384, 27, 90, and 10,000?

7. The dividend is 1,728,000, and the divisor 1,800. What is the quotient?

8. If the dividend is 5,443,200, and the several successive divisors are 9, 14, and 600, what is the final quotient?

9. A farmer's expenses and receipts one year were as follows:

Expenses.		*Receipts.*	
For Labor	$295	For Wheat	$419
" Seed	49	" Oats	385
" Agricultural Implements	136	" Corn	240
" Family Expenses	485	" Hay	176
" Interest Money	140	" Pork	117
		" Wool	95
		" Other Farm Products	58

Did he make or lose money that year, and how much? *$385.*

10. He sold his hay at $8 a ton. How much hay did he sell?

11. If 7 bricklayers are 67 days in putting up the walls of a machine-shop, and each man lays 1,950 bricks a day, how many bricks will there be in the walls of the building? *914,550.*

12. From a cistern containing 19,437 gallons of water, 13,294 gallons were drawn out, and afterward, during a rain storm, 7,483 gallons ran in. How much water was there in the cistern? *13,626 gallons.*

13. The live weight of an ox was 1,816 pounds. When dressed, the four quarters weighed respectively 271, 264, 275, and 287 pounds; the hide weighed 85 pounds, and the tallow 97 pounds. What was the difference between the live and dead weight of the ox? *537 pounds.*

14. One season a jobbing carpenter built 5 dwellings, which cost him $3,176, $5,194, $1,342, $6,950, and $788. He received for building them $3,875, $6,820, $1,280, $7,896, and $875. What were his season's profits? *$3,296.*

15. The cost of mounting and equipping a cavalry regiment of 1,037 men was $213,622. How much was the cost per man?

16. One year a stove manufacturer sold to a wholesale dealer, stoves as per annexed schedule.

To how much did the sales amount? See Manual.

53	*No.*	*10*	*Stoves,*	*at*	*$33*
169	"	*9*	"	"	*28*
214	"	*8*	"	"	*24*
192	"	*7*	"	"	*22*
73	"	*6*	"	"	*18*

$17,155

17. I bought a farm of 153 acres, at $95 an acre, and paid down $2,500. How much of the purchase money remained unpaid?

18. A man bequeathed to each of 2 sons $7,600; to a third son $1,500; to each of 3 daughters $3,775; and the balance of his estate, which amounted to $6,877, to other parties. But the will was set aside, and the property was divided equally among his children. How much did each receive? *$5,817.*

19. A merchant bought a piece of broadcloth containing 56 yards, for $266, and sold it at $6 a yard. How much was his profit?

20. In the Oakland Mill are 9 run of stone, each capable of grinding 100 bushels of wheat per day. In what time can 297,000 bushels of wheat be ground? *330 days.*

21. A dairy-man has fodder enough to keep his 35 cows 4 months. If he sells 7 cows, how many months will the fodder last the rest?

22. The receipts and expenditures of a church society for one year were as follows:

Receipts.	*Expenditures.*
Rent of 18 Pews, at $32, $---	Salary of Clergyman --$1.250
" 34 " " 26, ---	" " Chorister ---- 300
" 22 " " 22, ---	" " Sexton ------ 312
" 14 " " 15, ---	Expenses for Fuel ----- 174
" 8 " " 12, ---	" " Lights ---- 93
Free, 8 "	Contingent Expenses ---- 121

How did the account stand at the close of the year?

23. The United States Supreme Court consists of a Chief Justice, whose salary is $6,500 per annum, and 9 associate justices, whose salaries are $6,000 per annum each. How much do all their salaries amount to in one Presidential term? *$242,000.*

24. How many military companies of 98 men each can be formed from 3,675 recruits? *37, with 49 recruits left.*

25. A wood dealer sold 36 cords of hickory wood, at $6 a cord, 75 cords of maple wood, at $5 a cord, and 43 cords of soft wood, at $4 a cord. How much did he receive for the whole? *$763.*

26. A farmer who raised 984 bushels of oats, after retaining 48 bushels for seed, and enough to winter 5 horses, allowing 50 bushels to each horse, sold the balance. How many bushels did he sell? *686.*

27. A lady having $100, paid $58 for a set of furs, and $2 a yard for 17 yards of silk. How much money had she left? *$8.*

28. A grocer bought 2,880 pounds of coffee in 120-pound sacks. How many sacks did he buy?

29. A grocer bought 5 hogsheads of molasses that were billed to him at 140 gallons each; but the first was 17 gallons short, the second 5 gallons, the third 9 gallons, the fourth 4 gallons, and the fifth 2 gallons. How many gallons were in the 5 hogsheads?

30. A soldier enlisting for 3 years received a bounty of $949. He served 8 months as a private, at $13 a month; 8 months as a corporal, at $14 a month; 13 months as a sergeant, at $17 a month; and 7 months as an orderly sergeant, at $18 a month. What was the total amount of his pay? How much did it average per month? See Manual. $42 per month.

Table of Areas used in the next ten Problems.

	SQUARE MILES.		SQUARE MILES.
United States	3,001,002	New England	65,038
France	207,829	Michigan	56,243
England	50,922	Illinois	55,405
Island of Australia	2,980,770	Georgia	52,009
“ “ Borneo (about)	320,000	New York	50,519
“ “ Cuba	47,278	Tennessee	45,600
Texas	280,000	Ohio	39,964
California	159,000	Massachusetts	7,800

See Manual.

31. Into how many states, each as large as New York, could Australia be divided? *59, and 149 square miles remainder.*

32. How many states, each as large as Massachusetts, might be formed from Texas?

33. The Island of Borneo is how many times as large as Cuba?

34. How much larger is the State of Illinois than England?

35. The Island of Australia is how many times as large as New England? and how much larger or smaller than Tennessee is the remainder? *8,460 square miles.*

36. If all the territory of the United States were divided into as many states as possible, each as large as Michigan, the remainder forming another state, how many states would there be in the Union, and what would be the size of the small state? *54 states; 20,123 square miles.*

37. If a country consisted of 376 states, each as large as Massachusetts, would its area be greater or smaller than the area of the United States? *68,202 square miles.*

38. If Texas were divided into 4 states of equal size, how much larger than Ohio would each of them be?

39. How does 3 times the area of Georgia compare with the area of California?

40. Into how many countries could the United States be divided, and each have an area equal to the combined areas of France and England? *154,741 square miles more than 11 such countries.*

CHAPTER 2. DECIMALS.

SECTION I.

INDUCTION AND DEFINITIONS.

111. We have already learned (Chap. I., Sec. II.) that the places of the different orders of integral units—as ones, tens, hundreds, thousands, ten-thousands, and so on—increase in value from right to left in a tenfold ratio, or by the constant multiplier 10. Thus, 10 times ones are tens, 10 times tens are hundreds, 10 times hundreds are thousands, and so on.

We have also learned that the orders of units decrease from left to right in a tenfold ratio, or by the constant divisor 10. Thus, thousands ÷ 10 are hundreds, hundreds ÷ 10 are tens, tens ÷ 10 are ones, or simple units.

112. Continuing this division below ones, we obtain a new class of numbers, which are subject to the same general laws as are integers, and which differ from them in only one respect, namely; the value of a unit of any order is less than unity, or 1. Thus,

If ones	are divided by	10,	the resulting	units	are	*tenths;*
" tenths	"	10,	"	"	"	*hundredths;*
" hundredths	"	10,	"	"	"	*thousandths;*
" thousandths	"	10,	"	"	"	*ten-thousandths;*
" ten-thousandths	"	10,	"	"	"	*hundred-thousandths;*
" hundred-thousandths		10,	"	"	"	*millionths;*

and so on.

113. ***A Scale,*** in Arithmetic, is an established order of increase or decrease from any unit to higher or lower units in the same class of numbers.

114. A ***Decimal Scale*** is one in which the values of the orders of units increase by the constant multiplier, and decrease by the constant divisor, 10.

NOTES.—1. The term decimal is derived from the Latin *decem*, which signifies 10.

2. The scale of integers is a decimal scale.

115. A ***Decimal Unit*** is one of the equal decimal parts into which a thing, or the unit 1, is divided; as, 1 tenth, 1 hundredth, 1 thousandth, 1 ten-thousandth, and so on.

116. A ***Decimal*** is a number expressed by decimal units; as, 7 tenths, 258 thousandths.

NOTES.—1. A number consisting of an integer and a decimal is a *Mixed Number;* as, 8 and 25 hundreds.

2. Integers and decimals together form one general class of numbers, called *Decimal Numbers.*

SECTION II.

NOTATION AND NUMERATION.

117. If we divide an apple into 10 equal parts, each of the parts is 1 tenth of the apple. When any thing, or a 1, is divided into 10 equal parts, 1 of the parts is *1 tenth* of the thing or the 1, 2 of the parts are *2 tenths,* 3 of them are *3 tenths,* 4 of them are *4 tenths,* and so on. 10 tenths are 1.

1 tenth is written *.1*

2 tenths are written *.2*	6 tenths are written *.6*
3 " " " *.3*	7 " " " *.7*
4 " " " *.4*	8 " " " *.8*
5 " " " *.5*	9 " " " *.9*

118. In the number 111 the first or left-hand figure is 1 hundred, the second figure is 1 ten, and the third figure is 1 one. Since 1 ten is 1 tenth of 1 hundred, and 1 one is 1 tenth of 1 ten, it follows that

The value of any figure in a number is 1 tenth of the value of a like figure standing in the next place at the left. Hence,

119. The value of any figure written at the right of ones is tenths.

1 and 1 tenth are written *1.1*
19 and 3 tenths " " *19.3*
50 and 7 tenths " " *50.7*
276 and 9 tenths " " *276.9*

120. The *Decimal Point* is a period or point (.) placed before tenths. When placed between figures, the decimal point is always read *and.* Thus, 4.5 is read "4 and 5 tenths." See Manual.

In writing decimals, the decimal point must always be used.

EXERCISES.

1. Read .4, .8, .1, 7.3, 10.9, 392.6, 7198.2.
2. Write five tenths, one tenth, nine tenths.
3. Write 17 and 3 tenths; 28 and 6 tenths.
4. Write 240 and 9 tenths; 1006 and 5 tenths.
5. Write two tenths; five hundred sixty and four tenths.

121. If any thing, or a 1, be divided into tenths, and then each of the tenths be divided into 10 equal parts, there will be in the whole thing, or the 1, 10 times 10, or 100 equal parts; and each of the parts will be 1 hundredth of the whole thing, or of the 1. Hence,

1 hundredth is 1 tenth of 1 tenth. And, since the value of a figure in any place is 1 tenth of the value of a like figure in the next place at the left (see **118**), it follows that

The value of any figure written at the right of tenths is hundredths.

1 hundredth is written *.01*
2 hundredths are " *.02*
5 " " " *.05*
8 " " " *.08*

122. .37 is 3 tenths and 7 hundredths. But 3 tenths = 30 hundredths, and 30 hundredths + 7 hundredths = 37 hundredths.

Tenths and hundredths are read together as hundredths.

.23, or 2 tenths and 3 hundredths, is read 23 hundredths.
.57, or 5 tenths and 7 hundredths, is read 57 hundredths.
6.85, or 6 ones, 8 tenths, and 5 hundredths, is read 6 and 85 hundredths.

EXERCISES.

6. Read .43, .91, .04, 8.32, 5.09, 47.47, 5080.06.
7. Write 3 hundredths; 51 hundredths; 2 and 75 hundredths.
8. Write 15 and 15 hundredths; 328 and 11 hundredths.
9. Write 30 and 30 hundredths; 200 and 2 hundredths.

123. If any thing, or a 1, be divided into hundredths, and then each of the hundredths be divided into 10 equal parts, there will be in the whole thing, or the 1, 100 times 10, or 1,000 equal parts; and each of the parts will be 1 thousandth of the whole thing, or of 1. Hence,

1 thousandth is 1 tenth of 1 hundredth. And the value of any figure written at the right of hundredths is thousandths. (See **118**).

1 thousandth is written *.001*
4 thousandths are " *.004*
7 " " " *.007*
9 " " " *.009*

124. .278 is 2 tenths 7 hundredths and 8 thousandths, or 27 hundredths and 8 thousandths. But 27 hundredths = 270 thousandths, and 270 thousandths + 8 thousandths = 278 thousandths.

Tenths, hundredths, and thousandths are read together as thousandths.

.006 is read 6 thousandths.
.072 is read 72 thousandths.
.493 is read 493 thousandths.
19.136 is read 19 and 136 thousandths.

EXERCISES.

10. Read .43, .18, .02, 6.27, 342.51, 99.07.
11. Read .176, .584, .096, .204, .007, .901.
12. Read 4.23, 19.07, 70.219, 9.021, 817.108, 11.005.
13. Write 5 tenths and 6 hundredths, or 56 hundredths.
14. Write 93 hundredths; 6 hundredths.
15. Write 1 tenth 9 hundredths and 7 thousandths, or 197 thousandths.
16. Write 211 thousandths; 42 thousandths.
17. Write three hundred seven thousandths.
18. Write 30 and 19 hundredths.
19. Write 256 and 4 hundredths.
20. Write 193 and 5 thousandths.
21. Write 3,281 and 59 thousandths.
22. Write 10,000 and 208 thousandths.

125. A figure at the right of thousandths is *ten-thousandths;* and a decimal containing tenths, hundredths, thousandths, and ten-thousandths, is read as ten-thousandths.

.5763 is 5 tenths, 7 hundredths, 6 thousandths, and 3 ten-thousandths, and is read 5,763 ten-thousandths.

126. A figure at the right of ten-thousandths is *hundred-thousandths;* and a decimal containing tenths, hundredths, thousandths, ten-thousandths, and hundred-thousandths, is read as hundred-thousandths.

.57208 is 5 tenths, 7 hundredths, 2 thousandths, 0 ten-thousandths, and 8 hundred-thousandths, and is read 57,208 hundred-thousandths.

127. A figure at the right of hundred-thousandths is *millionths,* a figure at the right of millionths is *ten-millionths,* a figure at the right of ten-millionths is *hundred-millionths,* and so on.

128. When the right-hand figure of a decimal is millionths, the whole decimal is read as millionths; when the right-hand figure is ten-millionths, the decimal is read as ten-millionths; when the right-hand figure is hundred-millionths, the decimal is read as hundred-millionths, and so on. And in general,

The figures of a decimal are read the same as the figures of an integer, and to the whole decimal is given the local name of the last or right-hand figure.

.476298 is 476,298 millionths.
.5008721 is 5,008,721 ten-millionths.
.87396483 is 87,396,483 hundred-millionths.
.000084 is 84 millionths.
.0005008 is 5,008 ten-millionths.
.06070802 is 6,070,802 hundred-millionths.

129. .5 is 5 tenths. .50 is 50 hundredths, or 5 tenths and 0 hundredths. .500 is 500 thousandths, or 5 tenths, 0 hundredths, and 0 thousandths. That is, .5, .50, and .500 are all of the same value, namely, 5 tenths; consequently, ciphers on the right of a decimal do not change the places of the other figures. Hence,

I. *Ciphers may be annexed to any decimal, or decimal ciphers to any integer, without changing its value;* and

II. *Ciphers may be omitted from the right of any decimal, or decimal ciphers from the right of any integer, without changing its value.*

130. TABLE OF VALUES OF DECIMAL NUMBERS.

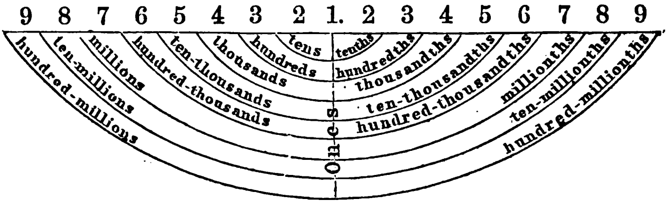
One	decimal	figure	expresses	*tenths.*
Two	decimal	figures	express	*hundredths.*
Three	"	"	"	*thousandths.*
Four	"	"	"	*ten-thousandths.*
Five	"	"	"	*hundred-thousandths.*
Six	"	"	"	*millionths.*
Seven	"	"	"	*ten-millionths.*
Eight	"	"	"	*hundred-millionths.*
Nine	"	"	"	*billionths.*

And so on.

See Manual.

131. Figures standing in places at equal distances to the right and left of ones have *names* that correspond to each other, as shown in the following

DIAGRAM OF DECIMAL NOTATION.



132. This diagram also shows that the value which any figure in a decimal expresses, is determined by the place it occupies.

133. From the illustrations and explanations now given, we deduce the following

Principles of Decimal Notation and Numeration.

I. *All places to the right of unity are decimals.*

II. *The values of the different places in a decimal increase from right to left, and decrease from left to right, in a tenfold ratio.*

III. *The place which any figure occupies in a decimal determines the value expressed by it in that decimal.*

IV. *The decimal point must always be placed before tenths.*

V. *In writing a decimal, all places not named must be filled by ciphers.*

VI. *Decimal ciphers may be annexed to, or omitted from, the right of any number, without changing its value.*

VII. *The names of places equi-distant to the left and right of unity differ only in their terminations, those at the left terminating in* ns *or* ds, *and those at the right in* ths.

VIII. *The figures of a decimal are read the same as the* *figures of an integer, the name of the place occupied by the right-hand figure of the decimal being pronounced after the last figure read.* See Manual.

134. These principles fully establish this

General Law of Decimal Numbers.

Integers and decimals form one class of numbers, in the decimal scale; and all like operations upon the two divisions of the class are governed by the same principles.

EXERCISES.

23. Read .1765; .3046; 92.1005; 100.0048.
24. Read .39417; .00009; 53.40206; 10.00538.
25. Read .476398; 11.000141; 904.204080; 21.600008.
26. Read .4598217; 19.3006009; 214.0380965.
27. Read .00000001; .70876941; 329000.80000185.
28. Read 3976.070009; 56.0085; 10006.000596.
29. Read 5.5682; 273.8760099; 1.000000007.
30. Write 291 ten-thousandths; write 706,095 millionths.
31. Write 508 millionths; write 217 hundred-millionths.
32. Write 90,085,765 hundred-millionths.
33. Write 5 ten-millionths; write 18 ten-billionths.
34. Write 3,750 and 17 ten-thousandths.
35. Write 7 thousand and 7 thousandths.
36. Write 2,548,006 and 905 millionths.
37. Write 19 and 19 billionths.
38. Write 297,641,879 trillionths; write 700,849 ten-billionths.
39. Write six hundred seventeen millionths.
40. Write six hundred and seventeen millionths.
41. Write four thousand seven hundred-thousandths.
42. Write four thousand and seven hundred-thousandths.
43. Write four thousand and seven hundred thousandths.
44. Write eight hundred nine thousand one hundred fifty-seven ten-millionths.
45. Write sixty-three million three hundred fifty-four thousand eight hundred seventy-seven billionths.
46. Write one thousand four hundred and ten thousandths.

See Manual.

SECTION III.

ADDITION.

135. Ex. What is the sum of 56.125, 9.356, and 123.25?

EXPLANATION. — Since only like orders of units in different numbers can be added (see **39**, II.), we write the numbers with like orders of units—both decimal and integral—in the same columns. The decimal points then stand in a column. We commence at the right, and add as in integers. Since the sum of thousandths is thousandths, the sum of hundredths is hundredths, and the sum of tenths is tenths, and there are thousandths, hundredths, and tenths in the given parts, there must also be thousandths, hundredths, and tenths in their sum. We therefore place the decimal point in the sum before the 7, and directly under the decimal points in the parts.

SOLUTION.

$$\begin{array}{r} 56.125 \\ 9.356 \\ 123.25 \\ \hline 188.731 \end{array}$$

136. *Rule for Addition of Decimals.*

I. *Write the numbers so that the decimal points shall stand in a column.*

II. *Add in the same manner as in integers, and place the decimal point in the sum directly under the decimal points in the parts.* See Manual.

PROBLEMS.

(1)	(2)	(3)	(4)	(5)
.321	.48	3.62	162.71	.0052
.746	.697	517.83	48.086	.02081
.984	.8	21.9	3915.3004	.016
.258	.5764	674.08	.721	.0000375

6. A farmer brought three loads of wood to market, the first load containing .8 of a cord, the second .75, and the third .9375. How many cords did he bring in all? *2.4875.*

7. A peddler traveled 6.75 miles one day, 4.6 miles the next, 7.384 miles the third, and 2.14 miles the fourth. How far did he travel in the four days? *20.874 miles.*

8. How many acres in four fields, there being 9.5 acres in the first, 11.4 acres in the second, 8.75 acres in the third, and 12.675 acres in the fourth? *42.325.*

9. A lady bought 16.25 yards of silk, 12.75 yards of alpaca, 6.5 yards of merino, 11.875 yards of delaine, and 23.5 yards of French calico. How many yards of goods did she purchase? *70.875.*

10. A farmer sold eight lots of hay, as follows: 7.637 tons, 3.5 tons, 17.396 tons, 5.824 tons, 12 tons, .95 of a ton, 8.0625 tons, and 6.4 tons. How much hay did he sell? *61.7695 tons.*

11. In six consecutive days a company of California miners obtained 3.5286 ounces, 1.4 ounces, 3.125 ounces, 7.0064 ounces, .65 of an ounce, and 2.72 ounces of gold. What was the whole amount for the six days? *18.43 ounces.*

12. Capt. Allen's farm consists of 7.4 acres of woodland, 16.275 acres of pasture, 23 acres of meadow, 6.025 acres of orchard and garden, and 72.3 acres of tilled land. How much land is there in his farm? *125 acres.*

13. At London the average fall of rain is, for

MONTHS.	INCHES.	MONTHS.	INCHES.	MONTHS.	INCHES.
January,	1.483	May,	1.853	September,	2.193
February,	.746	June,	1.83	October,	2.073
March,	1.44	July,	2.516	November,	2.4
April,	1.786	August,	1.453	December,	2.426

What is the average annual fall? *22.199 inches.*

14. A real-estate agent received for his services in selling seven farms, $137.25, $94.5, $216.375, $56.4, $113.7, $80.625, and $296.3. What were his total receipts? *$995.15.*

15. A grocer bought six hogsheads of molasses, containing 117.5 gallons, 124 gallons, 129.3175 gallons, 104.75 gallons, 130.0625 gallons, and 131.5625 gallons. How much molasses did he buy?

16. What is the sum of 967 thousandths, 54 hundred-thousandths, 953 and 5 tenths, 7 and 375 thousandths, 1000 and 1 ten-thousandth, 6 and 75 hundredths, 8 and 80,808 hundred-thousandths, and 483? *2,460 and 40,072 hundred-thousandths.*

SECTION IV.

SUBTRACTION.

137. Ex. 1. From 11.278 subtract 4.825.

EXPLANATION.—Since only like orders of units in different numbers can be subtracted the one from the other (see **52**, II.), we write the numbers with the units of the subtrahend, both decimal and integral, under like orders of units of the minuend. The decimal point of the subtrahend then stands directly under that of the minuend. We subtract as in integers. Since the difference between thousandths and thousandths is thousandths, the difference between hundredths and hundredths is hundredths, and the difference between tenths and tenths is tenths, and there are thousandths, hundredths, and tenths in the given numbers, there must also be thousandths, hundredths, and tenths in their difference. We therefore place the decimal point in the difference before the 4, and directly under the decimal points in the given numbers.

SOLUTION.

$$\begin{array}{r} 11.278 \\ \underline{4.825} \\ 6.453 \end{array}$$

Ex. 2. From 52 subtract 9.785.

EXPLANATION.—Since decimal ciphers may be annexed to a number without changing its value (see **133**, VI.), we annex ciphers to the minuend until it has as many decimal figures as the subtrahend, and then subtract and place the decimal point as in the Solution of Ex. 1.

SOLUTION.

$$\begin{array}{r} 52.000 \\ \underline{9.785} \\ 42.215 \end{array}$$

138. *Rule for Subtraction of Decimals.*

I. *Write the numbers with the decimal point of the subtrahend directly under that of the minuend.*

II. *Subtract in the same manner as in integers, and place the decimal point in the remainder directly under the decimal point in the subtrahend.* See Manual.

PROBLEMS.

	(1)	(2)	(3)	(4)
From	.3758	386.25	57.4628	2940
Subtract	.1974	50.7682	41.93	.0492

5. A merchant sold 19.25 yards of sheeting from a piece which contained 43.75 yards. How many yards were left? *24.5.*

6. A company that contracted to build a turnpike 17.5 miles long, have completed 9.875 miles. How much have they yet to build? *7.625 miles.*

7. One year a stock farmer put 43.5 tons of hay into his barns, and the following spring he had only 8.75 tons. How much had he fed to his stock? *34.75 tons.*

8. A seedsman having 73.625 bushels of choice potatoes, bought enough more to increase his stock to 120 bushels. How many potatoes did he buy? *46.375 bushels.*

9. The owner of a schooner sold .3125 of her to the captain. What part of the vessel did he still own? *.6875.*

10. There are 192.8125 barrels of water in a cistern which will hold 320.5 barrels. How much more water will the cistern hold?

11. A man bought a horse for $118.375, and afterward sold him for $130.25. What was his gain? *$11.875.*

12. A load of hay with the wagon weighed 2 and 65 thousandths tons, and the wagon weighed 1 and 9 hundredths tons. What was the weight of the hay? *975 thousandths of a ton.*

13. A woman sold a house and lot, which cost her $2250.5, for $1900.75. How much did she lose on it? *$349.75.*

14. A person traveled 1,200 miles in 4 weeks, going 276.5 miles the first week, 318.37 miles the second, and 294.2 miles the third. How far did he travel the last week? *310.93 miles.*

15. A vessel of 400 tons burthen, bound up the lakes, ships at Buffalo 93.4 tons of railroad iron, 56.81 tons of salt, and 211.7 tons of general merchandise. How much does she lack of a full cargo?

16. A man having three farms, containing, respectively, 296.5 acres, 145.75 acres, and 96 acres, sold to one man 72.5 acres, and to another 86 acres, and gave to each of his two sons 105.25 acres. How many acres had he left? *169.25.*

SECTION V.

MULTIPLICATION.

139. 2 times 4 are 8, 3 times 3 are 9, 5 times 7 are 35 (or 35 ones) = 3 tens and 5 ones. In each of these illustrations the two factors are ones and the product is ones.

The product of ones multiplied by ones is always ones.

140. In multiplying 24.3 by 2, the 8 of the product is obtained by multiplying the 4 ones of the multiplicand by the 2 ones of the multiplier. Hence, the 8 is ones, and the decimal point must be placed at the right of it.

Ex. 1.

$$\begin{array}{r} 24.3 \\ 2 \\ \hline 48.6 \end{array}$$

In multiplying 4.17 by 2.1, the product of the 4 ones of the multiplicand and the 2 ones of the multiplier is the 8 of the second partial product. Hence, that figure is ones, and the 8 of the final product is also ones.

Ex. 2.

$$\begin{array}{r} 4.17 \\ 2.1 \\ \hline 417 \\ 8.34 \\ \hline 8.757 \end{array}$$

Ex. 3.

$$\begin{array}{r} 4.26 \\ 2.13 \\ \hline 1278 \\ 426 \\ 8.52 \\ \hline 9.0738 \end{array}$$

In Ex. 3, the 8 of the third partial product is the product of the ones of the multiplicand and the ones of the multiplier. Hence it is ones, and the 9 of the final product is also ones.

In Ex. 4, the 8 in the second partial product is ones, and the 9 in the final product is also ones; and in Ex. 5, the 7 of the third partial product is ones, and also the 8 of the final product.

Ex. 4.

$$\begin{array}{r} 4.316 \\ 32.4 \\ \hline 17264 \\ 8.632 \\ 12948 \\ \hline 139.8384 \end{array}$$

Ex. 5.

$$\begin{array}{r} 2.5043 \\ 3.42 \\ \hline 50086 \\ 100172 \\ 7.5129 \\ \hline 8.564706 \end{array}$$

141. In Ex. 1 there is one decimal figure in one of the factors, and one decimal figure in the product. In Ex. 2 there are three decimal figures in the factors and three in the product. In Ex. 3 and 4 there are four decimal figures in the two factors and four in the product; and in Ex. 5 there are six decimal figures in the factors and six in the product. From these examples we learn that

The number of decimal figures in a product must equal the number of decimal figures in its factors.

142. By examining this Example we see that the 2 of the third partial product is the product of ones multiplied by ones, and therefore must be ones; and that

The product
- of ones and tenths is tenths;
- of ones and hundredths, and of tenths and tenths is hundredths;
- of ones and thousandths, and of tenths and hundredths is thousandths;
- of tenths and thousandths, and of hundredths and hundredths is ten-thousandths;
- of hundredths and thousandths is hundred-thousandths. Hence,

Ex.

		2.	1	0	3
			1.	3	2
		4	2	0	6
	6	3	0	9	
2	.1	0	3		
2	.7	7	5	9	6
ones	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths

The number of the decimal place in which the product of any two decimal figures belongs, counting from ones, is equal to the sum of the numbers of the decimal places of the two figures multiplied.

143. Since 3 times 0 ones is 0 ones, we place the decimal point in Ex. 1 at the right of the 0 in the product, as shown in (1). But the 0 may be omitted from the multiplicand without changing its value, and the product will then be .759, as shown in (2).

Ex. 1.

(1)	(2)
0.253	.253
3	3
0.759	.759

In multiplying 4 ten-thousandths by 2 thousandths, since the 4 is four places and the 2 is three places to the right of ones, the product, 8, must be seven places to the right of ones, and the other six places must be filled by ciphers. Hence,

Ex. 2.

.0004
.002
.0000008

When there are not as many figures in the product as there are decimal figures in the factors, decimal ciphers must be prefixed to the other figures to supply the deficiency.

144. Upon the principles deduced in Art. **141, 142, 143,** is based the

Rule for Multiplication of Decimals.

I. *Write the numbers and multiply as in integers.*

II. *Place the decimal point in the product, so that it shall contain as many decimal figures as both factors.*

PROBLEMS.

1. How many gallons of oil are there in 16 barrels, each containing 31.5 gallons? *504.*

2. In a certain manufactory 3.7 tons of coal are used each day. How much will be used in 26 days? *96.2 tons.*

3. A farmer sows 1.75 bushels of wheat to the acre. How much seed will he require to sow 19 acres? *33.25 bushels.*

(4)	(5)	(6)	(7)
33.125 miles	14.6	27.31	24753
27	2.7	4.5	3.16

8. Bought 137.5 acres of land, at $76.25 per acre. What did the whole cost? *$10484.375.*

9. If 62.5 tons of iron be required for the track of one mile of railroad, how much iron will it take for 371.75 miles?

10. One pound English money is worth $4.84 United States money. What is the value in United States money of 16.87 pounds English money? *$81.6508.*

11. What was the length of an army wagon-train that passed a given point in 4.08 hours, passing at the rate of 3.025 miles per hour? *12.342 miles.*

12. A mowing-machine company bought 137.5 tons of iron, at $32.75 a ton. To what did the purchase amount? *$4503.125.*

13. Brass is .8 copper and .2 zinc. How much copper and how much zinc must be used to make .875 of a ton of brass?
Copper, .7 of a ton; zinc, .175 of a ton.

14. A man owning .8 of a mill, sold .3125 of his share. What part of the mill did he sell? *.25 of it.*

15. If the length of a military step is 2.25 feet, how far will a soldier march in taking 1,762 steps? *3964.5 feet.*

16. One week a butcher bought 354 lambs, at $4.7 per head. How much did they cost him?

17. If the average rate of speed of a railroad freight train, including stops, is 11.88 miles per hour, how far will it run in .85 of an hour?

18. Limestone loses .35 of its weight when weighed in water. If a piece of limestone weighs 17.137 ounces in air, how much less will it weigh in water? *5.99795 ounces.*

19. A cubic inch of silver weighs 6.0613 ounces, and gold weighs 1.83865 times as much as silver. What is the weight of a cubic inch of gold? *11.144609245 ounces.*

(20)	(21)	(22)	(23)
72.65	.92	.000873	.00096
.6	.37	.26	.01298

24. What is the product of .0625 and .48? *.03.*

25. What is the weight of 25.75 feet of copper pipe, if one foot weighs .875 of a pound? *9.65625 pounds.*

26. If one man can mow 1.875 acres in a day, how many acres can 13 men mow in 7.5 days? *182.8125.*

27. I made 325 gallons of cider. How much had I left, after selling 9 barrels, each containing 31.5 gallons? *41.5 gallons.*

28. On invoicing his stock, a merchant finds that he has 7 pieces of cotton goods, of 43.75 yards each, 4 pieces of 46.5 yards each, 3 pieces of 39.5 yards each, one piece of 24.375 yards, and one of 19.675 yards. How many yards has he in all? *654.8.*

29. A farmer sowed three fields to rye. The first, of 13.5 acres, yielded 23 bushels per acre; the second, of 9 acres, yielded 30.25 bushels per acre; and the third, of 11.75 acres, yielded 24.44 bushels per acre. What was the total yield? *869.92 bushels.*

30. How many bushels of oats must a livery stable keeper buy to last 11 horses 19 weeks, if he feeds to each horse 2.625 bushels a week? *548.625.*

SECTION VI.

DIVISION.

145. We have learned, in Art. **95**, that in the division of integers any quotient figure must be of the same order of units as the right-hand figure of that part of the dividend used to obtain it.

If we divide 6 tenths, or .6, by 3, the quotient is 2 tenths, or .2. If we divide 6 hundredths, or .06, by 3, the quotient is 2 hundredths, or .02. The quotient of 6 thousandths, or .006, divided by 3, is 2 thousandths, or .002, and the quotient of .0006, divided by 3, is .0002. In other words,

(1)	(2)
$\underline{.6}\,\lfloor 3$	$\underline{.06}\,\lfloor 3$
$.2$	$.02$

(3)	(4)
$\underline{.006}\,\lfloor 3$	$\underline{.0006}\,\lfloor 3$
$.002$	$.0002$

When tenths are divided by an integer, the quotient is tenths;
" hundredths " " " " hundredths;
" thousandths " " " " thousandths;
" ten-thousandths " " " " ten-thousandths;
and so on.

When the divisor is an integer, any quotient figure will be of the same order of units, integral or decimal, as the right-hand figure of the partial dividend used to obtain it.

146. Ex. Divide 16.285 by 5.

SOLUTION.

$$\underline{16.285}\,\lfloor 5$$
$$3.257$$

EXPLANATION.—We write the terms, and commence at the left to divide, as in integers. Since the first partial dividend, 16, is ones, the first quotient figure, 3, must be ones, and the next quotient figure will be tenths. We therefore place the decimal point after the 3 ones, before writing any of the other figures of the quotient.

The decimal point must always be placed in the quotient, before writing the tenths' figure.

147. Ex. 1. Divide .0056 by 4.

SOLUTION.

$$\begin{array}{l|l} .0056 & 4 \\ \hline .0014 & \end{array}$$

EXPLANATION. — Since 4 is contained in 0 tenths 0 times, and in 0 hundredths 0 times, we write ciphers in the places of tenths and hundredths after the decimal point in the quotient.

Ex. 2. Divide .1272 by 8.

SOLUTION.

$$\begin{array}{l|l} .1272 & 8 \\ \hline .0159 & \end{array}$$

EXPLANATION. — Since 8 is contained in 1 tenth 0 times, we write a cipher in place of tenths after the decimal point in the quotient. Hence,

When the first decimal figure or figures of the dividend will not contain the divisor, a decimal cipher or ciphers must be written in the quotient.

148. Ex. 1. Divide 12.6 by 24.

SOLUTION.

$$\begin{array}{r|l} 12.6 & 24 \\ \cline{2-2} \underline{12\,0} & .525 \\ 6\,0 & \\ \underline{4\,8} & \\ 12\,0 & \\ \underline{12\,0} & \end{array}$$

EXPLANATION. — 12.6 ÷ 24 = .5, with a remainder of 6 tenths. 6 tenths = 60 hundredths (see **129**), and 60 hundredths ÷ 24 = 2 hundredths, with a remainder of 12 hundredths. 12 hundredths = 120 thousandths (see **129**), and 120 thousandths ÷ 24 = 5 thousandths. Hence,

When there is a remainder after using all the figures of the dividend, the division may be continued, each new partial dividend being formed by annexing a decimal cipher.

Ex. 2. Divide 31.5 by 8.

SOLUTION.

$$\begin{array}{l|l} 31.5 & 8 \\ \hline 3.9375 & \end{array}$$

EXPLANATION.—In this Solution we form each partial dividend after the second, by mentally annexing a decimal cipher to the partial remainder.

PROBLEMS.

1. A father divided 280.5 acres of land equally among 3 sons. How much land did each receive? *93.5 acres.*

2. In how many weeks will a man whose wages are $9 a week, earn $157.5? *17.5.*

3. If a ditcher digs 8 rods of ditch in one day, how long will it take him to dig 118 rods? *14.75 days.*

4. A farmer made 45 barrels of cider from 292.5 bushels of apples. How many apples did it take for a barrel of cider?

(5)	(6)	(7)	(8)
209.58 ⌊ 6	\$7209 ⌊ \$8	341.5 ⌊ 77	.1537 ⌊ 29

9. If 18 silver spoons weigh 33.75 ounces, what is the weight of 1 spoon? *1.875 ounces.*

10. If one sheet of paper makes 48 pages of a book, how many sheets will it take for a book of 348 pages? *7.25.*

11. A tailor cut 6 coats from 13.75 yards of broadcloth. How much cloth did he put into a coat?

149. 2 is contained in 6, 3 times; 2 tens in 6 tens, 3 times; 2 hundreds in 6 hundreds, 3 times; and so on. So, also, 2 tenths, or .2, is contained in 6 tenths, or .6, 3 times; 2 hundredths, or .02, is contained in 6 hundredths, or .06, 3 times; 2 thousandths, or .002, is contained in 6 thousandths, or .006, 3 times; and so on. That is,

When the divisor and dividend are of the same order of units, either integral or decimal, the quotient is ones.

This truth is shown in the following examples:

(Ex. 1)	(Ex. 2)	(Ex. 3)	(Ex. 4)	(Ex. 5)	(Ex. 6)
600 ⌊ *200*	*60* ⌊ *20*	*6* ⌊ *2*	*.6* ⌊ *.2*	*.06* ⌊ *.02*	*.006* ⌊ *.002*
3	*3*	*3*	*3*	*3*	*3*

150. Ex. 1. Divide 36.45 by .15.

SOLUTION.

$$\begin{array}{r|l} 36.45 & .15 \\ \underline{30} & \overline{243} \\ 6\,4 & \\ \underline{6\,0} & \\ 45 & \\ \underline{45} & \end{array}$$

EXPLANATION.—Since the right-hand figure of both divisor and dividend is of the same order of units (hundredths), the right-hand figure of the quotient must be ones, and consequently the entire quotient is an integer.

Ex. 2. Divide 2.68 by .25.

PARTIAL SOLUTION.

$$\begin{array}{l|l} 2.68 & .25 \\ \underline{25} & \overline{10} \\ 18 & \end{array}$$

SOLUTION.

$$\begin{array}{l|l} 2.68 & .25 \\ \underline{25} & \overline{10.72} \\ 180 & \\ \underline{175} & \\ 50 & \\ \underline{50} & \end{array}$$

Explanation.—The right-hand figure of both divisor and dividend being hundredths, the 10 of the quotient is an integer, as shown in the Partial Solution. But since there is a remainder, we place a decimal point after the 10, and continue the division by annexing decimal ciphers to the partial remainders. (See **148**).

Ex. 3. Divide 15.695 by 7.3.

SOLUTION.

$$\begin{array}{l|l} 15.6^{\vee}95 & 7.3 \\ \underline{14\,6} & \overline{2.15} \\ 1\,09 & \\ \underline{73} & \\ 365 & \\ \underline{365} & \end{array}$$

Explanation. — The quotient of 15.6 (the first three figures of the dividend) divided by 7.3, is an integer, because the right-hand figure of each term is tenths. We therefore place an inverted caret ($^{\vee}$) after the 6 of the dividend, to show what figures are used to obtain the integral part of the quotient. Placing the decimal point after the quotient figure, 2, we complete the division and obtain the entire quotient, 2.15.

The number of decimal figures in the quotient will equal the number of decimal figures left in the dividend, after taking from it as many decimal figures as there are decimal figures in the divisor.

Note.—When decimal ciphers are annexed to form partial dividends, they must be counted as decimal figures of the dividend.

151. Ex. Divide 42 by .56.

SOLUTION.

$$\begin{array}{l|l} 42.00 & .56 \\ \underline{39\,2} & \overline{75} \\ 2\,80 & \\ \underline{2\,80} & \end{array}$$

Explanation.—The right-hand figure of the divisor is hundredths; and as the right-hand figure of the dividend must also be hundredths to obtain ones for the quotient (see **149**), we annex two decimal ciphers to the dividend before dividing.

The dividend must contain at least as many decimal figures as the divisor.

152. Upon the principles deduced in Art. **145**, **147**, **148**, **150**, is based the

Rule for Division of Decimals.

I. When the divisor is an integer.

1. *If necessary, annex decimal ciphers to the dividend, till the figures of the dividend will contain the divisor.*

2. *Divide as in integers.*

3. *Place the decimal point in the quotient so that it shall contain as many decimal figures as the dividend.*

II. When the divisor is a decimal or a mixed number.

1. *Place an inverted caret after the figure of the dividend that is of the same order of units as the right-hand figure of the divisor.*

2. *Divide as in integers, and place the decimal point so that the quotient shall contain as many decimal figures as there are decimal figures at the right of the inverted caret in the dividend.*

PROBLEMS.

12. How many dress patterns, of 11.5 yards each, are there in a piece containing 46 yards of delaine? *4.*

13. If one length of 6-inch stove pipe can be made from 3.14 pounds of Russia iron, how many lengths can be made from 72.22 pounds? *23.*

(14)	(15)	(16)	(17)
75.6 ⌊.9	21.25 ⌊.4	99 ⌊24.75	3.985 ⌊159.4

18. How many casks, each holding 41.315 gallons, will be required to hold 11278.995 gallons of alcohol? *273.*

19. A merchant exchanged 35.0625 yards of cloth for wood, at the rate of 4.125 yards for 1 cord. How much wood did he receive? *8.5 cords.*

20. A miller received $3,009 for ship-stuffs, at $21.25 per ton. How many tons did he sell? *141.6.*

21. How long will it take to manufacture 1321.65 barrels of flour, at the rate of 53.4 barrels per day? *24.75 days.*

22. At Catskill, N.Y., on the 26th of July, 1860, the extraordinary fall of 18 inches of rain occurred in 7.5 hours. What was the average fall per hour? *2.4 inches.*

23. If 7 men cradle 116.55 acres of grain in 4.5 days, how many acres does 1 man cradle in 1 day? See Manual. *3.7.*

24. The winter term of a country school continued 13 weeks of 5.5 days each, and the aggregate attendance for the whole term was 3074.5 days. What was the average daily attendance? *43.*

153. Ex. How many goblets, each weighing 7.5 ounces, can a manufacturer make from 176 ounces of silver?

EXPLANATION.—Since he will not make the decimal part of a goblet, the result in this problem will be an integer; and the solution is complete when the ones of the quotient are obtained. Since the 260 is tenths, the 35 is tenths, and the true remainder is 3.5. Hence,

SOLUTION.

$$\begin{array}{r|l} 176 & 7.5 \\ \underline{150} & \overline{23} \\ 260 & \\ \underline{225} & \\ 3.5 & \end{array}$$

The right-hand figure of any remainder will always be of the same order of units, integral or decimal, as the last figure of the dividend used to obtain it.

PROBLEMS.

25. Into how many building lots, each containing .375 of an acre, can 5 acres of land be divided? *13, with .125 of an acre left.*

26. An oil refiner has on hand 22,240 gallons of oil. How many casks can he fill, if he puts 42.5 gallons in each cask? *523, and have 12.5 gallons left.*

27. A forwarder has 2,150 tons of freight to ship by canal. If 110.5 tons make one boat-load, how many boat-loads has he? *50.5 tons more than 19 boat-loads.*

28. If a teamster draws 1.125 cords of wood at a load, how many loads will 41.75 cords make? *37, and .125 of a cord more.*

29. How many potash kettles, each weighing 362.5 pounds, can be made from 20,500 pounds of iron? *Remainder, 200 pounds.*

UNITED STATES OF AMERICA
ONE DOL.
3 DOLLS 1854
UNITED STATES OF AMERICA TEN D.
UNITED STATES OF AMERICA TWENTY D.
UNITED STATES OF AMERICA 50 D C
2½ D
FIVE D.
ONE CENT
2 CENTS
5 CENTS
1 DOLLAR

SECTION VII.

UNITED STATES MONEY.

154. United States Money, or *Federal Money,* consists of dollars, cents, and mills.

10 mills are 1 cent.
100 cents are 1 dollar.

1 dollar is 100 cents.
1 cent is 10 mills.

The unit of United States Money is the dollar.

155. Since 100 cents are 1 dollar, 1 cent is 1 hundredth of a dollar; and since 10 mills are 1 cent, 1 mill is 1 tenth of a cent, or 1 thousandth of a dollar. Hence,

Cents may always be written as hundredths, and mills as thousandths of a dollar.

1 cent is written *$.01.*
47 cents are written *$.47.*
1 mill is written *$.001.*
50 cents 4 mills are written *$.504.*
25 dollars 5 cents 8 mills are written *$25.058.*
100 dollars 5 mills are written *$100.005.*

Note.—The Table of United States Money, as established by Act of Congress, August 8, 1786, is as follows:

10 mills are 1 cent;
10 cents " 1 dime;
10 dimes " 1 dollar;
10 dollars " 1 eagle.

But dimes are always read as tens of cents, and eagles as tens of dollars. Thus, 7 eagles 2 dollars 4 dimes 5 cents is $72.45, and is read "72 dollars 45 cents," or "72 and 45 hundredths dollars."

EXERCISES.

1. Read $.06, $.44, $.80, $3.15, $70.40, $9.08.
2. Read $.005, $.456, $.047, $.192, $.601, $.309.
3. Read $19.476, $500.104, $1.008, $297.027.
4. Write 20 cents; 5 cents; 93 cents.
5. Write 10 dollars 50 cents; 150 dollars 88 cents.
6. Write 4 mills; 26 cents 9 mills; 5 cents 3 mills.
7. Write 5 dollars 17 cents 5 mills.
8. Write 200 dollars 4 cents 8 mills.
9. Write 30 dollars 6 mills.

156. *Decimal parts of a dollar less than mills or thousandths are read as decimals of a mill.*

$.0005 is 5 tenths of a mill.
$.00025 is 25 hundredths of a mill.
$.0064 is 6 and 4 tenths mills.
$.3765 is 37 cents 6 and 5 tenths mills.
$45.40375 is 45 dollars 40 cents 3 and 75 hundredths mills.

EXERCISES.

10. Read $.0004, $.0056, $.00075, $.3715.
11. Read $.47675, $93.7564, $300.85354.
12. Write 5 tenths of a mill; 75 hundredths of a mill.
13. Write 5 and 8 tenths mills; 4 cents 2 and 9 tenths mills.
14. Write 56 cents 4 and 72 hundredths mills.
15. Write 8 dollars 10 cents 1 and 38 hundredths mills.

157. A *Coin* is a piece of metal on which certain characters are stamped, by authority of the General Government, making it legally current as money.

United States coins are made of gold, silver, nickel, and copper, as shown in the

COIN TABLE.

METALS.	NAMES OF COINS.	VALUES.	METALS.	NAMES OF COINS.	VALUES.
Gold,	50-dollar piece,	$50.00	Silver,	Dollar,	$1.00
	Double eagle,	20.00		Half-dollar,	.50
	Eagle,	10.00		Quarter-dollar,	.25
	Half-eagle,	5.00		Dime,	.10
	3-dollar piece,	3.00		Half-dime,	.05
	Quarter-eagle	2.50		3-cent piece,	.03
	Dollar,	1.00	Nickel,	5-cent piece,	.05
Copper,	2-cent piece,	.02		3-cent piece,	.03
	Cent,	.01		Cent,	.01

NOTES.—1. Gold coins of the values of $.50 and $.25 were coined by private assayers in California, the former in the years 1852–53, and the latter in 1854.

2. Half-cent copper coins have not been coined since the first issue of the nickel cent in the year 1857.

3. 3-cent pieces of copper and nickel were first issued in the year 1865, and 5-cent pieces of the same metals in 1866.

4. The 50-dollar piece shown in the cut, page 95, is about .8 as large across as the real coin; the other coins shown in the cut are full size.

See Manual.

158. *Alloy* is a baser metal mixed with a finer; as silver with gold, or copper with silver.

159. The United States gold and silver coins consist of 9 parts or .9 by weight of pure metal, and 1 part or .1 of alloy; the alloy of gold coins being equal parts by weight of silver and copper, and that of silver coins pure copper.

Nickel and copper coins are not alloyed. See Manual.

160. In final results of computations, and in business transactions

$.005	are written	$.00½,	and read	one half	cent.
$.0025	"	$.00¼,	"	one fourth	"
$.0075	"	$.00¾,	"	three fourths	"
$.00125	"	$.00⅛,	"	one eighth	"
$.00375	"	$.00⅜,	"	three eighths	"
$.00625	"	$.00⅝,	"	five eighths	"
$.00875	"	$.00⅞,	"	seven eighths	"

See Manual.

COMPUTATIONS OF U. S. MONEY.

161. Since the dollar is the unit of United States Money, (see **154**), and cents, mills, and parts of a mill are decimals of a dollar, it follows that

United States Money is added, subtracted, multiplied, and divided in the same manner as other decimals.

162. Ex. 1. What is the sum of \$275.10, \$18.37½, \$.883, and \$31?

EXPLANATION.—We write the numbers with dollars under dollars, cents under cents, and mills under mills; and then add the parts, and place the decimal point in the sum, as in Addition of Decimals.

SOLUTION.

$$\begin{array}{r} \$275.10 \\ 18.375 \\ .883 \\ 31 \\ \hline \$325.358 \end{array}$$

Ex. 2. From \$52.75 subtract \$10.96¾.

EXPLANATION. — We write dollars under dollars, and cents under cents; and then subtract, and place the decimal point in the remainder, as in Subtraction of Decimals.

SOLUTION.

$$\begin{array}{r} \$52.75 \\ 10.9675 \\ \hline \$41.7825 \end{array}$$

Ex. 3. Multiply \$45.625 by 5.6.

EXPLANATION. — We write the multiplier under the multiplicand; and then multiply, and place the decimal point in the product, as in Multiplication of Decimals. Omitting two decimal ciphers from the right of the product, (see **133**, VI.), the required product is \$255.50.

SOLUTION.

$$\begin{array}{r} \$45.625 \\ 5.6 \\ \hline 273750 \\ 228125 \\ \hline \$255.5000 \end{array}$$

Ex. 4. Divide \$1445.25 by 82.

EXPLANATION. — We write the divisor at the right of the dividend; and then divide, and place the decimal point in the quotient, as in Division of Decimals.

SOLUTION.

$$\begin{array}{l|l} \$1445.25 & \underline{82} \\ \underline{82} & \$17.625 \\ 625 & \\ \underline{574} & \\ 512 & \\ \underline{492} & \\ 205 & \\ \underline{164} & \\ 410 & \\ \underline{410} & \end{array}$$

Ex. 5. How many times are $5.06¼ contained in $567?

SOLUTION.

$$\begin{array}{r|l} \$567.0000 & \$5.0625 \\ \underline{50625} & 112 \\ 60750 & \\ \underline{50625} & \\ 101250 & \\ \underline{101250} & \end{array}$$

EXPLANATION.—We divide as in Division of Decimals. Since both dividend and divisor are concrete numbers (dollars), the quotient must be an abstract number. (See **97**).

163. In business transactions, when the mills in any *final result* are 5 or more, they are regarded as 1 cent; and when less than 5, they are rejected.

164. The commercial character @ signifies at, or by the yard, pound, gallon, bushel, or other unit named in the problem. Thus, "2 dozen eggs, @ $.28," signifies "2 dozen eggs, at $.28 a dozen."

PROBLEMS.

1. Martha paid $.87½ for a grammar, $.25 for a slate, $.75 for a reader, and $.12½ for a writing-book. What was the amount of her purchases? *$2.*

2. A farmer killed an ox, and sold the four quarters for $9.935, $9.62½, $8.11, and $8; the hide for $6.89; and the tallow for $8.92. How much did the ox bring him? *$51.48.*

3. One year a gentleman's income tax was $34.26; his state tax was $42.11; village tax, $18.04; school tax, $7.65; road tax, $.62½; and military tax, $1. What amount of taxes did he pay that year? *$103.68½.*

4. A owes to B, $374; to C, $47.50; to D, $193.1875; to E, $21.81; to F, $6.75; to G, $3.125; and to H, $11.0625. What is the amount of his indebtedness? *$657.435.*

What is the amount of each of the following bills?

5. *For Furniture.*	
1 Set of Chairs,	$7.50
1 Table, . . .	4.75
1 Rocking-chair,	3.25
1 Bedstead, . .	7.25
1 Mirror, . .	1.375

6. *For Hardware.*	
1 Plow, . .	$6.75
1 Spade, .	1.125
1 Hammer, .	.5625
1 Pitchfork,	.875
Nails, . .	1.375

7. *Traveling Expenses.*	
Railroad fare,	$18.625
Steamboat "	7.25
Carriage hire,	5.00
Hotel bills, .	31.875
Other expenses,	17.67

8. A man having $306.82 in bank, draws out $66.29. What is then his balance in bank? *$240.53.*

	(9)	(10)	(11)	(12)
From	$593.625	$1132.653	$251.	$1574
take	496.54	96	8.375	.856

13. A man gave me his note for $75, and he has since paid all but $24.50 of it. How much has he paid on the note? *$50.50.*

14. A dress-maker earned $34 in a month, and her expenses were $26.67. How much did she save? *$7.33.*

15. A grain buyer purchased a lot of wheat for $1078.25, and the following week sold it for $1219.125. How much did he clear on the wheat? *$140.87½.*

16. I paid $2841.375 for an interest in an iron-foundery, and afterward sold it for $3129.16. How much was my gain?

	(17)	(18)	(19)	(20)
Multiply	$21.25	$2.4375	$6.80	$4.625
by	46	.215	12.5	11.25

21. How much will 24 bushels of turnips come to, at $.375 a bushel? *$9.*

22. A lady bought 32 yards of carpeting, @ $1.12½. How much did it cost her? *$36.*

23. How much must I pay for 23 rolls of paper-hangings, @ $.375? *$8.62½.*

24. A farmer sold 396 pounds of wool, @ $.675. How much did it come to? *$267.30.*

25. A mechanic worked 4.9 days, for $1.875 per day. How much did he earn? *$9.18¾.*

26. If the interest on $1 for 1 year is $.07, what is the interest on $24.75? *$1.73¼.*

27. A drover bought a flock of 123 sheep, @ $2.5625. What was the cost of the flock? *$315.18¾.*

28. A milk-man's sales average 219 quarts a day, at $.05 a quart. What are his daily receipts?

29. How much do his sales amount to in a year, or 365 days? *$3996.75.*

30. I paid $19.9375 for iron castings, at $.0625 a pound. How many pounds of castings did I buy? *319.*

(31)	(32)	(33)	(34)
$5.25 ⌊ 14	$362.25 ⌊ 23	$180.40 ⌊ $.1025	$63 ⌊ 56

35. A shipment of 1,583 bushels of corn was sold for $890.43¾. What was the price per bushel? *$.56¼.*

36. A tanner paid $156.82½ for 25.5 cords of hemlock bark. How much was that a cord? *$6.15.*

37. Divide $6 into 150 equal parts.

38. How many bushels of potatoes can be bought for $57.3125, at $.875 per bushel? *65.5.*

39. A builder contracted to put up a brick dwelling for $3,725. The building materials cost him $1641.0625, and he paid out for labor $1296.50. Did he make or lose money on the contract, and how much? *He made $787.43¾.*

40. A man exchanged a horse worth $187.50, and a watch worth $64.875, for a span of horses worth $310, paying the balance in money. How much money did he pay? *$57.625.*

41. A farmer carried some pork to market, which he sold for $57.62½, and some poultry, which brought him $4.18¾. He paid out $13.50 for a coat, $4.48 for some groceries, and $29.74 for a bill of hardware. How much money had he left? *$14.09¼.*

42. A young man bought a farm of 84 acres, at $75 an acre, and made a cash payment on it of $1,750. How much did he run in debt? *$4,550.*

43. A man sold a quarter of beef, which weighed 156 pounds, at $.08½ per pound, and expended the money for nails, at $.05⅔ per pound. How many nails did he purchase? *234 pounds.*

44. The salary of the President of the United States is $50,000. How much is that per day, allowing 365 days to the year, and deducting the Sundays?

45. I bought a lot of teas for $376.75, and paid $31.18¾ for transportation on them. For how much must I sell them to make $103.12½?

46. A druggist bought 7 barrels of turpentine, each containing 31.5 gallons, at $1.37½ per gallon. What did the whole cost?

What is the sum of each of the following abstracts of business?

(47)

BOOT AND SHOE TRADE.
ONE DAY'S SALES.

1	Pair Calf Boots,	$6.50
1	" Stoga,	4.75
1	" Coarse,	2.25
1	" Ladies' Gaiters,	2.75
1	" Misses' "	1.75
1	" " Slippers,	1.25
1	" Children's Shoes,	.5625
1	" Gents' Slippers,	2.25
1	" Ladies' Rubbers,	.875
1	" Boys' Boots,	2.1875
1	" Misses' Rubbers,	.75
Repairing,		3.83

(48)

DRY GOODS TRADE.
ONE WEEK'S SALES.

Monday,	Cash,	$39.24
	Credit,	23.19
Tuesday,	Cash,	61.73
	Credit,	12.48
Wednesday,	Cash,	71.04
	Credit,	56.31
Thursday,	Cash,	58.98
	Credit,	60
Friday,	Cash,	49.06
	Credit,	87.50
Saturday,	Cash,	65.81
	Credit,	129.17

49. A house agent rents 7 tenements at $1.12½ a week, 5 at $1.25, and 11 at $1.50. What do the rents amount to in a year, or 52 weeks? *$1592.50.*

50. A merchant bought 3 barrels of sugar, containing, respectively, 236, 249, and 261 pounds, at $.09½ per pound. What was the amount of the bill? *$70.87.*

51. If 5 tons of coal are equal to 9 cords of wood for fuel, and a family burns 31.5 cords of wood in a year, how much will they save by changing from wood to coal, when wood is worth $4.25 per cord, and coal $6.80 per ton? *$14.87½ a year.*

52. The rates of telegraphing from New York to Washington are 50 cents for the first 25 words, and 5 cents for each additional word. At these rates, what will be the cost of sending a telegram of 117 words? *$5.10.*

53. A man bought 3 80-acre lots, and 1 40-acre lot, of Government land, at $1.25 an acre. He sold one half the land at 3 times, and the balance at 4 times, its original cost. For how much did he sell the land? *$1,225.*

54. A commission-merchant in Dubuque shipped 17 tons of prairie fowls to Philadelphia, where they were sold at $.145 per pound. How much did the shipment amount to, a ton being 2,000 pounds? *$4,930.*

SECTION VIII.

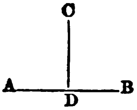
MEASUREMENT OF RIGHT-ANGLED SURFACES AND SOLIDS.

DEFINITIONS.

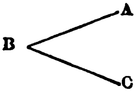
165. A *Line* is length or distance.

166. A *Straight Line* is the shortest distance between two points.

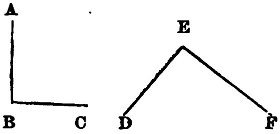
167. A *Perpendicular* is a line which stands upon another without inclining to either side. Thus, the line *CD* is perpendicular to the line *AB*.



168. An *Angle* is the difference of direction of two lines that meet in a point; as, the opening between the lines *AB* and *BC*.

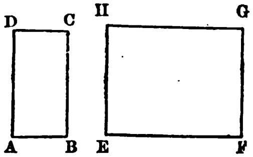


169. A *Right Angle* is one formed by two lines perpendicular to each other. Thus, the angles *ABC* and *DEF* are right angles.



170. A *Surface*, or *Superficies*, is a figure that has length and breadth.

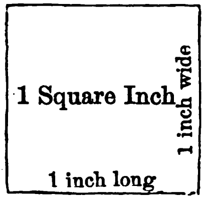
171. A *Rectangle* is a four-sided figure having only right angles. Thus, the surfaces *ABCD* and *EFGH* are rectangles.



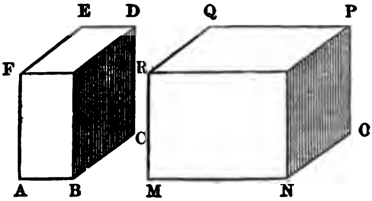
172. A *Square* is a figure bounded by four equal sides, and having four right angles.

173. A *Square Inch* is a square 1 inch long and 1 inch wide.

174. A *Square Foot* is a square 1 foot long and 1 foot wide; a *Square Rod* is a square 1 rod long and 1 rod wide; and a *Square Mile* is a square 1 mile long and 1 mile wide.

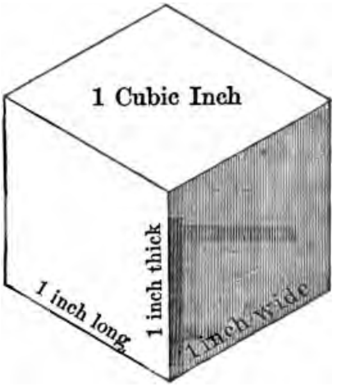


175. *Area* is the extent of any limited surface. Thus, if a figure extends over a surface of 15 square inches, its area is 15 square inches.



176. A *Solid* or *Body* is a figure that has length, breadth, and thickness.

177. A *Rectangular Solid* is a body that has six sides or surfaces, each of which is a rectangle. Thus, the solids *ABCDEF* and *MNOPQR* are rectangular solids.



178. A *Cube* is a body bounded by six equal square sides or surfaces.

179. A *Cubic Inch* is any body or portion of space 1 inch long, 1 inch wide, and 1 inch thick.

180. A *Cubic Foot* has six equal surfaces each 1 foot square; and a *Cubic Yard* has six equal surfaces each 1 yard square.

NOTES.—1. Length, width, and thickness are called *Dimensions*.

2. A line has one dimension, length; a surface has two dimensions, length and width; and a body has three dimensions, length, width, and thickness.

181. *Capacity* is the extent of any body or any portion of space having length, width, and thickness. Thus, if a body or a portion of space occupies 15 cubic feet, its capacity is 15 cubic feet.

NOTE.—Areas and capacities are also called *Contents*.

182. *Extension* is either length, area, or capacity.

Measures of Extension are of the three kinds named in the three following definitions:

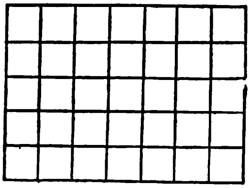
183. *Linear Measure* is the measure of lines.

184. *Superficial Measure,* or *Square Measure,* is the measure of surface.

185. *Solid Measure,* or *Cubic Measure,* is the measure of capacity.

CASE I.

Measurement of Surface.



186. Ex. How many square inches in one side of a piece of paper 7 inches long and 5 inches wide?

EXPLANATION.—If on the paper you draw lines just 1 inch apart, both lengthwise and crosswise, the surface of the paper will be divided into squares each 1 inch long and 1 inch wide. Since in each of the 5 rows there are 7 square inches, there are in all 5 times 7 square inches, or 35 square inches. We see from the figure that there are 5 rows of 7 square inches each, or 7 rows of 5 square inches each. (See **80**, V.) Hence,

The number of units in the area of any right-angled surface is equal to the number of units in the product of its two dimensions.

187. Ex. The contents of a certain field are 1,152 square rods, and the field is 36 rods long. How many rods wide is it?

SOLUTION.

$$\begin{array}{r|l} 1152 & 36 \\ \underline{108} & \overline{32} \\ 72 & \\ \underline{72} & \end{array}$$

EXPLANATION.—Since the field is 36 rods long, there are 36 square rods in a strip 1 rod wide. And since there are 1,152 square rods in the field, there must be as many strips, each 1 rod wide, as the number of times 36 square rods are contained in 1,152 square rods, which is 32. Since each strip is 1 rod wide, the lot must be 32 rods wide. Hence,

Either dimension of a right-angled surface is equal to the quotient obtained by dividing the area by the other dimension.

188. Upon the principles deduced in Arts. **186**, **187**, is based the

Rule for Measurement of Rectangular Surfaces.

I. To find the area.

Multiply the length by the breadth.

II. To find either dimension.

Divide the area by the other dimension.

PROBLEMS.

1. If upon a blackboard 19 feet long and 5 feet wide, I draw lines 1 foot apart, both lengthwise and crosswise of the board, into how many strips, lengthwise, will I divide the surface of the board? How many square feet will there be in 1 strip? How many square feet on the surface of the board? *95.*

2. How many square rods in a garden 8 rods long and 6 rods wide?

3. My slate is 12 inches long and 8.5 inches wide. How many square inches on one side? *102.*

4. How many yards of carpeting will it take to carpet a room 11.5 yards long and 5.5 yards wide? *63.25.*

NOTE.—Numbers expressing width and length are frequently written with the word "by," or the sign of Multiplication between them. Thus, 7 by 9 inches, or 7 × 9 inches, means 7 inches wide and 9 inches long.

5. How many square inches in a pane of 9 × 14 window-glass?

6. My hall is 2.75 by 8.5 yards. How much must I pay for oil-cloth to cover the floor, at $1.12½ per yard?

7. How many square feet in a city lot 75 × 125.3 feet?

8. How many square rods in a plat of ground 5.5 rods square?

9. The area of a door-way 31 inches wide is 2,604 square inches. What is its height? *84 inches.*

10. A farm in the form of a rectangle is 80 rods wide, and contains 19,200 square rods. What is its length? *240 rods.*

11. The ceiling of a room is 18 feet long, and its contents are 288 square feet. How wide is it? *16 feet.*

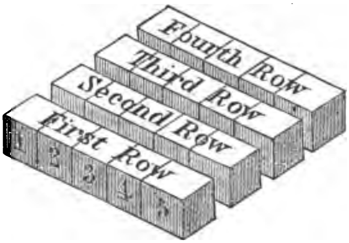
12. A carpenter put 3,375 feet of inch boards into the floor of a church 45 feet wide. What was the length of the church? *75 feet.*

13. I have 16,875 strawberry plants, in 75 equal rows. How many plants in each row? *225.*

CASE II.

Measurement of Capacity.

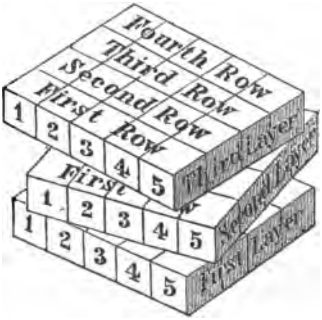
189. Ex. How many cubic feet in a block of stone 5 feet long, 4 feet wide, and 3 feet thick?



EXPLANATION.—If 5 blocks, each containing 1 cubic foot, be placed side by side, they will form a row 5 feet long, 1 foot wide, and 1 foot thick.

If 4 such rows be placed side by side, they will form a layer 5 feet long, 4 feet wide, and 1 foot thick.

If 3 such layers be placed one exactly upon the other, they will form a pile 5 feet long, 4 feet wide, and 3 feet thick. Hence,

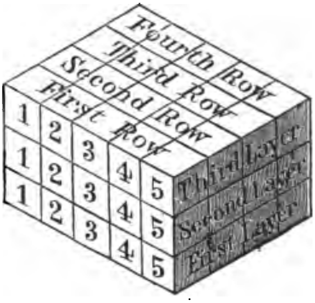


SOLUTION.

5 cubic feet.
4
20 cubic feet.
3
60 cubic feet.

4 times 5 cubic feet = 20 cubic feet, the number in 1 layer; and 3 times 20 cubic feet = 60 cubic feet, the number in the pile or in the block.

There are as many cubic feet in one row of these blocks as the pile is feet long, as many rows of blocks in one layer as the pile is feet wide, and as many layers in the pile as the pile is feet high.



The number of units in the capacity of any right-angled body or portion of space is equal to the number of units in the product of its three dimensions.

190. Ex. A paper-box maker made a package of 432 boxes, putting 8 boxes in each row, and 6 rows in each layer or tier. How many boxes high was the package?

Explanation.—Since in 1 row there were 8 boxes, in the 6 rows of 1 layer or tier there were 6 times 8 boxes, or 48 boxes. Since in the whole package there were 432 boxes, and in 1 layer 48 boxes, there were in the pile as many layers as the number of times 48 is contained in 432, which is 9. As there were 9 layers, each one box high, the package was 9 boxes high. Hence,

SOLUTION.

$$6 \times 8 = 48 \qquad \begin{array}{r|l} 432 & \underline{48} \\ \underline{432} & 9 \end{array}$$

Any one of the three dimensions of a right-angled body or portion of space is equal to the quotient obtained by dividing the capacity by the product of the other two dimensions.

191. Upon the principles in Arts. **189**, **190**, is based the

Rule for Measurement of Rectangular Solids.

I. To find the capacity.

Multiply the length, width, and thickness together.

II. To find any one of the three dimensions.

Divide the capacity by the product of the other two dimensions.

PROBLEMS.

14. A pile of bricks consists of 7 layers, and each layer contains 8 rows of 9 bricks each. How many bricks in the pile? *504.*

15. How many blocks, each measuring 1 cubic inch, can you put into a box 7 × 6 × 4 inches inside?

16. An embankment 12 feet high and 4 feet thick contains 6,000 cubic feet. How long is it? *125 feet.*

17. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord. How many cubic feet in a cord?

18. How many cubic feet in a stick of timber 35 feet long, 2 feet wide, and 1.5 feet thick? *105.*

19. The contents of a pile of wood 4 feet wide and 5 feet high are 1,280 cubic feet. What is its length? *64 feet.*

20. How many cubic yards of earth will be removed in digging a cellar 27 feet long, 24 feet wide, and 7 feet deep, there being 27 cubic feet in a cubic yard? *168.*

21. What is the capacity of a space 22.5 feet long, 6.4 feet wide, and 3.25 feet deep? *468 cubic feet.*

22. A music dealer found that a packing box that would hold a melodeon which was 18.5 inches wide and 28 25 inches high, must have a capacity of 21323.1 cubic inches. Allowing the lumber to be 1 inch thick, what were the outside dimensions of the box? *20.5 by 30.25 by 42.8 inches.*

23. In a granary is a bin 7 × 3 × 2.25 feet. What is its capacity?

24. The walls of a stone building 45 feet long and 24 feet wide, are 36 feet high and 1 foot thick. How many cubic feet of masonry in the walls, no allowance being made for openings? *4,824.*

25. How many cubic inches in a cubic foot?

26. How many cubic yards in a cubic mow of hay which measures 1 rod, or 5.5 yards, in each of its three dimensions? *166.375.*

27. Two of the dimensions of a stone column which contains 196.8 cubic feet, are 12.3 feet and 4 feet. What is the shape of one end of the column? *It is square.*

SECTION IX.

REVIEW PROBLEMS IN DECIMALS.

1. A contractor employed 37 laborers 56 days, 13 laborers 84 days, 12 laborers 43 days, and 17 laborers 21 days. What was the total amount of their wages, at $.87½ per day? *$3532.37½.*

2. A grocer buys 4 barrels of kerosene, each containing 31.5 gallons, for $55.12½, and he wishes to sell it at a profit of $.18¾ per gallon. At what price per gallon must he sell it?

3. At $7.87½ per bushel, how many bushels of grass seed can be bought for $66.93¾? *8.5.*

4. A fish dealer has 45 barrels of mackerel, which he wishes to repack into kits holding 12.5 and 25 pounds each, and to use an equal number of each size. How many kits must he use? *480.*

5. If 67.5 bushels of oats are required to feed one horse through the winter, how many horses can be wintered on 950 bushels? *14, with 5 bushels left.*

6. If in 1 hour 1,354 gallons of water run into, and 1010.8 gallons run out of, a reservoir which will hold 23,381 gallons, and the reservoir now contains 12999.2 gallons of water, in how many hours will it be full? *30.25.*

7. The terms of a weekly newspaper are, to single subscribers, $1.50; clubs of three, $3.75; clubs of five, $5.00; clubs of ten, $8.75. The paper has 694 single subscribers, 63 clubs of three, 47 clubs of five; 34 clubs of ten, and a free exchange list of 50 copies. What is the total circulation, and what are the receipts from subscriptions? *Circulation, 1,508; receipts, $1809.75.*

8. A ton of iron ore from Iron Mountain yields .56 of a ton of pure iron. How much iron will 736.72 tons of ore yield?

9. A drover bought 69 beeves at $28.75 a head, and sold 42 of them at $36.5 a head, and the rest at $37.75 a head. How much did he gain by the transaction? *$568.5.*

10. A and B start from the same place at the same time, and travel in opposite directions, A traveling at the rate of 23.16 miles per day, and B at the rate of 19.21 miles per day. How far will they be apart at the end of 17.4 days? *737.238 miles.*

11. In a school-room are 8 rows of double desks, and 7 desks in a row. How many pupils can be seated at the desks in the school-room? *112.*

12. A copper-plate engraver bought a plate of copper 16.25 by 25.2 inches, @ $.03 per square inch. How much did it cost him?

13. I bought a boat load of wood for $182, and by retailing it at $4.50 a cord, I gained $70. How many cords of wood were there in the load? *56.*

14. I borrowed some money, and paid $41.79 for the use of it, paying $.105 for the use of each dollar borrowed. How much money did I borrow? *$398.*

15. A government township of land is 6 miles square. How many square miles does it contain?

16. My garden, which is square, is inclosed by a tight board fence 8.5 feet high, and the fence contains 2,805 feet of lumber. What is the length of one side of my garden? *82.5 feet.*

17. One year a farmer's account of grain sold was as follows:

536	*bushels*	*oats------*	@	*$ .625*
319	"	*wheat----*	"	*2.125*
114	"	*barley----*	"	*1.375*
443	"	*corn------*	"	*1.125*
				$1,668

18. How many square feet in the four walls of a room 18 feet long, 13.25 feet wide, and 9.5 feet high?

19. A publisher's expenses in publishing an edition of 2,000 copies of a certain book are, for stereotyping, $515; paper, $365; binding, $370; engraving, $80. What must be the retail price of the book, that the author may receive a copyright of 8 cents per copy, the publisher's profit be 20.5 cents, and the retail bookseller's profit 30 cents per copy? *$1.25.*

20. Find the amount of six thousand one hundred nineteen millionths, four hundred eight and twenty-six thousandths, two million twenty thousand two hundred and seven hundred three ten-thousandths, and thirty thousand sixty-five hundred-millionths.

Two million twenty thousand six hundred eight and ten million two hundred seventy-one thousand nine hundred sixty-five hundred-millionths.

21. A steamship made a voyage in eight days, sailing the following distances: 217 miles, 198.85 miles, 246.7856 miles, 208 miles, 227.6987 miles, 200.045 miles, 241 miles, and 205.08675 miles. What was the length of the voyage? *1744.46605 miles.*

22. A farmer bought a yoke of oxen, which he fattened, slaughtered, and sold. The following is taken from his account of the entire transaction:

Disbursements.

Paid for 1 yoke oxen ------------------- $140.00
Cost of fattening the same ------------- 17.50 -- $157.50

Receipts.

4 hind quarters, 165, 169, 178, and 180 pounds, @ $.14
4 fore " 124, 125, 133, " 139 " " .12½
2 hides, 217 pounds --------------------- " .16½
164 pounds tallow ----------------------- " .15¼

Net profits ------------------------------- $65.32

23. From a stock of 214 tons of coal, a dealer sold in one week 6.65 tons, 9 tons, 8.775 tons, 9.27 tons, 5.45 tons, and 7.125 tons. How much coal had he left in his yard? *167.73 tons.*

24. Two men start from the same place, at the same time, and travel in the same direction, one at the rate of 23.75 miles, and the other of 19.5 miles per day. In how long a time will they be 64.6 miles apart?

25. How far apart will they be in the same time, if they travel in opposite directions? *657.4 miles.*

26. A merchant and a farmer bartered. The farmer sold to the merchant 37.25 pounds of butter @ $.27, and 21.5 dozen of eggs @ $.19; and the merchant sold to the farmer 12 yards of shirting @ $.25, and 13.5 yards of calico @ $.28. The balance was paid in money. How much was paid, and who paid it?

27. How many reams of commercial note paper each 3.5 × 5 × 8 inches in size, can be packed in a box the inside dimensions of which are 14 × 20 × 32 inches? *64.*

CHAPTER 3.
Compound Numbers.

SECTION I.

DEFINITIONS.

193. Some articles are bought and sold by the quart or gallon; some by the peck or bushel; some by the foot or yard; some by the acre; some by the cord; some by the pound or ton, and so on.

194. *Measure* is that by which extent, dimension, or quantity of matter is ascertained, whether it be length, breadth, thickness, or amount.

195. *Weight* is a measure of the amount of matter, or the quantity of heaviness, in a body.

196. Weight and measure are determined by processes called *Weighing* and *Measuring*, which consist in comparing the thing to be weighed or measured with some fixed standard. See Manual.

197. *Denomination* is the name of the unit of a concrete number; as, gallon, foot, pound, hour, dollar.

198. A *Denominate Number* is a number applied to a denomination; as, 9 quarts, 4 feet, $7.

199. A *Simple Number* is an abstract number; as, 6, 43, 915; or a concrete number of but one denomination; as, 78 men, 324 miles.

200. A *Compound Number* is a number expressed in two or more denominations; as, 4 pounds 10 ounces, 15 gallons 3 quarts 1 pint.

201. *Higher Denominations* are those which express the greater amount or quantity.

202. ***Lower Denominations*** are those which express the less amount or quantity. Thus, a peck is a higher denomination than a quart, and a lower denomination than a bushel.

NOTES.—1. A denominate number may be an integer, as 7 bushels; a decimal, as .75 of a mile; a mixed number, as 5.125 gallons; or a compound number, as 4 days 9 hours 20 minutes.

2. Compound numbers are sometimes called denominate numbers; but the term denominate number properly belongs to a concrete number of one denomination.

203. A ***Table,*** in Compound Numbers, is a regular arrangement of the denominations used to express a compound number.

204. A ***Denominate Unit*** is *one* of any denomination; as 1 pound, 1 foot, 1 quart.

205. A ***Standard Unit*** of a table is that unit which law or custom has established as the *one* from which the other denominations in the table are determined. Thus, the standard unit of length is the yard; the foot and inch being obtained by dividing this standard unit, and the rod and mile by multiplying it.

206. In computations in compound numbers, it is often necessary to change units of higher denominations to those of lower, as gallons to quarts or pints; or units of lower denominations to those of higher, as feet to rods or miles.

Reductions are processes of changing numbers from one denomination to another without changing values. They are of two kinds, *Reduction Descending* and *Reduction Ascending.*

207. ***Reduction Descending*** is the process of changing numbers from higher to lower denominations; and

208. ***Reduction Ascending*** is the process of changing numbers from lower to higher denominations.

209. The scale of decimal numbers being decimal, the successive orders of units increase and decrease uniformly by 10; while the scales of most compound numbers, being varying, the successive orders of units have no regular rate of increase and decrease. From this difference in their scales arises the *only* difference in computations in the two classes of numbers.

210. The different orders of units of a compound number, like those of an integer, increase in value from right to left, the higher denominations being written at the left.

211. The denominations of compound numbers are generally abbreviated, as shown in the tables. See Manual.

SECTION II.

NOTATION AND REDUCTIONS.

212. ***Money*** is the legal or recognized standard of value. It consists of coins, made of gold, silver, or other metal.

213. ***Treasury Notes*** are notes or bills issued by the General Government; and

214. ***Bank Notes*** or ***Bank Bills*** are notes or bills issued by a banking company.

215. ***Paper Money*** consists of notes or bills issued by the Government or a bank.

NOTES.—1. Treasury notes are payable to the bearer, at the General Treasury of the United States, at a time specified in them.

2. Bank notes are payable to the bearer, at the bank, on demand.

3. Paper money is a substitute for coin, and is regarded and circulated as money.

216. *Currency* is the coin, treasury notes, bank notes, and other substitutes for money, or recognized representatives of value, in circulation in trade and commerce.

NOTES.—1. Coin is commonly called *Specie Currency*, or *Specie;* and treasury and bank notes are called *Paper Currency.*

2. Every civilized nation has its own kind of money.

Table I.—United States Money.

217. *United States Money,* or Federal Money, is the money of the United States. Its denominations, as established by the General Government, are eagles, dollars, dimes, cents, and mills.

10 mills	are	1 cent.	1 eagle	is	10 dollars.
10 cents	"	1 dime.	1 dollar	"	10 dimes.
10 dimes	"	1 dollar.	1 dime	"	10 cents.
10 dollars	"	1 eagle.	1 cent	"	10 mills.

SCALE.—Decimal, or uniformly 10.

NOTE.—The subject of United States money has already been fully treated in Chap. II., Sec. VII. The Government table is given here, for the purpose of presenting all the tables of Compound Numbers in the same Chapter.

Table II.—Canada Money.

218. *Canada Money* is the money of Canada, or The New Dominion. Its denominations are dollars and cents.

100 cents are 1 dollar.	1 dollar is 100 cents.

SCALE.—Uniformly 100.

The coins are the 5-cent piece, 10-cent piece, and 20-cent piece.

NOTE.—In business transactions 20 cts. (cents) are called 1 s. (shilling), and 5 shillings 1 dollar.

Table III.—English Money.

219. *English Money,* or Sterling Money, is the money of Great Britain. Its denominations are pounds, shillings, pence, and farthings.

4 far., or qr. (farthings)	are	1 d. (penny.)	£1	is	20 s.
12 d. (pence)	"	1 s. (shilling.)	1 s.	"	12 d.
20 s.	"	£1 (pound.)	1 d.	"	4 far., or qr.

SCALE.—Ascending, 4, 12, 20; descending, 20, 12, 4.

Notes.—1. The abbreviation £ is always written before, and the other abbreviations are written after the numbers to which they give denomination or name. Thus, 3 pounds 5 shillings 4 pence is written £3 5 s. 4 d.

2. The coins are of gold, silver, and copper.

I. *Gold Coins.* — Sovereign = £1, half-sovereign = 10 s., guinea = 21 s. = £1 1 s., and half-guinea = 10 s. 6 d.

II. *Silver Coins.*—Crown = 5 s., half-crown = 2 s. 6 d., shilling = 12 d., and sixpence = 6 d.

III. *Copper Coins.*—Penny = 4 far., half-penny = 2 far., and farthing.

EXERCISES.

1. Read £4 7 s. 6 d.; £17 4 s. 3 d. 2 far.; 18 s. 9 d. 1 qr.
2. Write 25 pounds 10 shillings; £ eighteen three s.
3. Write 217 pounds 9 shillings 1 penny 2 farthings.
4. Write £ eight seventeen s. six d. three far.
5. Write 17 shillings 3 farthings; 11 pounds 9 pence 1 farthing.

220. Measures of Capacity are of three kinds, Liquid Measure, Dry Measure, and Cubic Measure.

Note.—Cubic Measure is also classed under Measures of Extension. (See **228.**)

Table IV.

Liquid Measure.

221. The table of liquid measure consists of the denominations gallons, quarts, pints, and gills. These denominations are used in measuring oil, molasses, syrups, wines, milk, and other liquids.

4 gi. (gills) are 1 pt. (pint.)	1 gal. is 4 qt.
2 pt. " 1 qt. (quart.)	1 qt. " 2 pt.
4 qt. " 1 gal. (gallon.)	1 pt. " 4 gi.

Scale.—Ascending and descending, 4, 2, 4.

In estimating the capacity of cisterns, reservoirs, etc.,

31.5 gal. are 1 bar., or bbl. (barrel.)
63 gal. " 1 hhd. (hogshead.)

NOTES.—1. The barrels and hogsheads used for commercial purposes are not fixed measures; the former containing from 30 to 45 gallons, and the latter from 60 to 125 gallons.

2. Physicians in prescribing, and apothecaries in mixing medicines that are liquids, divide the gallon according to the following

APOTHECARIES' FLUID MEASURES.

60 minims (or drops) are 1 fluid drachm.
8 fluid drachms " 1 fluid ounce.
16 fluid ounces " 1 pint.
8 pints " 1 gallon.

3. They also use the following measures, from vessels in common use: 4 tea-spoons are 1 table-spoon, 2 table-spoons are 1 ounce, 2 ounces are 1 wine-glass, 2 wine-glasses are 1 tea-cup, 4 tea-cups are 1 pint.

Since the vessels named are not made of uniform size, the values given must necessarily vary.

Table V.—Dry Measure.

222. The table of dry measure consists of the denominations bushels, pecks, quarts, and pints. These denominations are used in measuring grain, seeds, fruits, berries, several kinds of vegetables, lime, charcoal, and some other articles.

2 pt. are 1 qt.	1 bu. is 4 pk.
8 qt. " 1 pk. (peck.)	1 pk. " 8 qt.
4 pk. " 1 bu. (bushel.)	1 qt. " 2 pt.

SCALE—Ascending, 2, 8, 4; descending, 4, 8, 2.

NOTES.—1. Where fruit and vegetables are marketed by the basket or barrel, a peach basket should hold 16 qt. or 2 pk.; a potato basket, 24 qt. or 3 pk.; and a barrel, 3 potato baskets. Barrels made for marketing purposes commonly hold 100 qt.

2. In measuring grain, seeds, peas, beans, and small fruits, the measure must be *even* full. But in measuring large fruits, corn in the ear, coarse vegetables, and other bulky articles, the measure must be *heaping* full. 4 heaped measures must equal 5 even measures. See Manual.

EXERCISES.

6. Read 4 gal. 3 qt. 1 pt. 2 gi.; 15 gal. 1 pt.
7. Read 11 bu. 3 pk. 5 qt. 1 pt.; 8 bu. 7 qt.
8. Write 17 gallons 1 quart 1 pint 3 gills.
9. Write 260 bushels 3 pecks 4 quarts 1.5 pints.
10. Write four gal. three qt. two gi.; 3 bar. 15.5 gal.
11. Write eleven bu. one pk. six and seven-tenths qt.
12. Read 13.125 bu.; 41.75 gal.; .1 pk. 4.375 qt.; 1 gal. 1.8 pt.

REDUCTION DESCENDING.

223. Ex. 1. How many pints are 17 gallons?

EXPLANATION.—Since 17 gal. are 17 times 1 gal., and 1 gal. is 4 qt., 17 gal. are 17 times 4 qt., or 68 qt.; and since 68 qt. are 68 times 1 qt., and 1 qt. is 2 pt., 68 qt. are 68 times 2 pt., or 136 pt.

SOLUTION.

$$\begin{array}{r} 17\ \textit{gal.} \\ \underline{4\phantom{\ \textit{gal.}}} \\ 68\ \textit{qt.} \\ \underline{2\phantom{\ \textit{qt.}}} \\ 136\ \textit{pt.} \end{array}$$

Hence, *17 gal. = 136 pt.*

Ex. 2. How many quarts are 8 bu. 3 pk. 4 qt.?

EXPLANATION.—Since 8 bu. are 8 times 1 bu., and 1 bu. is 4 pk., 8 bu. are 8 times 4 pk., or 32 pk., and 32 pk. + 3 pk. are 35 pk. Since 35 pk. are 35 times 1 pk., and 1 pk. is 8 qt., 35 pk. are 35 times 8 qt., or 280 qt., and 280 qt. + 4 qt. are 284 qt.

FULL SOLUTION.

$$\begin{array}{l} 8\ \textit{bu.}\ 3\ \textit{pk.}\ 4\ \textit{qt.} \\ \underline{4} \\ 32 + 3 = 35\ \textit{pk.} \\ \underline{8} \\ 280 + 4 = 284\ \textit{qt.} \end{array}$$

Hence, *8 bu. 3 pk. 4 qt. = 284 qt.*

In the Common Solution we mentally added the 3 pecks with the pecks of the partial result, and the 4 quarts with the 40 quarts of the final result. This manner of solution is the one in common use.

COMMON SOLUTION.

$$\begin{array}{r} 8\ \textit{bu.}\ 3\ \textit{pk.}\ 4\ \textit{qt.} \\ \underline{4}\phantom{\ \textit{bu.}\ 3\ \textit{pk.}\ 4\ \textit{qt.}} \\ 35\ \textit{pk.}\phantom{\ 3\ \textit{pk.}\ 4\ \textit{qt.}} \\ \underline{8}\phantom{\ \textit{bu.}\ 3\ \textit{pk.}\ 4\ \textit{qt.}} \\ 284\ \textit{qt.}\phantom{\ 3\ \textit{pk.}\ 4\ \textit{qt.}} \end{array}$$

In Ex. 1 we reduced gallons to quarts, and quarts to pints—that is, a higher to a lower denomination—by multiplication ; and in Ex. 2 we reduced bushels and pecks to quarts—that is, higher denominations to a lower denomination—by multiplication. Hence,

A denominate number is reduced to lower denominations by multiplication.

PROBLEMS.

1. Reduce £32 to shillings. To pence. *7,680 d.*

2. How many mills are there in 249 dollars 43 cents ?

3. In $93 4 s. 17 cts., Canada currency, how many cents ? *9,397.*

4. £45 8 s. 6 d. are how many farthings ? *43,608.*

5. At 1 s. a yard, how many yards of sheeting can be bought for £11 10 s. ? *230.*

6. Reduce 9 s. 7.25 d. to farthings.

7. How many pence in £231 8 d. ?

8. How many gills are there in 15 gallons ?

9. Reduce 27 gal. 3 qt. 2 gi. to gills.

10. A grocer has 87 gal. 1.5 qt. of cider which he wishes to put into pint bottles. How many bottles must he use ? *699.*

SOLUTION OF PROBLEM 7.

£231 8 d.
20
4620 s.
12
55448 d.

11. 18 bushels are how many quarts ?

12. Two brothers picked 6.375 bushels of berries. How many quarts of berries had they ? *204.*

13. A seedsman put up 2 bu. 4 qt. of marrowfat peas in pint papers. How many papers did he fill ?

14. Reduce 126 bu. 1 pt. to pints.

15. How many quart boxes will be required to hold 7 bu. 3 pk. 7 qt. of strawberries ? *255.*

16. £6.5 are how many shillings ? How many pence ?

17. A dealer in findings sold 13 bu. 3 pk. 6 qt. of shoe-pegs by the quart. How many quarts did he sell ?

SOLUTION OF PROBLEM 14.

126 bu. 1 pt.
4
504 pk.
8
4032 qt.
2
8065 pt.

REDUCTION ASCENDING.

224. Ex. 1. How many bushels are 192 quarts?

EXPLANATION.—Since every 8 qt. are 1 pk., and 8 qt. are contained in 192 qt. 24 times, 192 qt. are 24 pk. And since every 4 pk. are 1 bu., and 4 pk. are contained in 24 pk. 6 times, 24 pk. are 6 bu.

SOLUTION.

$\underline{192\text{ qt.}}\;\lfloor 8\text{ qt.}$

24 times.

192 qt. = 24 pk.

$\underline{24\text{ pk.}}\;\lfloor 4\text{ pk.}$

6 times.

24 pk. = 6 bu.

Hence, *192 qt. = 6 bu.*

Ex. 2. How many bushels are 637 quarts?

EXPLANATION.—Since every 8 qt. are 1 pk., and 8 qt. are contained in 637 qt. 79 times with a remainder of 5 qt., 637 qt. are 79 pk. 5 qt. And since every 4 pk. are 1 bu., and 4 pk. are contained in 79 pk. 19 times with a remainder of 3 pk., 79 pk. are 19 bu. 3 pk.

FULL SOLUTION.

$\underline{637\text{ qt.}}\;\lfloor 8\text{ qt.}$

79 times and 5 qt. rem.

637 qt. = 79 pk. 5 qt.

$\underline{79\text{ pk.}}\;\lfloor 4\text{ pk.}$

19 times and 3 pk. rem.

79 pk. = 19 bu. 3 pk.

Hence, *637 qt. = 19 bu. 3 pk. 5 qt.*

In the Full Solution we have written all the numbers mentioned in the explanation; but in the Common Solution we have written only the denominate numbers. This manner of solution is the one in common use.

COMMON SOLUTION.

$\underline{637\text{ qt.}}\;\lfloor 8\text{ qt.}$

$\underline{79\text{ pk. }5\text{ qt.}}\;\lfloor 4\text{ pk.}$

19 bu. 3 pk.

Hence, *637 qt. = 19 bu. 3 pk. 5 qt.*

In Ex. 1 we reduced quarts to pecks, and pecks to bushels—that is, a lower to a higher denomination—by division; and in Ex. 2 we reduced quarts to bushels, pecks, and quarts—that is, a lower denomination to higher denominations—by division. Hence,

A denominate number is reduced to higher denominations by division.

PROBLEMS.

18. Reduce 256,327 mills to dollars, cents, and mills.
256 dollars 32 cents 7 mills.

19. 1680 d. are how many £?

20. How much will 765 bushels of lime cost at 1 s. a bushel?

21. How many gallons are 486 pints? *60.75, or 60 gal. 3 qt.*

22. How many times can you fill a gallon measure from 1,024 gills of alcohol?

23. Reduce 733 pints dry measure to higher denominations.
11 bu. 1 pk. 6 qt. 1 pt.

24. How many bushels are 7,280 quarts of shelled corn?
227.5, or 227 bu. 2 pk.

25. One season a gardener sold 3,975 quart boxes of strawberries. How many bushels did he sell?

26. How many gallons of sweet-oil will an apothecary use in filling 500 gill bottles?
15.625, or 15 gal. 2 qt. 1 pt.

SOLUTION OF PROBLEM 25.

3 9 7 5 qt. ⌊ *8 qt.*
4 9 6 pk. 7 qt. ⌊ *4 pk.*
1 2 4 bu.

Hence, *3975 qt. = 124 bu. 7 qt.*

27. A grocer buys 187 qt. of walnuts. How many bushels does he buy?

28. A cask is emptied in 892 minutes, by a pipe which discharges 1 pint of water per minute. What is the capacity of the cask?

225. Upon the principles deduced in Arts. **223**, **224**, is based the

Rules for Reductions of Compound Numbers.

I. For Reduction Descending.

1. *Multiply the number of the highest denomination given, whether integer, decimal, or mixed number, by that number of the next lower denomination which equals* 1 *of this higher, and to the product add the given lower denomination.*

2. *In the same manner, reduce this result to the next lower denomination; and so continue until the given number is reduced to the required denomination.*

II. For Reduction Ascending.

1. *Divide the number of the given denomination, whether integer, decimal, or mixed number, by that number of this denomination which equals* 1 *of the next higher, writing the quotient as so many of the higher denomination, and the remainder as so many of the denomination divided.*

2. *In the same manner, reduce this quotient to the next higher denomination; and so continue until the given number is reduced to the required denomination.*

3. *Write the last quotient and the several remainders in their order, from the highest denomination to the lowest, for the required result.*

PROBLEMS.

29. How many quart cupfuls in 5 gal. 3 qt. of milk?

30. Reduce 4,879 far. to higher denominations. *£5 1 s. 7.75 d.*

31. A 10-qt. pail holds how many gills? How many gallons?

32. Reduce 167,824 qr. to higher denominations. *£174 16 s. 4 d.*

33. How many times can a 2-quart measure be filled from a keg which contains 7.5 gallons of vinegar?

Solve the above problem in 4 different ways.

34. Reduce .6875 of a gal. to gills. *22 gi.*

35. 6784.8 d. are how many pounds? *£28.27, or £28 5 s. 4.8 d.*

36. One week a woman who kept a fruit stand sold 19.5 bushels of peanuts by the half-pint measure. How many measurefuls did she sell?

37. Reduce 3 qt. to the decimal of a bushel. *.09375 bu.*

38. One day a hostler at a hotel stable fed out 129 half-peck measures of oats. How many bushels did he feed out?

39. In The New Dominion 3,287 cents are how many dollars, shillings, and cents? *$32 4s. 7 cts.*

40. 7.3125 s. are how many farthings? *351.*

41. A lady made 3 gal. 2 qt. of strawberry wine, which she put into pint bottles. How many bottles of wine had she?

42. A housekeeper filled 549 quart cans with cherries. How many cherries did she use? *17 bu. 5 qt.*

43. If I measure 3 bu. 1 pk. 5 qt. of walnuts in a quart measure, how many times will I fill the measure?

44. Reduce 2 hhd. 1 bbl. 15.25 gal. to quarts.

45. How many bushels of potatoes in 12,250 quarts?

46. From a peach orchard 1500 bu. 2 pk. of peaches were sold, at an average of 75 cents a basket. How much was received for them? *$2250.75.*

Table VI.

Linear Measure.

226. The table of linear or line measure consists of the denominations miles, rods, yards, feet, and inches. These denominations are used in measuring distances, and also the dimensions of things, as their length, width, thickness, height, and depth.

12 in. (inches)	are	1 ft. (foot.)	1 mi.	is	320 rd.
3 ft.	"	1 yd. (yard.)	1 rd.	"	5.5 yd.
5.5 yd.	"	1 rd. (rod.)	1 yd.	"	3 ft.
320 rd.	"	1 mi. (mile.)	1 ft.	"	12 in.

SCALE.—Ascending, 12, 3, 5.5, 320; descending, 320, 5.5, 3, 12.

Topographical, civil, and military engineers express distances less than a mile by yards or feet.

1760 yd., or 5280 ft. are 1 mi.

In measuring goods sold by the linear yard, the yard is divided into quarters, eighths, and sixteenths.

2.25 in.		are 1 sixteenth.
2	sixteenths, or 4.5 in.	" 1 eighth.
2	eighths, or 9 in.	" 1 qr. (quarter), or 1 fourth of a yard.
4	qr.	" 1 yd.

Mariners use the following denominations :

6 feet are 1 fathom, in measuring depths at sea.
120 fathoms are 1 cable's length, for short distances.
1 nautical mile, or knot, is 1.15 common or English miles.
1 " league is 3 nautical miles, or 3.45 English miles.

In geographical and astronomical calculations,

1 geographic mi. is 1.15 statute mi.;
3 " " are 1 l. (league.)
60 " " or } " 1 deg. (degree) of latitude, or of
69.16 statute " } longitude on the equator.

NOTES.—1. The knot is used in measuring the speed of vessels.

2. The nautical mile (or knot) and league are the same as the geographic mile and league.

3. The length of a degree of latitude is not quite uniform. 69.16 miles is the average length, and is the one adopted by the U. S. Coast Survey.

4. In measuring the height of horses, 4 inches are 1 hand, the measure being taken directly over the shoulder.

5. In clock-making, 6 points are 1 line, and 12 lines are 1 inch.

6. In measuring the length of the foot, 3 barleycorns, or sizes, are 1 inch.

7. The sacred cubit, mentioned in the Bible, is 21.888 inches.

8. The old road measures, 40 rd. are 1 fur. (furlong), and 8 fur. are 1 mi., are now but little used.

Table VII.—Square Measure.

227. The table of square or surface measure consists of the denominations square miles, acres, square rods, square yards, square feet, and square inches. These denominations are used in computing the area of land, flooring, plastering, and other surfaces. See Manual.

144 sq. in. (square in.)	are	1 sq. ft.	1 sq. mi.	is	640 A.
9 sq. ft.	"	1 sq. yd.	1 A.	"	160 sq. rd.
30.25 sq. yd.	"	1 sq. rd.	1 sq. rd.	"	30.25 sq. yd.
160 sq. rd.	"	1 A. (acre.)	1 sq. yd.	"	9 sq. ft.
640 A.	"	1 sq. mi.	1 sq. ft.	"	144 sq. in.

SCALE.—Ascending, 144, 9, 30.25, 160, 640; descending, 640, 160, 30.25, 9, 144.

Builders use the following units in estimating their work:

100 sq. ft. are 1 square of roofing or flooring.
6 shingles, laid 6 in. to the weather, cover 1 sq. ft.
100 lath, or 1 bunch, cover 5 sq. yd. of surface.

NOTES.—1. Glazing and stone-cutting are estimated by the square foot.

2. Painting, plastering, paper-hanging, ceiling, and paving are estimated by the square yard.

3. Brick-laying is estimated by the sq. yd., or the square of 100 sq. ft. In either case the work is understood to be 12 in. or 1.5 bricks thick. Brick-laying is also estimated by the thousand bricks.

Table VIII.—Cubic Measure.

228. The table of cubic or solid measure consists of the denominations cubic inches, cubic feet, and cubic yards. These denominations are used in computing the solidity of timber, stone, portions of earth, and many other articles; and in estimating the capacity of bins, boxes, etc., when their dimensions are given in denominations of linear measure. See Manual.

1728 cu. in. (cubic in.) are 1 cu. ft.
27 cu. ft. " 1 cu. yd.

1 cu. yd. is 27 cu. ft.
1 cu. ft. " 1728 cu. in.

SCALE.—Ascending, 1728, 27; descending, 27, 1728.

NOTES.—1. On public works a cubic yard of earth is a standard load.

2. Railroad and transportation companies estimate light freight by the space it occupies, in cubic feet; and heavy freight, by actual weight.

3. In estimating the tonnage of ships and other vessels, 100 cu. ft. of space are 1 T. of shipping.

4. A perch of stone or of masonry is 16.5 ft. long, 1 ft. high, and 1 ft. thick, or 16.5 cu. ft.

5. A common brick is 8 × 4 × 2 inches, or 64 cu. in.

6. In every cubic foot of bricks, piled solid, are 27 bricks; and in every cubic foot of brick wall, laid in mortar, are 22.5 bricks.

7. Five courses of bricks in the height of a wall are called a foot.

8. Brick-layers, masons, and joiners make a deduction of one half the space occupied by the windows and doors in the walls of buildings.

9. In computing the cubic contents of the walls of foundations and buildings, brick-layers and masons multiply the *girth* (*i. e.*, the distance round the outside of the walls), height, and thickness together. By this method of measuring, the corners are measured twice.

Table IX.—Wood Measure.

229. The table of wood measure consists of the denominations cords, cord feet, and cubic feet. These denominations are chiefly used in measuring wood. Rough stone is also commonly sold by the cord. A pile of wood 8 feet long, 4 feet wide, and 4 feet high is 1 cord; and 1 foot in length of such a pile is 1 cord foot.

16 cu. ft. are 1 cd. ft. (cord ft.)
8 cd. ft., or 128 cu. ft. } " 1 cd.

1 cd. { is 128 cu. ft., or 8 cd. ft.
1 cd. ft. is 16 cu. ft.

Scale.—Ascending, 16, 8; descending, 8, 16.

EXERCISES.

13. Read 5 mi. 2 yd. 2.7 ft.; 54 rd. 1.37 yd.

14. Read 5 A. 15.25 sq. yd.; 3 sq. mi. 30 sq. rd. 5.5 sq. yd.

15. Read 11 cu. yd. 8 cu. ft. 512 cu. in.; 5 cu. yd. 60 cu. in.

16. Read 45 cd. 7 cd. ft. 13 cu. ft; 31 cd. 5.75 cd. ft.

17. Write 6 miles 85 rods 2 yards 2 feet 8 inches.

18. Write twelve acres sixty square rods one hundred six square inches.

19. Write 16 cubic yards 18 cubic feet 350 cubic inches.

20. Write 15 cords 4 cord feet 13 cubic feet.

21. Write 90 square rods 14 square yards 5 square feet 108 square inches.

22. Write 15 miles 200 rods; 11 cubic yards 1,576 cubic inches.

PROBLEMS.

47. How many feet are 7 mi. 108 rd. 3 yd. 1 ft.? *38,752.*

48. Reduce 25 sq. rd. 3 sq. yd. 8 sq. ft. to square inches. *985,140.*

49. In 7 cu. yd. 19 cu. ft. 960 cu. in., how many cubic inches?

50. Reduce 18 cd. 5 cd. ft. 8 cu. ft. to cubic feet. *2,392 cu. ft.*

51. Reduce 115,372 inches to higher denominations. *1 mi. 262 rd. 3 yd. 2 ft. 4 in.*

52. 176,169 sq. in. are what units of higher denominations? *4 sq. rd. 14 sq. yd. 8 sq. ft. 57 sq. in.*

53. Reduce 1,001,100,100 cu. in. to higher denominations. *21,457 cu. yd. 1 cu. ft. 580 cu. in.*

54. Change 12,875 cu. ft. of wood to cords.

55. How many planks, averaging 1 ft. wide, will be required for a plank road 7 mi. 284 rd. long? *41,646.*

56. How many 1-inch blocks will be required to make a pile that shall contain 23 cu. yd. 18 cu. ft.?

57. A farmer planted 1 hill of corn on every square yard of ground in a field of 13 A. 96 sq. rd. How many hills did he plant? *65,824.*

58. How many loads must a teamster draw, to move 131 cd. of stone, if he draws 1 cd. ft. at a load?

59. 2 mi. 125 rd. 1.5 ft. are how many feet?

60. Reduce 126,720 in. to miles.

61. How many acres in a tract of land 6 miles square? *23,040.*

62. In 4,305,780 sq. yd. there are how many square miles?

63. How much wood in a pile 160 ft. long, 4 ft. wide, and 9 feet high?

64. 25 cd. 7 cd. ft. 12.75 cu. ft. are how many cubic feet?

65. .9 of a foot are how many inches? *10.8.*

66. Reduce 100.8 sq. rd. to the decimal of an acre. *.63 A.*

67. .0015 mi. is what decimal of a rod? *.48.*

68. How many cubic yards are 468,018 cu. in.? *10.03125.*

69. How many acres in a field 125 rd. long and 80 rd. wide?

70. One year my potato crop yielded 1 bushel to the square rod, and the total yield was 1,145 bushels. How much land did I plant to potatoes? *7 A. 25 sq. rd.*

71. A dealer in real estate owns 5 rectangular lots of land of 1 acre each; and the fronts of the lots, or their widths on the street, are 2 rd., 4 rd., 5 rd., 8 rd., and 10 rd. What are their respective depths?

72. A gentleman used 5,560 tiles, each 1 ft. long, in underdraining his land. How much tile drain did he put down?

73. If Mississippi River deposits 1 inch of sediment at the bottom of the Gulf of Mexico each year, how much will it raise the bottom of the gulf in 1000 years? *27 yd. 2 ft. 4 in.*

74. How many cubic yards of earth must be removed, in digging a cellar 52 ft. × 28 ft. × 8 ft.?

75. How many cords of rough stone in a pile 47 ft. × 14 ft. × 5.5 ft.?

76. How many acres in a county of 20 townships each 6 miles square?

77. How many square inches are 150 sq. rd. 95 sq. in.?

78. 5 mi. 109 rd. 4 yd. = how many feet? *28210.5.*

79. If you measure only the length and height of a pile of 4-foot wood, how many feet of surface measure will you allow for 1 cord? How many cords of such wood are there in a pile 75 ft. long and 7 ft. high? *16 cd. 3 cd. ft. 4 cu. ft.*

80. If the pile is 4 feet high, how many feet of running measure (that is, length) will be a cord? How many cords are there in such a pile 185.6 ft. in length?

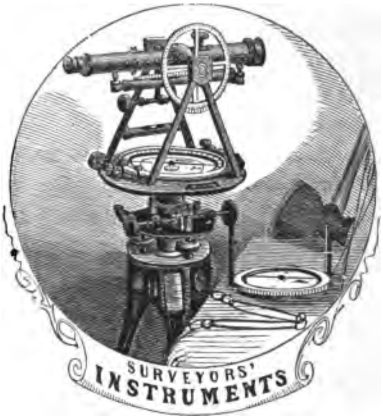
81. How much wood in a pile of 8-foot wood 87.5 ft. long and 5.5 ft. high?

82. How many rods of fence will be required to inclose a tract of land 2 mi. 45 rd. long and 225 rd. wide? 5,220 *1,820.*

83. A company of immigrants purchased a tract of western land 4.8 mi. × 1.75 mi. in extent. How many acres in the tract?

84. Allowing that 4 persons can stand on 1 square yard of ground, how many people can stand in a street 15 rd. long and 35 ft. wide? *3,850.*

Table X.—Surveyors' Measures.



230. A Gunter's chain is 4 rods or 66 feet long, and consists of 100 links, each 7.92 inches long. This chain is used by surveyors in measuring the dimensions or boundary lines of land.

The table of surveyors' measures consists of the denominations miles, chains, and links, used in measuring boundary lines; and townships, square miles or sections, square chains, poles or square rods, and square links, used in computing the area of lands.

1st. In Measuring Dimensions:

100 l. (links)	are	1 ch. (chain.)	1 mi.	is	80 ch.
80 ch.	"	1 mi.	1 ch.	"	100 l.

SCALE.—Ascending, 100, 80; descending, 80, 100.

2d. In Computing Areas:

625 sq. l.	are	1 P. (pole) or sq. rd.	1 Tp.	is	36 sq. mi.
16 P.	"	1 sq. ch.	1 sq. mi or Sec.	"	640 A.
10 sq. ch.	"	1 A.	1 A.	"	10 sq. ch.
640 A.	"	1 sq. mi. or Sec. (section.)	1 sq. ch.	"	16 P.
36 sq. mi.	"	1 Tp. (township.)	1 P.	"	625 sq. l.

SCALE.—Ascending, 625, 16, 10, 640, 36; descending, 36, 640, 10, 16, 625.

NOTES.—1. Since 100 links are 1 chain, 1 link is .01 of a chain, 35 links are .35 of a chain, and so on. Hence, links may be written either as hundredths of a chain; thus 15.44 ch.; or chains and links as a compound number; thus, 15 ch. 44 l.

2. Since a chain is 4 rods long, 25 links are 1 rod. The denomination rod is seldom used in linear chain measure.

3. Civil engineers on railroads and canals commonly use an engineers' chain, which consists of 100 links each 1 foot long.

4. Sections, or square miles of Government lands, are divided into 8 equal parts, and each part, or 80 acres, is often called a *lot of land.* Two lots, or 160 acres, are called a *quarter section.* See Manual.

5. Formerly 40 rods were called a *rood*, and 4 roods an acre. The denomination rood, so common in old deeds, mortgages, and surveys, is now but little used.

EXERCISES.

23. Read 6 ch. 44 l.; 3 mi. 52.81 ch.

24. Read 3 A. 7 sq. ch. 12 P. 480 sq. l.

25. Write 1 square mile 217 acres 6 square chains 9 poles 145 square links.

26. Write 45 chains 22 links both as a compound number and as a denominate mixed decimal number.

27. Write eighty acres seven square chains forty-two square links.

PROBLEMS.

85. Reduce 3 mi. 75 ch. 12 l. to links. *31,512 l.*

86. 25,000,000 l. are how many miles? *3,125.*

87. Change 8 A. 14 P. 462.5 sq. l. to square links.

88. Reduce 236,754 square links to higher denominations.

89. The front of a certain city lot measures 21 l. 3.75 in. How many inches front has it? How many feet?

90. How many acres in a farm which is 215 rods long by 140 rods wide? *188.125.*

91. The area of a certain piece of land is 9 sq. ch. 11.25 P. What is its area in square rods?

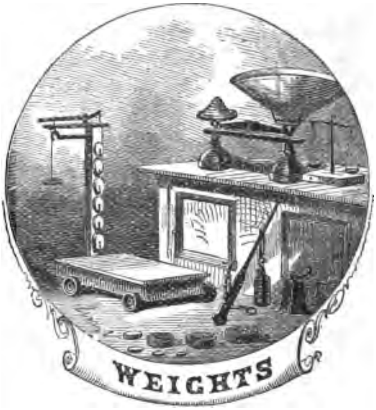
92. In chaining the route for a proposed railroad, the engineers applied a Gunter's chain 7,254 times. What was the length of the route? *90.675 mi.*, or *90 mi. 54 ch.*

93. Walworth Co., Wis., consists of 16 Government townships. How many acres in the county?

94. 2.376 in. is what decimal of a chain? *.003.*

Table XI.

Avoirdupois Weight.



231. The table of avoirdupois weight consists of the denominations tons, hundred-weight, pounds, and ounces. These denominations are used in weighing most kinds of produce, provisions, groceries, metals, coal, and many other articles.

16 oz. (ounces) are 1 lb. (pound.)	1 T. is 20 cwt.
100 lb. " 1 cwt. (hundred-weight.)	1 cwt. " 100 lb.
20 cwt. " 1 T. (ton.)	1 lb. " 16 oz.

SCALE.—Ascending, 16, 100, 20; descending, 20, 100, 16.

In wholesale transactions in coal, iron, and iron ore, and in invoices passing the United States Custom-Houses, of all English goods sold by weight,

28 lb. are 1 qr. (quarter.)	1 T. is 20 cwt.
4 qr. " 1 cwt.	1 cwt. " 4 qr.
20 cwt. " 1 T.	1 qr. " 28 lb.

The ton of 2,000 lb. is commonly called the *short ton*, or the *net ton;* and the ton of 2,240 lb. (20 cwt. = 80 qr. = 2,240 lb.), is called the *long ton*, or the *gross ton.*

The following units or denominations are in common use:

56 lb. are 1 bu. of salt, at the New York State Salt Works.
280 lb. (5 bu.) " 1 bar. " " " " " "
100 lb. " 1 cental of grain.
100 lb. " 1 cask of nails or raisins.
196 lb. " 1 bar. of flour.
200 lb. " 1 bar. of beef or pork.

NOTES.—1. There is no such thing as a quarter, of 25 lb. When the quarter is named, it always means 28 lb.

2. In theory, 16 dr. (drams) are 1 oz. But the dram is neither used nor *recognized in business.*

3. Hundred-weight and pounds may be read together as pounds, or the pounds may be read as hundredths of a hundred-weight. Thus, 4 cwt. 56 pounds is 456 lb., or 4.56 cwt.; 5 T. 17 cwt. 9 lb. is 5 T. 1709 lb., or 5 T. 17.09 cwt.

232. The weight of a bushel of the principal kinds of grain and seeds has been fixed by statute in many of the States, as shown in the following

AVOIRDUPOIS BUSHEL TABLE.

	Cal.	Conn.	Del.	Ill.	Ind.	Iowa.	Ky.	La.	Me.	Mass.	Mich.	Minn.	Mo.	N. C.	N. H.	N. J.	N. Y.	Ohio.	Or.	Penn.	Vt.	W. T.	Wis.
Barley, . . .	50			48	48	48	48	32		46	48	48	48	48		48	48	48	46	47	46	45	48
Buckwheat,	40	45		40	50	52	52			46	42	42	52	50		50	48		42	48	46	42	42
Clover seed,				60	60	60	60				60	60	60			64	60	60	60			60	60
Indian corn,	52	56	56	52	56	56	56	56		56	56	56	56	54		56	58	56	56	56	56	56	56
Oats, ...	32	28		32	32	35	100 to 3 bu.	32	30	30	32	32	35	30	30	30	32	32	34	32	32	36	32
Rye, ...	54	56		54	56	56	56	52		56	56	56	56			56	56	56	56	56	56	56	56
Timothy s'd				45	45	45	45						45				44						46
Wheat, . . .	60	56	60	60	60	60	60	60		60	60	60	60	60		60	60	60	60	60	60	60	60

Table XII.—Troy Weight.

233. The table of Troy weight consists of the denominations pounds, ounces, pennyweights, and grains. These denominations are used in weighing the precious metals and jewels, and in philosophical experiments.

24 gr. (grains)	are 1 pwt. (pennyweight.)	1 lb.	is 12 oz.
20 pwt.	" 1 oz.	1 oz.	" 20 pwt.
12 oz.	" 1 lb.	1 pwt.	" 24 gr.

SCALE.—Ascending, 24, 20, 12; descending, 12, 20, 24.

NOTE.—Physicians in prescribing, and apothecaries in mixing, medicines that are dry, divide the Troy pound according to the following table of

APOTHECARIES' WEIGHT.

20 gr. (grains)	are 1 sc. or ℈ (scruple.)
3 ℈	" 1 dr. or ʒ (dram.)
8 ʒ	" 1 oz. or ℥ (ounce.)
12 ℥	" 1 ℔ (pound.)

Dry medicines are sold by avoirdupois weight.

EXERCISES.

28. Read 5 T. 16 cwt.; 33 T. 1 cwt. 54 lb. 7 oz.; 2 T. 375.25 lb

29. Read 4 lb. 7 oz. 10 pwt. 20 gr.

30. Write fifty tons two hundred seven and five tenths pounds.

31. Write 6 pounds 4 ounces 19 pennyweights 12 grains.

PROBLEMS.

95. 7 T. 15 cwt. 45 lb. 9 oz. are how many ounces? *248,729.*

96. Change 1,999 oz. to hundred-weight. *1 cwt. 24 lb. 15 oz.*

97. How many grains are 1 lb. 9 oz. Troy weight? *10,080.*

98. Reduce 5,190 grains to ounces. *10 oz. 16 pwt. 6 gr.*

99. One day, 9 T. 56 lb. of Oswego corn-starch were packed in pound papers. How many papers were put up?

100. A jeweler made 456 finger rings, each containing 4.25 pwt. of gold. How much gold did he use?

101. Reduce .6 gr. to the decimal of an ounce.

102. 11 oz. 11 pwt. 11 gr. are how many grains?

103. One year the Lebanon Shakers put up 2 T. 16 cwt. 95.75 lb. of garden seeds in papers, each containing .25 of a pound. How many papers of seeds of this weight did they put up?

104. .00021 T. is what decimal of a pound? *.42 lb.=6.72 oz.*

105. One spring a Vermont farmer made 161,268 oz. of maple sugar. How many tons did he make? *5 T. 79.25 lb.*

106. Reduce .003125 lb. Troy to the decimal of a grain.

107. What is the length of a roll of gold wire that weighs 2 lb. 9 oz., if it weighs 1 gr. to the foot? *3 mi.*

108. How many pounds of bluing will a manufacturer use in putting up 845,000 1-ounce boxes?
52812.5 lb. = 26 T. 812 lb. 8 oz.

109. A wholesale dealer bought 2 T. 8 cwt. of carpet tacks in 8-oz. papers. How many papers of tacks did he buy? *9,600.*

110. .375 lb. = how many pennyweights?

111. 1,250 flat-irons, weighing 5 lb. each, weigh how many tons?
3.125 T. = 3 T. 250 lb.

Table XIII.—Time.

234. *Time* is a limited portion of duration.

The table of time consists of the denominations centuries, years, months, weeks, days, hours, minutes, and seconds. These denominations are used in expressing portions of time or duration of different lengths.

The day and the year are the natural divisions of time, the other denominations, except centuries, being parts of one or the other of these.

60 sec. (seconds)	are	1 min. (minute.)			
60 min.	are	1 h. (hour.)	1 century	is	100 yr.
24 h.	"	1 da. (day.)	1 leap-year	"	52 wk. 2 da., or 366 da.
7 da.	"	1 wk. (week.)			
52 wk. 1 da., or 365 da.	"	1 common yr. (year.)	1 common yr.	"	52 wk. 1 da., or 365 da.
52 wk. 2 da. or 366 da.	"	1 leap-yr.	1 da.	"	24 h.
			1 h.	"	60 min.
100 yr.	"	1 century.	1 min.	"	60 sec.

Scale.—Ascending, 60, 60, 24, 365 or 366, 100; descending, 100, 366 or 365, 24, 60, 60.—The 7 is omitted from the scale.

The length of a solar year is 365 da. 5 h. 48 min. 48 sec.

The following years are leap-years, of 366 days each:

I. *Every centennial year that is exactly divisible by 400;* as 400, 800, 1600; 2000, 2400, 2800. And,

II. *Every year not a centennial year that is divisible by 4;* as 1804, 1808, 1812; 1876, 1880, 1892.

For explanation of the Calendar, see Manual.

The year is divided into 12 calendar months, and these are divided into 4 seasons.

SEASONS.			MONTHS.	ABBREVIATIONS.	DAYS.
Winter.	1st	mo.	January,	Jan.	31
	2d	"	February,	Feb.	28 or 29
Spring.	3d	"	March,	Mar.	31
	4th	"	April,	Apr.	30
	5th	"	May,		31
Summer.	6th	"	June,		30
	7th	"	July,		31
	8th	"	August,	Aug.	31
Autumn.	9th	"	September,	Sept.	30
	10th	"	October,	Oct.	31
	11th	"	November,	Nov.	30
Winter.	12th	"	December,	Dec.	31

NOTES.—1. February has 28 days in a common year, and 29 in a leap-year.

2. In most business transactions, 30 days are regarded as a month.

EXERCISES.

32. Read 5 yr. 4 mo. 15 da.; 3 wk. 5 da. 10 h. 45 min. 30 sec.
33. Read 12 yr. 134 da. 17.35 h.
34. Write 4 years 9 months 3 days 30 minutes 15 seconds.
35. Read June 10, 1869; Aug. 17, 1812; Dec. 21, 1875.

PROBLEMS.

112. How many minutes in the three spring months of a common year? *132,480.*

113. How many hours from Independence day at noon, to Christmas day at noon? *4,176.*

114. Reduce 573,596 min. to higher denominations. *56 wk. 6 da. 7 h. 56 min.*

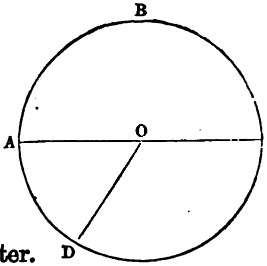
115. How many seconds in a solar year?

116. How long a time will it take a clock that ticks once every second, to tick one million times? *11 da. 13 h. 46 min. 40 sec.*

117. My watch ticks 4 times each second. How many times will it tick in a leap-year?

118. 3 wk. 6 da. 23 h. 30 min. 45 sec. are how many seconds?

Table XIV.—Circular and Angular Measure.



235. A *Circle* is a round, plane surface.

See Manual.

236. A *Circumference* is the bounding line of a circle.

237. An *Arc* is any part of the circumference of a circle.

238. A *Diameter* is the distance across a circle through its center.

239. A *Radius* is the distance from the center to the circumference of a circle.

NOTE.—The radius of a circle is always equal to one half its diameter.

240. If the surface about a point in a plane be divided into 360 equal parts or spaces, by lines drawn from the point, 360 equal angles will be formed, and any one of these angles will be a *degree.* And since an angle is the difference of direction of two lines, or the opening between two lines that meet in a point (see **168**), it follows that

A *Degree* is one of the 360 equal angles which will just fill the space about a common point in a plane.

241. The lines which form the sides of these angles may be of any length; and if about their common point of meeting as a center, a circumference be drawn, cutting all these lines, it will be divided into 360 equal parts, and one of these parts will be the measure of a degree, or of the angle at the center of the circle. Hence,

The *Measure of an Angle* at the center of a circle is that part of the circumference included between the sides of the angle.

242. If the circumference of any circle be divided into 360 equal parts, each of these parts is also called a degree.

NOTE.—Since circles may be great or small, the degrees in their circumferences will be correspondingly great or small. An angle of 1 degree is constant; while the measure of the angle, or 1 degree in a circumference, varies with every change in the dimensions of the circle.

243. The table of circular and angular measure consists of the denominations circles, degrees, minutes, and seconds. These denominations are used

By *Surveyors*, in determining the directions or bearings of land boundaries and other lines;

By *Navigators*, in determining the position of vessels at sea; and

By *Geographers* and *Astronomers*, in determining latitude, longitude, and the position and motion of the heavenly bodies; and in computing difference of time.

60″ (seconds) are 1′ (minute.)
60′ " 1° (degree.)
360° " 1C. (circumference.)

1 C. is 360°
1° " 60′
1′ " 60″

SCALE.—Ascending, 60, 60, 360; descending, 360, 60, 60.

NOTES.—1. A right angle or a *quadrant* is an angle of 90°, and is always included between two lines perpendicular to each other; its measure is one fourth of a circumference. Hence, we say 4 right angles or quadrants are 1 circumference.

2. Navigators call one sixth of a circumference a *sextant*. Hence, in navigation, 60° are 1 sextant; and 6 sextants are 1 circumference.

3. Astronomers divide the zodiac, or the sun's apparent path in the heavens, into 12 equal parts, of 30° each (for the 12 months of the year), which they call signs. Hence, in astronomical calculations,

30° are 1 S. (sign), and 12 S. are 1 great circle of the heavens.

4. A minute of the circumference of the earth is 1 geographic mile, which is 1.15 English miles, or 1 mi. 48 rd. See Manual.

EXERCISES.

36. Read 10° 40′ 35″; 8 S. 25°; 72° 0′ 23.75″.

37. Write 19 degrees 53 minutes 42 seconds.

38. Write one hundred five degrees twenty-six geographic miles.

PROBLEMS.

119. Reduce 47° 13′ to seconds. *169,980″.*

120. How many degrees in 59,300″? *16° 28′ 20″.*

121. When a planet has moved 1,426,444″ in the heavens, has it described more or less than one complete revolution in its orbit? *36° 14′ 4″ more than 1 revolution.*

122. When the sun has seemed to pass over 8 S. 62.25° of the zodiac, how many seconds has he seemed to move?

123. How many seconds are .015°? *54″.*

124. 3 quadrants 57° 58′ are how many minutes?

125. 67,875″ = how many degrees? *18° 51′ 15″.*

126. Reduce 3′ to the decimal of a degree. *.05°.*

Table XV.

Counting.

244. The table of counting consists of the denominations ones, dozens, gross, and great gross. These denominations are used in counting several classes of articles for market purposes.

12 DOZ SCHOOL SLATES
ONE HALF REAM COMMERCIAL NOTE WRITING PAPER Co. HOLYOKE, MASS.
ONE GROSS BUTTONS
MISCELLANEOUS TABLES

12 ones or things	are	1 doz.	(dozen)	1 great gro.	is	12 gro.
12 doz.	"	1 gro.	(gross.)	1 gro.	"	12 doz.
12 gro.	"	1 grt. gro.	(great gro.)	1 doz.	"	12 things.

SCALE.—Ascending and descending, uniformly 12.

NOTES.—1. Six things of a kind are often called a *set;* as a set of chairs, spoons, forks, plates, etc.

2. Twenty things of a kind are sometimes called a *score;* as a score of times, three score of years, etc.

Table XVI.—Paper.

245. The table of paper consists of the denominations bales, bundles, reams, quires, and sheets. These denominations are used in the paper trade.

24 sheets are 1 quire.
20 quires “ 1 rm. (ream.)
2 rm. “ 1 bundle.
5 bundles “ 1 bale.

1 bale is 5 bundles.
1 bundle “ 2 rm.
1 rm. “ 20 quires.
1 quire “ 24 sheets.

SCALE.—Ascending, 24, 20, 2, 5; descending, 5, 2, 20, 24.

NOTE.—Paper is bought at wholesale by the bale, bundle, and ream; and at retail by the ream, quire, and sheet.

Table XVII.—Copying.

246. Lawyers' clerks and copyists in public offices are often paid by the folio for making copies of legal papers, records, and documents.

72 words are 1 folio, or sheet of common law.
90 “ “ 1 “ “ “ “ chancery.

Table XVIII.—Books.

247. This table consists of the terms used to indicate the number of leaves of a book made from one sheet of paper.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 leaves,	a folio,	4 pp. (pages.)
4 “	a quarto or 4to,	8 “
8 “	an octavo or 8vo,	16 “
12 “	a duodecimo or 12mo,	24 “
16 “	a 16mo,	32 “
18 “	an 18mo,	36 “
24 “	a 24mo,	48 “
32 “	a 32mo,	64 “
64 “	a 64mo,	128 “

NOTE.—A sheet of medium size print paper is 23 × 36, 24 × 37.5, or 25 × 38 inches. These are the sizes commonly used for printed books.

EXERCISES.

39. Read 5 grt. gro. 11 gro. 4 doz.; 6 gro. 5 doz. 8 steel pens.
40. Write five rm. fifteen quires eleven sheets.
41. Write 19 gross 7 dozen; 8 great gross 7.75 dozen.

PROBLEMS.

127. How many gross are 2,156 buttons? *14 gro. 11 doz. 8.*

128. How many crayons are there in 25 boxes, each containing 1 gross? *3,600.*

129. How many gross of screws will a joiner use in the 26 working-days of a month, if he uses 56 screws per day? *10 gro. 1 doz. 4.*

130. On inventorying his goods, a hardware merchant finds that he has 7 gro. 8.5 doz. wardrobe hooks on hand. What number of hooks has he? *1,110.*

131. One day a paper dealer sold 6 bales 1 bundle 4 rm. of manilla wrapping paper. How many sheets of paper did he sell?

132. 7,260 sheets of foolscap paper are how many reams? *15 rm. 2.5 quires.*

133. How many sheets of print paper in a 12mo book of 456 pp.?

134. One month a lady copied 648.5 common-law folios for a lawyer, at $.10 per folio. How much did she receive?

135. A stationer has 6 grt. gro. 11 doz. lead pencils. How many pencils has he?

136. One winter a wood turner manufactured 56,870 clothespins, which he packed in boxes of 1 great gross each. How many boxes had he? *10 gro. 11 doz. 2 pins more than 32 boxes.*

137. A printed case in the Supreme Court (or Chancery) contained 456,120 words. How much was the printer's bill, at $.12½ per folio? *$633.50.*

138. How many reams of print paper will be required to supply 3,250 subscribers with a weekly newspaper one year? *352 rm. 1 quire 16 sheets.*

139. A village grocer shipped 5,160 eggs to the city, in 5 barrels. How many eggs did he pack in a barrel? *86 doz.*

140. A bookseller's stock of steel pens consists of 7 packages of 1 doz. boxes each, 9 boxes of a broken package, and 5 doz. 8 pens of an opened box. How many pens has he, each full box containing 1 gross? *13,460.*

141. A publisher issued an edition of 5,000 copies of an 18mo book of 216 pp. How much paper did he use, allowing 1 quire to each ream for waste? *65 rm. 15 quires 19 sheets.*

Table XIX.

Government Standard Units of Measures and Weights.

218. The standard units of the money, measures, and weights now in use, as adopted by the United States Government in the year 1834, and from which the other denominations in the respective tables are determined, are—

TABLES.	UNITS.	VALUES.
United States Money,	Dollar,	.900 silver, .100 alloy.
Lines, Surfaces, and Solids,	Yard,	3 feet, or 36 inches.
Liquid Measure,	Gallon,	231 cubic inches.
Dry Measure,	Bushel,	2150.42 cubic inches.
Troy Weight,	Pound,	5,760 grains.
Avoirdupois Weight,	Pound,	7,000 Troy grains.

See Manual.

NOTES.—1. The yard in use at the Custom-Houses is divided decimally into tenths and hundredths.

2. For ordinary purposes, 2150.4 cu. in. are called a bushel.

3. In the actual standard weights used by the General Government, the Troy ounce is divided decimally into tenths, hundredths, thousandths, and ten-thousandths. The values of these divisions are shown in the margin of this note.

1 oz. = 480 gr.
.1 " = 48 "
.01 " = 4.8 "
.001 " = .48 "
.0001 " = .048 "

Table XX.—Comparative Values.

219. I. Of Measures of Capacity.

DENOMINATIONS.	LIQUID MEASURE.	DRY MEASURE.
1 gal.,	231 cu. in.,	268.8 cu. in. (.5 pk.)
1 qt.,	57.75 " "	67.2 " "
1 pt.,	28.875 " "	33.6 " "

II. Of Weights.

DENOMINATIONS.	TROY WEIGHT.	AVOIRDUPOIS WEIGHT.
1 lb.,	5,760 gr.,	7,000 gr.
1 oz.,	480 "	437.5 "

NOTES.—1. Multiplying the number of cubic inches in a liquid gallon by 7, and the number in a dry gallon by 6, we find that 7 liquid gallons contain 4.2 cubic inches more than 6 dry gallons. Hence, in ordinary computations, it is sufficiently accurate to estimate 7 liquid gal. = 6 dry gal.

$$\begin{array}{r} 231 \\ 7 \\ \hline 1617 \end{array} \qquad \begin{array}{r} 268.8 \\ 6 \\ \hline 1612.8 \end{array}$$

2. Multiplying the number of grains in a pound Troy by 175, and the number in a pound avoirdupois by 144, we have

5,760 × 175 = 7,000 × 144. Hence,
175 pounds Troy = 144 pounds avoirdupois.

PROBLEMS.

142. How many more cubic inches in 568.5 gallons dry measure, than in the same number of gallons liquid measure ? *21489.3.*

143. If you dip 32 quarts of water into a tub that will hold 32 quarts of wheat, how much will the tub lack of being full ?

The above problem can be solved in at least five different ways.

144. What is the difference in the weight of 42.375 pounds of lead and 42.375 pounds of silver ?

145. 100 pounds avoirdupois are how many pounds Troy ?

146. A brewer has a vat that will hold 5,000 gallons of beer. How many bushels of barley will it hold ?

147. A grocer buys 3 bu. of chestnuts, at $5.00 a bushel, wooden measure, and retails them at $.20 a quart, tin measure. How much does he gain ? *$7.40.*

NOTE 3.—Among many retailers, dry measure is called *Wooden Measure*, and liquid measure, *Tin Measure*.

148. An express agent in Washington collected charges at the Mint, for transporting 562 pounds, commercial weight, of silver from California. How many Mint pounds of silver were transported ? *682 ℔. 11 oz. 16 pwt. 16 gr.*

250. *The Metric System of Weights and Measures.*

In the year 1866, the Congress of the United States passed a bill authorizing the use of a new system of weights and measures. In this system the principal denomination is the *Metre*, from which all the other denominations in all the tables are derived. Hence, this system is called the *Metric System.*

The principal denomination for the Measure of Surface is the *Are;* for the Measure of Capacity, the *Litre;* and for Weight, the *Gram.* See Manual.

The lower denominations in each table are tenths, hundredths, or thousandths of these; and their names are formed by prefixing *deci, centi,* or *milli* to the name of the principal denomination.

The higher denominations are 10, 100, 1,000, or 10,000 times the principal denomination of any table; and their names are formed by prefixing *deka, hecto, kilo,* or *myria* to the name of that principal denomination.

TABLE OF DENOMINATIONS AND THEIR RELATIVE VALUES.

PREFIXES FOR LOWER DENOMINATIONS.			NAMES OF PRINCIPAL DENOMINATIONS.		PREFIXES FOR HIGHER DENOMINATIONS.		
Milli	(mill-ee)	.001 of	METRE	(mee-ter)	*Deka*	(dek-a)	10
Centi	(sent-ee)	.01 of	ARE	(āre)	*Hecto*	(hec-to)	100
Deci	(des-ee)	.1 of	LITRE	(li-ter)	*Kilo*	(kill-o)	1,000
			GRAM		*Myria*	(mĭr-e-a)	10,000

MEASURES OF LENGTH.

10 millimetres	are	1 centimetre.	1 millimetre	is	.001 metre.
10 centimetres	"	1 decimetre.	1 centimetre	"	.01 metre.
10 decimetres	"	1 metre.	1 decimetre	"	.1 metre.
10 metres	"	1 dekametre.	**1 METRE**	"	*39.37 inches.*
10 dekametres	"	1 hectometre.	1 dekametre	"	10 metres.
10 hectometres	"	1 kilometre.	1 hectometre	"	100 metres.
10 kilometres	"	1 myriametre.	1 kilometre	"	1,000 metres.
			1 myriametre	"	10,000 metres.

MEASURES OF SURFACE.

	1 centare is .01 are.
100 centares are 1 are.	**1 ARE** " 100 sq. metres, or *119.6 sq. yd.*
100 ares " 1 hectare.	1 hectare " 100 ares.

MEASURES OF CAPACITY.

10 millilitres are 1 centilitre.	1 millilitre is .001 litre.
10 centilitres " 1 decilitre.	1 centilitre " .01 litre.
10 decilitres " 1 litre.	1 decilitre " .1 litre.
10 litres " 1 dekalitre.	**1 LITRE** " 1 cu. decimetre, or *.908 dry qt.* *1.0567 liquid qt.*
10 dekalitres " 1 hectolitre.	1 dekalitre " 10 litres.
10 hectolitres " 1 kilolitre, or stere.	1 hectolitre " 100 litres.
	1 kilolitre or stere " 1000 litres.

WEIGHT.

10 milligrams are 1 centigram.	1 milligram is .001 gram.
10 centigrams " 1 decigram.	1 centigram " .01 gram.
10 decigrams " 1 gram.	1 decigram " .1 gram.
10 grams " 1 dekagram.	**1 GRAM** " *15.432 grains.*
10 dekagrams " 1 hectogram.	1 dekagram " 10 grams.
10 hectograms " 1 kilogram.	1 hectogram " 100 grams.
10 kilograms or kilos " 1 myriagram.	1 kilogram or **1 KILO** " 1000 grams, or *2.2046 pounds.*
10 myriagrams " 1 quintal.	1 myriagram " 10 kilos.
10 quintals " 1 millier, or tonneau.	1 quintal " 100 kilos.
	1 millier " 1,000 kilos.

NOTE.—The weights and measures of this system have not yet come into use in this country. They are in general use in France, Belgium, Spain, and Portugal; and their use has been legalized by Great Britain, Italy, Norway, Sweden, Greece, Mexico, and most of the South American governments.

SECTION III.

ADDITION.

251. Ex. What is the sum of 1 wk. 4 da. 18 h. 45 min., 5 wk. 14 h. 30 min., 2 da. 20 h. 25 min., 9 wk. 4 da. 11 h., and 3 wk. 9 h. 8 min.?

SOLUTION.

1 wk.	*4 da.*	*18 h.*	*45 min.*
5	*0*	*14*	*30*
	2	*20*	*25*
9	*4*	*11*	
3	*0*	*9*	*8*
19 wk.	*6 da.*	*1 h.*	*48 min.*

EXPLANATION.—Since only like orders of units in different numbers can be added (see **39**, II.), we write the numbers with like denominations — or orders of units — in the same columns. Then, commencing at the right, we add the units of each denomination, in order, from the lowest to the highest. The sum of the minutes is 108, or 1 h. 48 min. We write the 48 min. as the minutes of the sum, and add the 1 h. with the hours of the given numbers. The sum of all the hours is 73, or 3 da. 1 h. We write the 1 h. as the hour of the sum, and add the 3 da. with the days of the given numbers. The sum of all the days is 13, or 1 wk. 6 da. We write the 6 da. as the days of the sum, and add the 1 wk. with the weeks of the given numbers. The sum of all the weeks is 19, which we write as the weeks of the sum. The result, 19 wk. 6 da. 1 h. 48 min., is the sum required.

In integers and decimals, the units of each order are added separately, and 1 is added or *carried* to the next higher order, for every 10 in the sum of the order added. (See **39**, III.)

In compound numbers the units of each denomination—or order—are added separately, and 1 is added or carried to the next higher denomination, for as many units in the sum of the denomination added, as equal 1 of the next higher denomination. That is,

I. *In addition of integers and decimals, the carrying unit is uniformly 10;* and,

II. *In addition of compound numbers, the carrying unit in any denomination is that number in the scale which equals 1 of the next higher denomination.*

252. Upon these principles is based the

Rule for Addition of Compound Numbers.

I. *Add the units of each denomination separately, and when the sum is less than a unit of the next higher denomination, write it in the result.*

II. *When the sum of the units of any denomination is equal to one or more units of the next higher denomination, write the excess in the result, and add the number of units of the higher denomination with the given units of that denomination.*

PROBLEMS.

1. In four days a gardener sold 12 bu. 3 pk., 10 bu. 2 pk., 8 bu. 1 pk., and 7 bu. 3 pk. of peas. How many peas did he sell?

2. A milkman sells in 6 successive days 200 qt. 1 pt., 220 qt., 215 qt. 1 pt., 208 qt., 199 qt., and 187 qt. 1 pt. of milk. How many gallons does he sell? *307 gal. 2 qt. 1 pt.*

(3)

26 lb.	8 oz.	18 pwt.	12 gr.
15	6	11	.0
	9	0	22
14	3	17	9

(4)

18°	16′	21.3″
25	56	45
87	45	39.75

(5)

£17	5s.	6d.	
32	15	9	
487	00	11	1 far.
	16	3	2

6. One winter, a wood-cutter chopped 7 piles of wood, that measured 29 cd. 6 cd. ft. 8 cu. ft., 38 cd. 4 cd. ft., 31 cd. 2 cd. ft. 4 cu. ft., 43 cd. 7 cd. ft., 21 cd. 3 cd. ft. 4 cu. ft., 34 cd. 7 cd. ft., and 38 cd. 2 cd. ft. How much wood did he chop? *238 cd.*

7. A coal dealer bought at the Scranton coal mines in Pennsylvania 108 T. 13 cwt. 1 qr. 16 lb. of stove coal, 87 T. 7 cwt. 2 qr. 20 lb. of chestnut coal, 76 T. 19 cwt. 3 qr. 4 lb. of large egg coal, 69 T. 3 qr. of small egg coal, and 41 T. 19 cwt. 22 lb. of lump coal. How much coal of all kinds did he buy? *384 T. 3 qr. 6 lb.*

8. From a cask of vinegar, 8 gal. 3 qt. 1 pt. were drawn one day, 7 gal. 1 qt. the second, 12 gal. 1 pt. the third, and 9 gal. 3 qt. 1.5 pt. the fourth. How much was drawn from the cask?

9. A dairy-man makes 515 lb. 8 oz. of butter in June, 499 lb. 12 oz. in July, 496 lb. 15 oz. in August, and 489 lb. 9 oz. in September. How much butter does he make in the four months?

10. How much land in 5 farms which contain 235 A. 125.75 sq. rd., 99 A. 18 sq. rd., 545 A. 88.25 sq. rd., 127 A. 43.5 sq. rd., and 333 A. 142.375 sq. rd.?

11. The cargo of a canal-boat consisted of 23 T. 17 cwt. 25 lb. of pig-iron, 18 T. 9 cwt. 48 lb. of agricultural implements, 14 T. 14 cwt. of flour, and 3 T. 7 cwt. 28 lb. of salt. What was the weight of the cargo? *60 T. 8 cwt. 1 lb.*

12. The ceiling of a room contains 33 sq. yd. 8 sq. ft. 94 sq. in. of plastering, the side walls 22 sq. yd. 8 sq. ft. 120 sq. in. each, and the two end walls 21 sq. yd. 120 sq. in. each. How much plastering is there in the room? *122 sq. yd. 142 sq. in.*

13. I bought meat as follows:

March 8,	2 lb. 11 oz. of steak;	3 lb. 9 oz. of mutton.	
" 10,	3 lb. 6 oz. " "	2 lb. 6 oz. " "	
" 13,	1 lb. 15 oz. " "	3 lb. 12 oz. " "	

How much meat of each kind did I buy?

14. Mr. Young has a farm of six sides, which measure 41 ch. 56 l., 15 ch. 94 l., 27 ch. 22 l., 18 ch. 10 l., 26 ch. 40 l., and 48 ch. 72 l. long. What length of fence is required to inclose it?

15. In grading up a building lot, I used the earth dug from four cellars. The first cellar was 32 × 22 ft., the second 28 × 20 ft., the third 24 × 18 ft., and the fourth 28 × 18 ft., and each of them was 4 feet deep. How much earth did I use? *325 cu. yd. 25 cu. ft.*

16. What is the sum of 4 mi. 280 rd. 2 yd. 2 ft., 6 mi. 130 rd. 1 yd. 2 ft., 1 mi. 96 rd. 5 yd. 1 ft.?

Explanation.—The sum of the feet is 5, or 1 yd. 2 ft. The sum of the yards is 9, or 1 rd. 3.5 yd. Writing the 3 yd. under the column of

SOLUTION.

4 mi.	*280 rd.*	*2 yd.*	*2 ft.*	
6	*130*	*1*	*2*	
1	*96*	*5*	*1*	
		3 yd.	*2 ft.*	
			1	*6 in.*
12 mi.	*187 rd.*	*4 yd.*	*0 ft.*	*6 in.*

yards, we reduce the .5 yd. to units of lower denominations. .5 yd. = 1.5 ft., and .5 ft. = 6 in. We write the 1 ft. under the 2 ft. already obtained, and the 6 in. at the right of the 1 ft. Then, commencing again at the right, we add the inches, feet, and yards of the two partial results, after which we proceed with the remaining denominations of the given numbers.

17. Add 17 mi. 196 rd. 5 yd. 1 ft., 3 mi. 12 rd. 3 yd. 2 ft. 6 in., 1 mi. 76 rd. 4 yd. 1 ft. 4 in., and 2 mi. 156 rd. 5 yd. 2 ft. 8 in.

24 mi. 123 rd. 3 yd.

SECTION IV.

SUBTRACTION.

253. Ex. 1. From 17 yd. 1 ft. 9 in. take 9 yd. 2 ft. 5 in.

SOLUTION.

17 yd. 1 ft. 9 in.
9 2 5
7 yd. 2 ft. 4 in.

EXPLANATION.—Since only like orders of units in different numbers can be subtracted the one from the other (see **52**, II.), we write the numbers with the denominations of the subtrahend under like denominations—or orders of units—of the minuend. Then, commencing at the right, we subtract the units of each denomination of the subtrahend from the like denomination of the minuend, in order, from the lowest to the highest. 5 in. from 9 in. leave 4 in., which we write as the inches of the remainder. Since we can not subtract 2 ft. from 1 ft., and since the difference will not be affected by adding the same number to both minuend and subtrahend (see **52**, III.), we add 3 ft. to the 1 ft. of the minuend, and 1 yd. (= 3 ft.) to the 9 yd. of the subtrahend. We then subtract 2 ft. from 4 ft., and 10 yd. from 17 yd., writing the 2 ft. and the 7 yd. as the feet and yards of the remainder. The result, 7 yd. 2 ft. 4 in., is the remainder required.

Ex. 2. From 3 lb. 12 pwt. take 7 oz. 14 pwt. 9.25 gr.

SOLUTION.

3 lb.	*0 oz.*	*12 pwt.*	
7	*14*		*9.25 gr.*
2 lb.	*4 oz.*	*17 pwt.*	*14.75 gr.*

Explanation.—In solving this example, we first add 24 gr. to the minuend, and 1 pwt. (= 24 gr.) to the 14 pwt. of the subtrahend, and subtracting 9.25 gr. from 24 gr., we obtain 14.75 gr. We next add 20 pwt. to the 12 pwt. of the minuend, and 1 oz. (= 20 pwt.) to the 7 oz. of the subtrahend, and subtracting 15 pwt. from 32 pwt., we obtain 17 pwt. We then add 12 oz. to the minuend, and 1 lb. (= 12 oz.) to the subtrahend, and subtracting 8 oz. from 12 oz., we obtain 4 oz. Finally, subtracting 1 lb. from 3 lb., we obtain 2 lb. The result, 2 lb. 4 oz. 17 pwt. 14.75 gr., is the remainder required. See Manual.

In integers and decimals, when the units of any order in the subtrahend are more than those of the like order in the minuend, 10 units of the same order are added to the minuend, and 1 unit of the next higher order is added to the subtrahend.

In compound numbers, when the units of any denomination in the subtrahend are more than those of the like denomination in the minuend, as many units of the same denomination as equal 1 unit of the next higher denomination are added to the minuend, and 1 unit of the next higher denomination is added to the subtrahend.

That is, whenever the units of any order or denomination in the subtrahend are more than those of the same order in the minuend,

I. *In integers and decimals, 10 units of the same order are really added to both terms;* and,

II. *In compound numbers, as many units of the same denomination are really added to both terms as equal 1 unit of the next higher denomination.*

254. Upon these principles is based the

Rule for Subtraction of Compound Numbers.

I. *Subtract the units of each denomination of the subtrahend separately from those of the same denomination of the minuend, writing the difference for the units of the same denomination in the result.*

II. *When the units of any denomination in the subtrahend are more than those of the like denomination in the minuend, add to the minuend as many units of the same denomination as equal 1 unit of the next higher denomination, and to the subtrahend 1 unit of the next higher denomination.*

PROBLEMS.

1. If my trees yield 11 bu. 3 pk. 6 qt. of cherries, and I wish 3 bu. 1 pk. 4 qt. for my own use, how many cherries will I have to sell? *8 bu. 2 pk. 2 qt.*

2. From a bin containing 432 bu. 3 pk. of wheat, 256 bu. 1 pk. 6 qt. have been taken. How much remains in the bin? 176 " 1 " 2

	(3)	(4)
From	17 gal. 2 qt. 1 pt. 2 gi.	5 sq. mi. 180 A. 4 sq. ch.
Subtract	9 3 1 3	1 576 8
	7 gal. 2 qt. 1 pt. 3 gi.	3 sq. mi. 243 A. 6 sq. ch.

5. A grocer bought a jar of butter, weighing 56 pounds. After selling 17 lb. 14 oz., how much had he left? 39 " 2

6. A jeweler used 5 oz. 8 pwt. 15 gr. of gold in making 8 oz. 5 pwt. 12 gr. of jewelry. How much alloy was used?

7. A load of hay with the wagon weighs 1 T. 9 cwt. 65 lb., and the wagon alone weighs 11 cwt. 36 lb. What is the weight of the hay?

8. One morning, at a wood yard on Mississippi River, there were on hand 1,753 cd. 2 cd. ft. of wood, and during the day 119 cd. 6 cd. ft. were sold to passing steam-boats. How much wood was in the yard at night? *1,633 cd. 4 cd. ft.*

9. Providence is situated in longitude 71° 24′ 48″ west, and San Francisco in longitude 122° 23′ west. How much farther west is San Francisco than Providence? *50° 58′ 12″.*

10. From a farm that contained 213 A. 40 sq. rd., I sold 98 A. 128 sq. rd. How much land remained in the farm? *114 A. 72 sq. rd.*

11. From a cellar containing 289 cu. yd. 17 cu. ft. of earth, 175 cu. yd. 25 cu. ft. were taken. How much earth remained in the cellar?

12. From a cask containing 42 gal. 3 qt. of vinegar, I sold 15 gal. 3 qt. 1.6 pt. How much had I left?

13. Two masons put on 533 sq. yd. 3 sq. ft. of wall, one of them plastering 299 sq. yd. 6 sq. ft. How much did the other plaster?

14. A coal dealer having bought 545 T. 3 qr. of coal, sold 26 T. 2 qr. 25 lb. How much had he left?

15. A stationer bought 30 gro. 4 doz. lead-pencils, and immediately afterward sold 9 gro. 8 doz. 6 of them. How many pencils had he left? *20 gro. 7 doz. 6.*

16. An excavation 45 × 22 × 8 ft. was to be made, and 215 cu. yd. 28.25 cu. ft. of it has been excavated. How much remains to be done?

17. From 17 mi. 4 rd. 2 yd. 1 ft., subtract 14 mi. 4 yd. 2 ft.

Explanation.—2 ft. from 4 ft. leave 2 ft. 5 yd. from 7.5 yd. (2 yd. + 5.5 yd.) leave 2.5 yd. = 2 yd. 1 ft. 6 in. We write the 2 yd. under the yards of the given numbers, and, since the 1 ft. 6 in. belongs with the feet and inches of the final result, we add them to that part of the result already found, and obtain 3 yd. 0 ft. 6 in. We then finish the solution as already taught. See Manual.

SOLUTION.

17 mi.	*4 rd.*	*2 yd.*	*1 ft.*	
14	*0*	*4*	*2*	
		2 yd.	*2 ft.*	
			1	*6 in.*
3 mi.	*3 rd.*	*3 yd.*	*0 ft.*	*6 in.*

18. From 25 mi. 2 ft. 11 in., take 132 rd. 3 yd. 8.75 in.
24 mi. 187 rd. 3 yd. 8.25 in.

19. The length of gas pipe in use in a certain city last year was 23 mi. 194 rd. 2 yd., and now it is 25 mi. 46 rd. 1 yd. How much pipe has been laid during the year? *1 mi. 171 rd. 4 yd. 1 ft. 6 in.*

20. What is the difference between 31 rd. 5 yd. 2 ft. 11 in. and 32 rd. 1 ft. 4 in.? *1 in.*

255. Difference between any two Dates.

Ex. How many years, months, and days elapsed between May 21, 1869, and Sept. 14, 1871?

EXPLANATION.—Since the later of two dates is always expressed by a greater compound number than the earlier, we subtract the compound number expressing the earlier date from that expressing the later, writing the number of the year, month, and day of each date in order, as shown in the Solution. Whenever the number of days in the subtrahend is greater than that in the minuend, we call thirty days a month. (See **234**, Note 2).

SOLUTION.

1871 yr.	*9 mo.*	*14 da.*
1869	*5*	*21*
2 yr.	*3 mo.*	*23 da.*

PROBLEMS.

21. A note dated May 11, 1864, was paid Sept. 25, 1865. How long did it remain unpaid? *1 yr. 4 mo. 14 da.*

22. The civil war in the U. S. commenced April 12, 1861, and closed May 26, 1865. What length of time did it continue?

23. A note given May 22, 1868, was paid Aug. 10, 1869. How long did the note run?

24. George Washington died Dec. 14, 1799, aged 67 yr. 9 mo. 22 da. What was the date of his birth? *Feb. 22, 1732.*

25. How much time has passed since his death?

26. Mary was 13 yr. 8 mo. 12 da. old July 15, 1867. What was the date of her birth? *Nov. 3, 1853.*

27. A note was given Aug. 12, 1865, payable Feb. 4, 1866. How long had it to run? *5 mo. 22 da.*

28. Henry was born Dec. 5, 1852. How old was he March 19, 1868? *15 yr. 3 mo. 14 da.*

29. March 7, a mason contracts to build five bridge abutments that shall contain 856 pch. 8 cu. ft. of stone work, and to have the work done July 1. May 24, he has finished 594 pch. 12 cu. ft. How much work remains to be done, and how much time has he in which to finish it? *261 pch. 12.5 cu. ft.; 1 mo. 7 da.*

SECTION V.

MULTIPLICATION.

256. Ex. How much is 7 times 38 bu. 1 pk. 5 qt.?

EXPLANATION.—We write the multiplier under the lowest denomination of the multiplicand, and multiply the units of each denomination of the multiplicand by the multiplier, in order, from the lowest to the highest. Since the product is always of the same kind or denomination as the true multiplicand (see **80**, IV.), 7 times 5 qt. are 35 qt., or 4 pk. 3 qt. We write the 3 qt. as the quarts of the product, and reserve the 4 pk. to be added with the pecks of the product. 7 times 1 pk. are 7 pk., and 7 pk. + 4 pk. = 11 pk., or 2 bu. 3 pk. We write the 3 pk. as the pecks of the product, and reserve the 2 bu. to be added with the bushels of the product. 7 times 38 bu. are 266 bu., and 266 bu. + 2 bu. = 268 bu., which we write as the bushels of the product. The result, 268 bu. 3 pk. 3 qt., is the product required.

SOLUTION.

38 bu. 1 pk. 5 qt.
7
268 bu. 3 pk. 3 qt.

In integers and decimals, the units of each order are multiplied separately, and 1 is added to the product of the next higher order, for every 10 in the product of the two figures multiplied.

In compound numbers the units of each denomination are multiplied separately, and 1 is added or carried to the product of the next higher denomination, for as many units in the product of the denomination multiplied, as are equal to 1 of the next higher denomination. That is,

I. *The carrying unit in multiplication of integers and decimals is 10, the same as in addition;* And

II. *The carrying unit in multiplication of compound numbers, is determined from the table or scale to which the compound number belongs, the same as in addition.* Hence,

257. ***Rule for Multiplication of Compound Numbers.***

I. *Multiply the units of each denomination by the whole multiplier, as in integers and decimals.*

II. *Carry from a lower to a higher denomination in the product, for that number in the table or scale corresponding to the denomination multiplied, as in addition of compound numbers.*

PROBLEMS.

1. How much seed wheat will it take to seed 93 acres, using 1 bu. 3 pk. 4 qt. to the acre? *174 bu. 1 pk. 4 qt.*

2. How much will 25 doz. pocket-knives cost, at £3 2 s. 6 d. a dozen? *£78 2 s. 6 d.*

(3)	(4)	(5)
22 cd. 7 cd. ft. 12 cu. ft.	2° 43′ 19″	5 mi. 37 ch. 56 l.
9	127	14

6. What is the weight of 6 sets of silver forks, each fork weighing 1 oz. 15 pwt. 12 gr.? *5 lb. 3 oz. 18 pwt.*

7. A stone-mason contracts to build the cellar walls for 13 dwellings. If it takes 7 cd. 5 cd. ft. 4 cu. ft. for each cellar, how much stone will it take for all of them? *99 cd. 4 cd. ft. 4 cu. ft.*

8. If in digging the cellars, 76 cu. yd. 15 cu. ft. of earth be taken from each, how much earth will be taken from all of them? *995 cu. yd. 6 cu. ft.*

9. A butcher slaughtered 18 sheep, and their average weight was 35 lb. 15 oz. What was their total weight?

10. A train of 63 coal cars was loaded at a coal mine in Pennsylvania, 3 T. 5 cwt. 2 qr. of coal being put upon each car. How much coal did the train carry?

11. Multiply 27 mi. 218 rd. 4 yd. 2 ft. 8 in. by 145. *4,014 mi. 58 rd. 4 yd. 2 ft. 8 in.*

12. If 5 men can make 38 rd. 5 yd. of post-and-rail-fence in a day, how much fence can they build in 30 days? *3 mi. 207 rd. 1 yd. 1 ft. 6 in.*

13. A piece of land near a city was divided into 38 lots, each containing 50 sq. rd. 24 sq. yd. How much land was there in the piece? *12 A. 10 sq. rd. 4.5 sq. yd.*

14. If a steam-boat runs a mile in 4 min. 30 sec., in how long a time will it make a trip of 295 miles? *22 h. 7 min. 30 sec.*

15. If a housekeeper uses on an average 1 gal. 3 qt. 2 gi. of molasses in a month, how much will she use in a year? *21 gal. 3 qt.*

16. A farmer drew 45 loads of hay to market, and each load weighed 1 T. 375 lb. How much hay did he draw?

17. If school is in session 5 h. 25 min. each day, how long is it in session during a term of 17 weeks of 5 school days each?

18. If the rate of speed of a railroad train is 25 mi. 315 rd. an hour, how far will it run in 24 hours?

19. How much wine in eight casks, if each contains 28 gal. 2 qt. 1.5 pt.?

20. Charles is 7 yr. 251 da. old, and his grandfather is 9 times as old as he. How old is his grandfather?

SECTION VI.

DIVISION.

CASE I.

258. The Divisor an Abstract Number.

Ex. Divide 46 mi. 126 rd. 1.5 yd. by 9.

SOLUTION.

46 mi. 126 rd. 1.5 yd. | *9*
5 mi. 49 rd. 3 yd. 0 ft. 8 in.

EXPLANATION.—We write the dividend and divisor as in integers, and commencing at the left, we divide the units of each denomination of the dividend by the divisor, in order, from the highest to the lowest. The dividend being a concrete number, the quotient must be a concrete number (see **109**, IV.); the quotient arising from dividing the units of any denomination must be of the same denomination (see **109**, V.); and any partial remainder must be of the same denomination—or order of units—as the partial dividend used (see **109** VIII.). One ninth of 46 mi. is 5 mi. with a remainder of 1 mi. We write the 5 mi. as the miles of the quotient, reduce the 1 mi. remainder to rods, and to it add the 126 rd., making 446 rd. One ninth of 446 rd. is 49 rd. with a remainder of 5 rd. We write the 49 rd. as the rods of the quotient, reduce the 5 rd. remainder to yards, and to it add the 1.5 yd., making 29 yd. One ninth of 29 yd. is 3 yd. with a remainder of 2 yd. We write the 3 yd. as the yards of the quotient, and reducing the 2 yd. remainder to feet, we have 6 ft. One ninth of 6 ft. is no whole feet; we therefore write 0 ft. in the quotient, and reducing the 6 feet to inches, we have 72 in. One ninth of 72 in. is 8 in., which we write as the inches of the quotient. The result, 5 mi. 49 rd. 3 yd. 8 in., is the quotient required.

In integers and decimals, the units of each order are divided separately, and the units in any partial remainder are called 10 times as many units of the next lower order.

In compound numbers the units of each denomination are divided separately, and the units in any partial remainder are changed to units of the next lower denomination by reduction. That is,

I. *In division of integers and decimals, any partial remainder is tens of the order of units next lower than the partial dividend used;* And,

II. *In division of compound numbers, each unit of any partial remainder is as many times* 1 *of the next lower denomination, as there are ones of the lower denomination in a unit or* 1 *of the partial dividend used.*

PROBLEMS.

1. A silver-ware manufacturer used 5 lb. 6 oz. 12 pwt. of silver in making 9 goblets. How much silver did he use for each?

2. In settling an estate, a farm of 184 A. 46.25 sq. rd. was divided equally among 15 heirs. How much land did each heir receive?
12 A. 45.75 sq. rd.

3. A farmer cut 11 T. 17 cwt. of hay from 6 acres of meadow. What was the yield per acre? *1 T. 1,950 lb.*

(4) 56 gal. 2 qt. 1 pt. 1 gi. ⌊ 7

(5) 46 mi. 230 rd. 4.5 yd. ⌊ 9

6. If 2,864 cu. yd. 24 cu. ft. of stone are used in making 586 rd. of Macadamized road, how much stone will be used in making 1 rd.? *4 cu. yd. 24 cu. ft.*

CASE II.

259. The Divisor a Concrete Number.

$4 are contained in $12, 3 times; $.04 in $.12, 3 times; 4 lb. in 12 lb., 3 times; 4 oz. in 12 oz., 3 times; etc. That is,

We can divide dollars by dollars, cents by cents, pounds by pounds, ounces by ounces, etc. But we can not divide dollars by cents, nor pounds by ounces. For $12 ÷ $.04 = neither $3 nor $.03; so, also, 12 lb. ÷ 4 oz. = neither 3 lb. nor 3 oz. Hence,

Only concrete numbers of the same denomination can be divided, the one by the other.

Ex. 1. How many times are 4 lb. 9 oz. contained in 27 lb. 6 oz.?

Explanation.—Since both dividend and divisor are compound numbers, we reduce them to simple concrete numbers of the same denomination, (ounces), and divide as in integers.

SOLUTION.

27 lb. 6 oz. = 438 oz.
4 lb. 9 oz. = 73 oz.

438 oz. | 73 oz.
438 | 6

Ex. 2. Divide 86 yd. 1 ft. by 8 ft. 4 in.

Explanation.—Since the lowest denomination in either term is inches, we first reduce both terms to inches, and then divide as in integers and decimals.

SOLUTION.

86 yd. 1 ft. = 3,108 in.
8 ft. 4 in. = 100 in.

3,108 in. | 100 in.
31.08

Ex. 3. 105 wk. are how many times 3 wk. 4 da.?

Explanation. — After dividing all the units of the dividend, we annex a decimal cipher to the remainder, and continue the division as in decimals.

SOLUTION.

105 wk. = 735 da.
3 wk. 4 da. = 25 da.

```
735 da. | 25 da.
50      |------
---     | 29.4
235
225
---
 100
 100
 ---
```

PROBLEMS.

7. I have a measure that is 5 ft. 8 in. long. How many times the length of my measure is a pole 18 yd. 2 ft. 18 in. long?

8. If a man feeds his horse 1 pk. 6 qt. of oats a day, how long will 5 bu. 1 pk. last him? *12 days.*

9. A joiner used 2 gro. 7 doz. 2 screws in hanging and trimming the doors of a house, using 1 doz. 5 screws to each door. How many doors were in the house? *22.*

10. At how many loads can a teamster draw 93 cd. 1 cd. ft. 8 cu. ft. of wood, drawing 1 cd. 2 cd. ft. 8 cu. ft. at a load?

11. How many demijohns, each containing 3 gal. 2 qt. 1 pt., can be filled from 97 gal. 3 qt. 1 pt. of wine? 27.

260. Upon the principles deduced in **258**, **259**, is based the

Rule for Division of Compound Numbers.

I. When the divisor is an abstract number.

1. *Divide the units of each denomination separately, and write the several results for the same denominations of the quotient.*

2. *Reduce each partial remainder to the next lower denomination, and add to it the units of that denomination, for the next partial dividend.*

II. When the divisor is a compound or a concrete number.

Reduce both dividend and divisor to the lowest denomination contained in either, and divide as in integers and decimals.

PROBLEMS.

3\ 12. Divide 71 mi. 237 rd. 3 yd. 1 ft. 6 in. by 9.
7 mi. 310 rd. 4 yd. 2 ft.

3\ 13. A wood-chopper cut 63 cd. 3 cd. ft. of wood in 26 days. How much did he cut per day? *2 cd. 3.5 cd. ft.*

3\ 14. An importer paid £663 15 s. for 50 gold watches. What did they cost apiece? *£13 5 s. 6 d.*

3\ 15. A farmer raised 488 bu. 1 pk. of barley from 14 acres of land. What was the yield per acre?
34.875 bu., or 34 bu. 3 pk. 4 qt.

3\ 16. If a vessel sails 250 mi. in 2 da. 4 h. 5 min., what is the average rate of speed? *1 mi. in 12 min. 30 sec.*

17. If a paver in 24 days can put down 76 sq. rd. 15 sq. yd. of pavement, how much can he put down in one day?
3 sq. rd. 5 sq. yd. 6 sq. ft.

3\ 18. If a train of 27 cars carry 57 T. 1 cwt. 2 qr. 24 lb. of iron ore, what is the average per car?

19. A family consumes 7 lb. 11 oz. of meat each week; how long will it take them to consume 192 lb. 3 oz.? *25 weeks.*

20. In 256,728 cu. in. how many gallons liquid measure? How many quarts dry measure?

21. If 6 men in 12 days mow 86 A. 64 sq. rd. of grass, how much will 1 man mow in 1 day? *1 A. 32 sq. rd.*

22. If a railroad train runs 144 mi. 291 rd. in 5.5 h., how far does it run per hour? *26 mi. 111 rd. 1 ft. 6 in.*

23. How many silver spoons, each weighing 1 oz. 9 pwt., can be made from 12 lb. 18 pwt. of silver?

24. How long must a field be to contain 14 A., if it is 35 rd. wide?

25. How many bars of railroad iron, each 18 ft. long, will be required for a railroad 521 miles long?

26. How many rolls of wall-paper, 20 in. wide and 9 yd. long, will be required to paper the walls of a room 14 by 16 ft. and 9 ft. high, no allowance being made for openings in the walls? *12.*

SECTION VII.

REVIEW PROBLEMS IN COMPOUND NUMBERS.

1. How many yards of carpeting will be required to cover a floor 24 ft. long and 18 ft. wide? *48.*

2. A note dated June 17, 1865, was made payable January 10, 1866. How long had it to run?

3. How many bushels of wheat can a farmer store in a molasses hogshead which will hold 128 gallons? (See **248**, Note 2.) *13 bu. 3 pk.*

4. A manufacturer of patent medicine puts 1ʒ 2℈ 10 gr. of calomel into each bottle of medicine. How much calomel does he use for 50 dozen bottles? *11℔. 5℥ 4ʒ.*

5. If I start at latitude 15° 35′ 40″ north, and travel due north 2,159 geographic miles, in what latitude will I be? *51° 34′ 40″ north.*

6. What is the capacity in bushels of a bin 8 ft. long, 7 ft. wide, and 6 ft. deep? *270.*

7. A gardener raised 31 bu. 1 pk. 5 qt. of marrowfat peas for seed, and put them up in papers, each holding .25 pt. How many papers did he put up?

8. How many pieces of ribbon, each 1 eighth of a yard long, can be cut from 5 pieces, each containing 35 yd. 3 qr.?

9. An apothecary pays $3.50 per pound avoirdupois for 6 pounds of rhubarb. If he sells it in prescriptions, at the rate of $.60 an ounce, how much does he gain?

10. What is the weight of 1,250 barrels of flour? *122.5 T. = 122 T. 10 cwt.*

11. How many pump logs, each 12 ft. long, will it require to bring water to my house from a spring 1.375 miles distant?

12. How many bunches of lath will be required for the walls of a room 18 ft. long, 15 ft. wide, and 12 ft. high?

13. In digging a ditch 120 rd. long and 3 ft. wide, 1,320 cu. yd. of earth were removed. How deep is the ditch? *6 ft.*

14. How many more minutes in the summer months than in the winter months of a common year?

15. How many bunches, each containing 500 shingles, will be required to cover a roof, each side of which is 75 ft. long and 22.5 ft. wide? *40.5.*

16. How many bricks lying flatwise will be required for a walk 25 rd. 4 ft. long and 5 ft. wide? *9371.25.*

17. What is the difference between a figure which contains .5 of a sq. ft. and one which is .5 of a foot square?

18. How many times will a wagon wheel 12 ft. 6 in. in circumference turn round in going 11 mi. 28 rd.?

19. How many more farthings in £19 4.5 d. than in £17 19 s. 3.875 far.?

20. A merchant bought 15 pieces of merino, each piece containing 42 yd., at 2 s. per yd. What was the amount of his bill?

21. A stationer paid 7 s. a gross for 1,080 pen-holders. How much did they cost him? *£2 12 s. 6 d.*

22. I own a tract of Western land 215 rd. long and 140 rd. wide. How many acres in the tract? *188.125.*

23. When hay sells at $1.37½ per hundred, what is the price per ton?

24. How many cords of stone will be required to lay a wall 218 ft. long, 3 ft. high, and 1.5 ft. thick? *7 cd. 5 cd. ft. 5 cu. ft.*

25. A pile of wood 183 × 6 × 4 ft. contains how many cords?

26. .02875 bu. is what decimal of a quart? *.92 qt.*

27. How many bricks will be required for the two side walls of a building 50 ft. long, 20 ft. high, and 1 ft. thick?

28. What is the capacity of a cistern 14 ft. long, 11 ft. wide, and 7.5 ft. deep? *137 hhd. 9 gal.*

29. A pile of wood which contains 12.25 cd., is 56 ft. long and 8 ft. high. What is its width?

30. A man owns 3.5 A. of land. If he lays it out into village lots, each 5 by 8 rd., how many lots will he have?

31. What is the entire weight of 15 bar. of pork, 12 bar. of flour, 6 casks of nails, and 8 bar. of N. Y. salt? *4 T. 192 lb.*

32. How many quart cupfuls, tin measure, in 2 bu. 3 pk. of chestnuts. (See **249**, Note 3.)

33. How much will 18 barrels of pork cost, at $.12½ per pound?

34. A milkman furnishes 420 qt. of milk daily to the customers on his route. How many barrels of milk does he furnish in 2 weeks?

35. A man bargained for 3 qt. of blackberries daily during the blackberry season, which lasted 21 days. How many berries did he buy?

36. How many square yards in the four walls of a room which is 40 ft. long, 30 ft. wide, and 20 ft. high?

37. How many square yards in the ceiling of the same room?

38. A coal dealer bought 225 tons of coal, at $4.75 per gross ton, and retailed it for $6.50 per net ton. What was his gain? *$569.25.*

39. How much land is there in a field that is 11 × 13 ch.?

40. Reduce 1,000,000 cu. in. to higher denominations.

41. Reduce 1,000,000 sq. in. to higher denominations.
25 sq. rd. 14.75 sq. yd. 5 sq. ft. 64 sq. in.,
or 25 " 15 " 3 " 28 "

42. A farmer raised 5 acres of potatoes, which yielded 175 bushels to the acre. He sold them to a grocer for $2.75 a barrel of 2 bu. 3 pk. How much did he receive for them?

43. How many days were there in the last century?

44. How many seconds in the circumference of a cart wheel?

45. A printer used 3 rm. 2 quires 12 sheets of paper for quarter-sheet posters. How many posters did he print? *6,000.*

46. 10,000 silver dollars, of 412.5 gr. each, weigh how many pounds? *716 lb. 1 oz. 15 pwt.*

47. A farmer sold 25 T. 625 lb. of hay for $8.37½ per load of 1,125 lb. How much did he receive?

48. If you were to count 80 1-dollar bills every minute, for 10 hours a day, how long would you be in counting $1,000,000?

49. A fruit-dealer bought 3 bu. 3 pk. (=3.75 bu.) of cranberries, at $6.75 per bushel, and retailed them at $.25 per quart, tin measure. What was his profit? *$9.68¾.*

50. A produce dealer buys 5,600 bu. of oats in St. Louis, Mo., @ $.37½, and sells them in New Haven, Conn., @ $.75. How much are his profits, freights being $.31¼ per bu., payable at St. Louis?
$1,400.

CHAPTER 4.

FACTORS AND MULTIPLES.

SECTION I.

DEFINITIONS.

261. An *Exact Divisor* of a number is any integral factor of that number. Thus, 2, 3, 4, and 6 are exact divisors of 12.

262. An *Even Number* is one that is exactly divisible by 2; as 2, 4, 20, 36, 758.

263. An *Odd Number* is one that is not exactly divisible by 2; as 1, 3, 7, 29, 245.

264. A *Composite Number* is one that can be separated into factors. Thus, 18 is a composite integer, and its factors are 2 and 9, or 3 and 6, or 2, 3, and 3.

265. A *Prime Number* is one that can not be separated into integral factors; as, 3, 5, 7, 29, 257.

NOTE.—When the factors of a number are prime numbers, they are called *Prime Factors.* Thus, 4 and 6, or 3 and 8, or 2 and 12, are factors of 24; but the prime factors of 24 are 2, 2, 2, and 3.

266. A *Common Divisor* of two or more numbers is any factor found in each of them. Thus, 4 is a common divisor of 24, 36, and 48.

NOTE.—Two or more numbers are *prime to each other* when they have no common factor. The number 1 is not regarded as a factor.

267. The *Greatest Common Divisor* of two or more numbers is the greatest factor found in each of them. Thus, 12 is the greatest common divisor of 24, 36, and 48.

268. A *Multiple* or *Exact Dividend* is a number of which a given number is a factor. Thus, 27 is a multiple of 9.

269. A *Common Multiple* of two or more numbers is a number of which each of the given numbers is a factor. Thus, 32 is a common multiple of 4, 8, and 16.

270. The *Least Common Multiple* of two or more numbers is the least number of which each of the given numbers is a factor. Thus, 30 is the least common multiple of 3, 5, 10, and 15.

SECTION II.

CHANGES OF DIVIDEND AND DIVISOR.

271. The value of the quotient depends upon the values of both dividend and divisor. Hence, any change in either of these terms must produce a change in the quotient.

CASE I.

Changes of Dividend.

272. The quotient of 30 ÷ 5 is 6. If we multiply the dividend by 2, and divide the product (60) by 5, the quotient is 12, or 2 times 6. Again, if we multiply the dividend by 7, and divide as before, the quotient is 42, or 7 times 6. Hence,

$\underline{30}\ \lfloor 5$	$\underline{2 \times 30}\ \lfloor 5$	$\underline{7 \times 30}\ \lfloor 5$
6	$\underline{60}\ \lfloor 5$	$\underline{210}\ \lfloor 5$
	$12 = 2 \times 6$	$42 = 7 \times 6$

Multiplying the dividend multiplies the quotient.

273. The quotient of 30 ÷ 3 is 10. If we divide the dividend by 2, and then divide the quotient (15) by 3, the result is 5, or 10 ÷ 2. Again, if we divide the dividend by 5, and then divide the quotient (6) by 3, the result is 2, or 10 ÷ 5. Hence,

$\underline{30}\ \lfloor 3$	$\underline{30 \div 2}\ \lfloor 3$	$\underline{30 \div 5}\ \lfloor 3$
10	$\underline{15}\ \lfloor 3$	$\underline{6}\ \lfloor 3$
	$5 = 10 \div 2$	$2 = 10 \div 5$

Dividing the dividend divides the quotient.

CASE II.

Changes of Divisor.

274. The quotient of 120 ÷ 15 is 8. If we multiply the divisor by 2, and then divide 120 by the product (30), the quotient is 4, or 8 ÷ 2. Again, if we multiply the divisor by 4, and divide as before, the quotient is 2, or 8 ÷ 4. Hence,

120 ⌊ 15 *8*	*120 ⌊ 2 × 15* *120 ⌊ 30* *4 = 8 ÷ 2*	*120 ⌊ 4 × 15* *120 ⌊ 60* *2 = 8 ÷ 4*

Multiplying the divisor divides the quotient.

275. The quotient of 90 ÷ 18 is 5. If we divide the divisor by 2, and then divide 90 by the quotient (9), the result is 10, or 2 times 5. Again, if we divide the divisor by 6, and then divide the dividend (90) as before, the result is 30, or 6 times 5. Hence,

90 ⌊ 18 *5*	*90 ⌊ 18 ÷ 2* *90 ⌊ 9* *10 = 2 × 5*	*90 ⌊ 18 ÷ 6* *90 ⌊ 3* *30 = 6 × 5*

Dividing the divisor multiplies the quotient.

CASE III.

Like Changes of Dividend and Divisor.

276. The quotient of 32 ÷ 8 is 4. If we multiply both dividend and divisor by 2, and divide the new dividend (64) by the new divisor (16) the quotient is 4, the same as before. Again, if we multiply both terms by 5, and then divide as before, the quotient is still 4. Hence,

32 ⌊ 8 *4*	*2 × 32 ⌊ 2 × 8* *64 ⌊ 16* *4*	*5 × 32 ⌊ 5 × 8* *160 ⌊ 40* *4*

Multiplying both dividend and divisor by the same number does not change the quotient.

277. It has already been shown, in Art **107**, that

Dividing both dividend and divisor by the same number does not change the quotient.

278. These three cases establish the

General Principles of Division.

I. *The quotient is multiplied by multiplying the dividend or dividing the divisor.*

II. *The quotient is divided by dividing the dividend or multiplying the divisor.*

III. *The quotient is not changed by either multiplying or dividing both dividend and divisor by the same number.*

See Manual.

CASE IV.

Cancellation.

279. Ex. What is the quotient of 4 × 75 divided by 4 × 3?

EXPLANATION. — When division is exact, the quotient consists of that factor of the dividend not common to both dividend and divisor. In this example 4 is a factor of both dividend and divisor. And, since the quotient is not changed by dividing both dividend and divisor by the same number (278, III.), we divide 4 × 75 and 4 × 3 by 4,—or, which is the same thing, we omit the factor 4 from both terms,—and divide 75, the remaining factor of the dividend, by 3, the remaining factor of the divisor.

SOLUTION.

$$\begin{array}{r|l} \underline{4\times 75} & \underline{4\times 3} \\ \underline{75} & 3 \\ 25 & \end{array}$$

Cancellation is the process of omitting or striking out equal factors from the dividend and divisor.

280. From the solution and explanation of the last example, we see that

A factor is cancelled by dividing both dividend and divisor by that factor.

Ex. 1. Divide 210 by 35.

EXPLANATION.—We cancel the common factor, 5, by dividing both dividend and divisor by 5; and then divide the new dividend, 42, by the new divisor, 7.

SOLUTION.

$$\begin{array}{rr|ll} \textit{Dividend,} & \underline{210} & \underline{35} & \textit{Divisor.} \\ \textit{New Dividend,} & \underline{42} & 7 & \textit{New Divisor.} \\ & 6 & \textit{Quotient.} & \end{array}$$

Ex. 2. Divide 21 × 64 by 56.

SOLUTION.

$$\begin{array}{r|l} 21\times64 & 56 \\ \hline 3\times64 & 8 \\ \hline 3\times8 & 1 \\ \hline 24 & \end{array}$$

Explanation.—Canceling the common factor, 7, we have 3 × 64 for a new dividend, and 8 for a new divisor. Then, canceling the common factor, 8, we have 3 × 8 for a dividend and 1 for a divisor. The product of 3 × 8, or 24, is the required quotient.

Note.—From this solution and explanation we learn that, *When either dividend or divisor is canceled, a* 1 *belongs in its place.*

Ex. 3. Divide the product of 24, 80, 9, and 12.8, by the product of 3.2, 144, and 16.

Explanation.—The factors of the divisor may be written at the right of those of the dividend, as shown in the First Solution; or under those of the dividend, as shown in the Second Solution. When a factor is canceled, we draw an oblique line across it.

FIRST SOLUTION.

$$\cancel{24}\times\overset{5}{\cancel{80}}\times\overset{3}{\cancel{9}}\times\overset{\overset{2}{\cancel{4}}}{\cancel{12.8}}\ \Big\lfloor\ \cancel{3.2}\times\overset{\overset{\cancel{2}}{\cancel{6}}}{\cancel{144}}\times\cancel{16}$$

$$5\times3\times2=30$$

SECOND SOLUTION.

$$\frac{\cancel{24}\times\overset{5}{\cancel{80}}\times\overset{3}{\cancel{9}}\times\overset{\overset{2}{\cancel{4}}}{\cancel{12.8}}}{\cancel{3.2}\times\underset{\underset{\cancel{2}}{\cancel{6}}}{\cancel{144}}\times\cancel{16}}=5\times3\times2=30$$

PROBLEMS.

1. The dividend is 714, and the divisor 42. What is the quotient?

2. What is the quotient of 21 × 13 divided by 7?

3. How many times is 11 × 15 contained in 825?

4. Divide 28 × 7.2 by 16. *12.6.*

5. The factors of a dividend are 8, 25, and .45; and of a divisor, 15, 2, and 1.2. What is the quotient? *2.5.*

6. How many tons of hay at $10 a ton, must be given in exchange for 16 tons of coal at $5 a ton?

7. In how many days can 45 men do as much work as 63 men can do in 35 days? *49.*

8. If a mechanic can earn $88 in 28 days, how much can he earn in 42 days?

9. I rent a store 18 months for $2,400. What is the rent per year?

10. If 15 acres of land produce 420 bushels of wheat, how much wheat will 35 acres produce, at the same rate?

11. A ship's crew of 39 men have provisions enough to last them 76 days. If the crew is increased to 57 men, how long will the provisions last them? *52 days.*

12. In building a church, 9 bricklayers laid 407,880 bricks in 55 days. At the same rate, how many bricks can 11 bricklayers lay in 60 days?

13. If a telegram of 2,790 words can be transmitted in 45 minutes, how many words can be telegraphed in 72 minutes? *4,464.*

SECTION III.

PROPERTIES OF COMPOSITE NUMBERS.

281. The right-hand figure of any even number is 0, 2, 4, 6, or 8. Hence,

PROPERTY I. *Any number is divisible by 2, when its right-hand figure is 0, 2, 4, 6, or 8.* (See **262.**)

282. The right-hand figure of the product of any even number of times 5 is 0; thus $2\times5=10$, $6\times5=30$, $14\times5=90$. The right-hand figure of any odd number of times 5 is 5; thus, $3\times5=15$, $7\times5=35$; $19\times5=95$. Hence,

PROPERTY II. *Any number is divisible by 5, when its right-hand figure is 0 or 5.*

283. Any number expressed by more than one figure may be separated into two parts, one of which is a multiple of some power of 10, and the other is ones. Thus, $56=50+6$, or 5 times 10^1 and 6 ones; $256=200+56$, or 2 times 10^2 and 56 ones; $3256=3000+256$, or 3 times 10^3 and 256 ones; and so on. Hence,

PROPERTY III. *Any number is divisible by any power of 2 or 5, when the number expressed by as many of its right-hand figures as equal the index of the power is divisible by the same power of 2 or 5.*

284. The number 54,678, and the sum of the numbers 5, 4, 6, 7, and 8 (the digits of 54,678), are divisible by 3. The number 12,348, and the sum of the numbers 1, 2, 3, 4, and 8, are divisible by 9. Hence,

PROPERTY IV. *Any number is divisible by 3 or 9, when the sum of its digits is divisible by 3 or 9.*

285. We can divide 84 by 3, and the result (28) by 7. We can also divide 84 by 21, the product of 3 times 7. Again, we can divide 756 by 4, the result (189) by 3, and this result (63) by 7. We can also divide 756 by all the successive divisors, 4, 3, and 7, and by their product, 84. Hence,

PROPERTY V. *Any number which is divisible by two or more factors successively, is also divisible by each of the factors, and by their product.*

NOTE.—A number may be divisible by two or more factors, and not be divisible by their product. Thus, 24 is divisible by 8 and by 12, but not by their product, 96.

286. If we divide any number by one of its prime factors, and divide the result by another prime factor, and so on, until the quotient is 1, we shall use all the prime factors of the number for divisors. Thus, $72 \div 2 = 36$, $36 \div 2 = 18$, $18 \div 2 = 9$, $9 \div 3 = 3$, $3 \div 3 = 1$. The factors used as divisors are 2, 2, 2, 3, 3; and their product, 72, is divisible by the product of any number of these factors. Hence,

PROPERTY VI. *Any number is divisible by the product of any two or more of its prime factors.*

287. The factors of 6 (2 and 3) are contained in 12 $(= 2 \times 6)$, 18 $(= 3 \times 6)$, 30 $(= 5 \times 6)$, 54, 96, or any number of times 6. Hence,

PROPERTY VII. *Any factor of a composite number is contained in any number of times that number.*

288. Since 35 is 5 times 7, and 21 is 3 times 7, 35 + 21, or 56, is 5 times 7 + 3 times 7, or 8 times 7; and 35 − 21, or 14, is 5 times 7 − 3 times 7, or 2 times 7. Hence,

PROPERTY VIII. *Any factor common to two numbers is also a factor of their sum, and of their difference.*

NOTE.—These properties apply more generally to numbers in the decimal scale; but to a limited extent to compound numbers also.

SECTION IV.

PRIME NUMBERS.

289. All even numbers except 2, and all odd numbers ending in 5 except 5, are composite. (See **262**, **282**.) Hence,

The right-hand figure of a prime number is 1, 3, 7, or 9.

290. Since 2 is a factor of 4 and of 6, it is a factor of 2 + 6 and of 4 + 6, and of 2 or 4 + any number of times 6; and, since 3 is a factor of 6, it is a factor of 3 + 6, and of 3 + any number of times 6. (See **287**, **288**.)

All the remainders that can be obtained in dividing numbers by 6 are 1, 2, 3, 4, and 5; and we have just shown that, when the remainder is 2, 3, or 4, the number divided is composite. Hence,

When any prime number is divided by 6, the remainder is 1 or 5.

PROBLEMS.

1. Which of the numbers 19, 45, 67, 91 are prime numbers?

2. Of the numbers 103, 126, 131, 217, which are prime numbers?

3. Which of the numbers 111, 133, 14.7, 149, 219, 3.42, are composite numbers?

4. Which of the seven numbers 293, 371, 385, 440, 524, 617, and 713 are prime, and which composite numbers?

5. Determine which are prime, and which composite, of the numbers 911, 973, 103.3, 10.57, 3373, 3.407, 358.41.

6. Find all the prime numbers less than 100.

SECTION V.

COMMON DIVISORS.

CASE I.

Prime Factors or Divisors.

291. Ex. 1. What are the prime factors of 1,260?

EXPLANATION.—Since the right-hand figure of 1,260 is 0, we divide by the prime number 2 (see **281**); and for the same reason, we divide the quotient (630) by 2. Since the right-hand figure of the second quotient (315) is 5, we next divide by the prime number 5 (see **282**). Since the sum of the digits of the third quotient (63) is divisible by 3, we divide this quotient by 3 (see **284**); and for the same reason, we divide the fourth quotient (21) by 3. The last quotient (7) is a prime number. The product of the divisors 2, 2, 5, 3, 3, and the last quotient, 7, is 1,260; and hence they must be all the prime factors of that number. (See **57**.)

SOLUTION.

$$\begin{array}{r|l} 1260 & 2 \\ \hline 630 & 2 \\ \hline 315 & 5 \\ \hline 63 & 3 \\ \hline 21 & 3 \\ \hline 7 & \end{array}$$

Ex. 2. What are all the factors or divisors of 30?

EXPLANATION.—We first find all the prime factors of 30 to be 2, 5, and 3, and each of these is a divisor of 30 (see **285**). Since 2, 3, and 5 are prime factors of 30; 6, or 2 times 3, 10, or 2 times 5, and 15, or 3 times 5 are also divisors of 30 (see **286**). Hence, all the factors or divisors of 30 are 2, 3, 5, 6, 10, and 15.

SOLUTION.

$$\begin{array}{r|l} 30 & 2 \\ \hline 15 & 5 \\ \hline 3 & \end{array}$$

$$2 \times 3 = 6$$
$$2 \times 5 = 10$$
$$3 \times 5 = 15$$

292. *Rule for finding Prime Factors.*

I. *Divide the number by any prime factor.*

II. *Divide the quotient in the same manner; and so on, till a quotient is obtained that is a prime number. The divisors and the last quotient will be the prime factors required.*

PROBLEMS.

1. Find the prime factors of 540. *2, 2, 5, 3, 3, 3.*
2. What are the prime factors of 1,650 and 1,755?
3. Separate 1,836 into its prime factors. *2, 2, 3, 3, 3, 17.*
4. Separate 945 and 3,990 into their prime factors.
5. What are all the factors or divisors of 84? *There are ten.*
6. What are the prime, and what the component factors of 164? *There are 3 prime, and 2 component factors.*

CASE II.

Common Factors or Divisors.

293. Ex. 1. Find a common divisor of 15, 25, and 40.

EXPLANATION.—We separate the given numbers 15, 25, and 40, into their prime factors, and find that 5 is a divisor of each number. (See **266**.)

SOLUTION.

$$15=5\times3$$
$$25=5\times5$$
$$40=5\times2\times2\times2$$

Ex. 2. Find all the common divisors of 54 and 72.

EXPLANATION. — We separate the numbers 54 and 72 into their prime factors, as in Ex. 1, and we find that 2, 3, and 3 are common factors, and are therefore common divisors (see **266**). Since 2, 3, and 3 are common factors, 6 or 2 times 3, 9 or 3 times 3, and 18 or 2 times 3 times 3 are also common factors (see **286**). Hence, 2, 3, 3, 6, 9, and 18 are all the common divisors of 54 and 72.

SOLUTION.

$$54=2\times3\times3\times3$$
$$72=2\times2\times2\times3\times3$$

$$2\times3=6$$
$$3\times3=9$$
$$2\times3\times3=18$$

PROBLEMS.

7. What number is a common divisor of 21 and 36? *3.*
8. Find a common divisor of 4.5 and 10.5. *1.5.*
9. What are the prime common factors of 20, 32, 56, and 18?
10. Find all the common divisors of 36, 42, and 90.
11. How many common divisors have 64, 112, 48, and 144? *Four.*

CASE III.

Greatest Common Factor or Divisor.

FIRST METHOD.

294. Ex. What number is the greatest common divisor of 252, 210, and 168?

SOLUTION.

$$\begin{aligned} 252 &= 2\times 2\times 3\times 3\times 7 \\ 210 &= 2\times 5\times 3\times 7 \\ 168 &= 2\times 2\times 2\times 3\times 7 \\ 2\times 3\times 7 &= 42. \end{aligned}$$

EXPLANATION.—We separate the numbers 252, 210, and 168 into their prime factors, as in Case II. (see **293**), and we find, that 2, 3, and 7 are the only prime factors common to all the given numbers. Since 2, 3, and 7 are all the common prime factors of the given numbers, their product, 42, is the greatest common factor of the given numbers, and hence is the greatest common divisor required. (See **267**.)

SECOND METHOD.

295. Ex. What is the greatest common divisor of 21 and 77?

SOLUTION.

$$\begin{array}{rr|rr|r|} & & & 77 & 21 \\ & & & \underline{63} & 3 \\ & & 21 & 14 & \\ & & \underline{14} & 1 & \\ \underline{14} & 7 & & & \\ 2 & & & & \end{array}$$

EXPLANATION.—Since the common divisor of two numbers can not be greater than the less number, we divide 77, the greater number, by 21, the less, and obtain a remainder of 14. If 14 is a divisor of 21, it is also a divisor of 77, which equals 14 + 3 times 21 (see **287**). Dividing 21, the first divisor, by 14, the first remainder, we obtain a remainder of 7. Now, if 7 is a divisor of 14, it is a divisor of 21, which equals 7 + 14, and of 77, which equals 7 + 5 times 14 (see **287**). Dividing 14, the last divisor, by 7, the last remainder, we find that 7 is a divisor of 14. Hence, 7 is a common divisor of 21 and 77.

Having proved that 7 is a common divisor of 21 and 77, we must now prove that it is their greatest common divisor.

Since any number that is a common divisor of 21 and 77, must be a divisor of 14 (see **288**), a number greater than 14 can not be a common divisor of 21 and 77. Again, any number that is a common divisor of 14 and 21, must be a divisor of 7 (see **288**). Hence, 7, the greatest common divisor of itself and 14, is the greatest common divisor of 21 and 77.

PROBLEMS.

12. What is the greatest common divisor of 56 and 84? *28.*

13. What is the greatest common divisor of .102 and .153? *51.*

14. Find the greatest common divisor of 96, 120, and 168.

15. What is the length of the longest line that will exactly measure two fences, one 96 rods and the other 76 rods long?

296. The explanations and solutions given in **294**, **295**, are sufficient to establish the following

Rule for finding a Greatest Common Divisor.

I. *Separate the numbers into their prime factors.*

II. *Multiply together all the factors that are common. The product will be the greatest common divisor.* Or,

Divide the greater number by the less, the first divisor by the first remainder, the second divisor by the second remainder, and so on, until an exact divisor is obtained. This divisor will be the greatest common divisor.

NOTES.—1. By the Second Method, if more than two numbers are given, we must first find the greatest common divisor of two of them, then of their greatest common divisor and another of the numbers, and so on, till all the given numbers have been used. The last common divisor obtained will be the greatest common divisor of all the given numbers.

2. Only abstract numbers, or like concrete numbers of the same denomination, can have a common divisor.

3. The common divisor of two or more concrete numbers may be either an abstract or a concrete number. (Sec **98**, I., II., III.)

PROBLEMS.

16. What number is a common divisor of $25 and $60? *5, or $5.*

17. Find a common divisor of 16 A. and 28 sq. rd.

18. What is the greatest common divisor of 135 and 225?

19. How many common divisors have 1 pk. and 6 qt.? *One.*

20. The sides of my garden are 168 ft., 280 ft., 182 ft., and 252 ft. What is the greatest length of boards that I can use in fencing it, without cutting any of them? *14 feet.*

21. If 283.5 yd. Wamsutta, 567 yd. N. Y. Mills, and 445.5 yd. Lawrence Mills sheetings are in whole pieces of the greatest possible equal length, how many yards are there in each piece? *40.5.*

SECTION VI.

COMMON MULTIPLES.

CASE I.

Common Multiples or Dividends.

297. Ex. What number is a common multiple or dividend of 15 and 24?

SOLUTION.
15 × 24 = 360

EXPLANATION.—Since a common multiple of 15 and 24 is a number of which both 15 and 24 are factors (see **269**), and since any product must be a multiple of any set of factors which will produce it (see **57**, **285**), we multiply 15 and 24 together. The product, 360, is the common multiple required.

PROBLEMS.

1. Find a common multiple of 3, 4, and 6.

2. Find a common multiple of 5, 7, 32, and 10.

3. What number is a common multiple of 4.8, 9, and 5.25?
226.8, or any integral number of times 226.8.

4. What number is a common multiple of $15, $2, and $8.50?

5. Find a common multiple of 1 bu. 3 pk., 1 pk. 4 qt., and 5 qt. 1 pt. (= 5.5 qt.) *115 bu. 2 pk.*

CASE II.

Least Common Multiples or Dividends.

298. 24 is a common multiple of 4 and 6, because all the factors of 4 (2 and 2) and of 6 (2 and 3) are also prime factors of 24 (2, 2, 2 and 3). But 2, 2, and 3 are all the prime factors required to produce both 4 and 6; and a number that contains only the prime factors 2, 2, 3, will also contain 4 and 6 (see **285**). Multiplying these three factors together, we have 2 × 2 × 3 = 12; and since 12 contains all the prime factors of 4 and 6, and no other factors, it is their least common multiple.

Ex. Find the least common multiple of 18, 24, and 30.

SOLUTION.

18=2×3×3
24=2×2×2×3
30=2×3×5

Prime Factors required.

2×2×2×3×3×5=360

EXPLANATION.—Since the least common multiple of 18, 24, and 30 must contain only the prime factors of these numbers (see **270**), we separate each of the numbers into its prime factors. Since 24 has the greatest number of prime factors, we next, for convenience, write all the factors of 24 (2×2×2×3) in a line. Then, comparing the factors of 18 with these factors, we find that we have all the factors but a 3; and we write a 3 with the prime factors required. Again, comparing the factors of 30 with the prime factors required, we find that we have all the factors but a 5; and we write a 5 with the prime factors required. We now have all the prime factors of the given numbers, and no others; and multiplying them together, we obtain 360, their least common multiple.

PROBLEMS.

6. What is the least common multiple of 8, 12, and 14? *168.*
7. What is the least common multiple of $16 and $20?
8. Find the least common multiple of .6 and .8. *.24.*
9. What is the least number of which 75, 225, and 500 are factors?

299. *Rules for Finding Multiples.*

I. For a Common Multiple.

Multiply the numbers together. The product, or any number of times the product, will be a common multiple.

II. For the Least Common Multiple.

1. *Separate the numbers into their prime factors.*

2. *Multiply together all the prime factors of that number having the greatest number of prime factors, and those prime factors of the other numbers not found in the factors of the number taken. The product will be the least common multiple.*

NOTE.—Only abstract numbers, or like concrete numbers of the same denomination, can have a common multiple. See Manual.

PROBLEMS.

10. Find a common multiple of 36, 18, 24, and 12.

11. Find the least common multiple of the numbers given in Problem 10.

12. Find the least common multiple of 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72. *144.*

13. What number is the least common multiple of 9, 60, 45, 72, 15, 35, 18, 12?

14. Find the least common multiple of the nine digits. *2,520.*

15. What is the least common multiple of 2 yd. 1 ft., and 2 ft. 8 in.? *3 rd. 2 yd. 6 in.*

16. What is the smallest sum of money for which a person can purchase, either oxen at $85 each, or cows at $35 each? *$595.*

17. A can hoe a row of corn in a certain field in 30 minutes, B can hoe a row in 20 minutes, and C in 35 minutes. What is the least number of rows that each can hoe, in order that all may finish together?

18. In a factory are three wheels, which revolve once in 25, 30, and 50 seconds respectively. What is the least time in which all of them will make an exact number of revolutions? *2 min. 30 sec.*

CHAPTER 5. FRACTIONS.

SECTION I.

NOTATION AND PRINCIPLES.

300. The number *one half* may be obtained by dividing 1 into 2 equal parts; *one third*, by dividing 1 into 3 equal parts; *one fourth*, by dividing 1 into 4 equal parts; *one fifth*, by dividing 1 into 5 equal parts; and so on.

Again, *two* thirds may be obtained by dividing 2 into 3 equal parts, or by dividing 1 into three equal parts and taking two of those parts.

Two fourths may be obtained by dividing 2 into 4 equal parts; and *three* fourths, by dividing 3 into 4 equal parts.

Two fifths, *three* fifths, and *four* fifths may be obtained by dividing 2, 3, and 4, respectively, into 5 equal parts.

Halves, thirds, fourths, and fifths are written thus:

1 half, $\frac{1}{2}$,
1 third, $\frac{1}{3}$, 2 thirds, $\frac{2}{3}$,
1 fourth, $\frac{1}{4}$, 2 fourths, $\frac{2}{4}$, 3 fourths, $\frac{3}{4}$,
1 fifth, $\frac{1}{5}$, 2 fifths, $\frac{2}{5}$, 3 fifths, $\frac{3}{5}$, 4 fifths, $\frac{4}{5}$.

When 1 is divided into 6 equal parts, the parts are *sixths;* when into 7 equal parts, they are *sevenths;* when into 8 equal parts, *eighths;* and when into 9 equal parts, *ninths*.

Sixths, sevenths, eighths, and ninths are written thus:

1 sixth, $\frac{1}{6}$, 2 sevenths, $\frac{2}{7}$, 1 eighth, $\frac{1}{8}$, 2 ninths, $\frac{2}{9}$,
3 sixths, $\frac{3}{6}$, 3 sevenths, $\frac{3}{7}$, 3 eighths, $\frac{3}{8}$, 5 ninths, $\frac{5}{9}$,
5 sixths, $\frac{5}{6}$; 6 sevenths, $\frac{6}{7}$; 5 eighths, $\frac{5}{8}$; 6 ninths, $\frac{6}{9}$.

Numbers which express one of the equal parts of an integer; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{37}$; or which express an equal part of two or more integers, or two or more equal parts of a one; as, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{8}{21}$, $\frac{37}{256}$; form a class of numbers called Fractions. Hence,

301. A *Fractional Unit* is one of the equal parts into which a thing, or the unit 1, is divided. And

302. A *Fraction* is a number consisting of one or more fractional units.

A fractional unit may be either abstract or concrete. Thus, the fractional unit of $\frac{3}{4}$ is $\frac{1}{4}$, of \$$\frac{3}{4}$ is \$$\frac{1}{4}$, of $\frac{3}{4}$ ft. is $\frac{1}{4}$ ft., of $\frac{3}{4}$ lb. is $\frac{1}{4}$ lb.

A fraction is expressed by two numbers, written one under the other, with a horizontal line between them.

303. The *Terms* of a fraction are the two numbers used to express it. Thus, the terms of $\frac{5}{7}$ are 5 and 7.

304. The *Denominator* of a fraction is that term which expresses the number of equal parts into which *one* is divided; it is written below the horizontal line. And

305. The *Numerator* is that term which expresses the number of equal parts indicated by the fraction; it is written above the line. Thus, in the fraction $\frac{4}{5}$, the 5 is the denominator, and expresses that a unit or 1 is divided into 5 equal parts, and 4 is the numerator, and expresses that 4 of the equal parts (fifths) are indicated by the fraction.

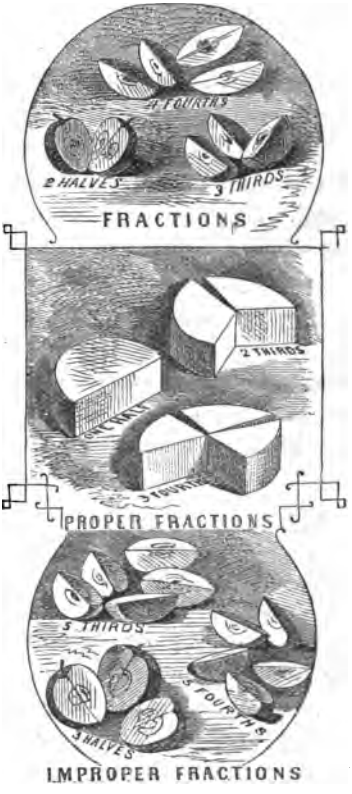
306. The *Reciprocal* of a number is the quotient of a *one* divided by that number. Thus, the reciprocal of 7 is $\frac{1}{7}$, of \$13 is \$$\frac{1}{13}$, of 25 bu. is $\frac{1}{25}$ bu.

307. The value of a fraction depends upon the relative values of its numerator and denominator.

I. *When the numerator and denominator are equal, the value of the fraction is 1;* because as many fractional units are expressed as equal an integral unit or 1. Thus, $\frac{7}{7}$, $\frac{15}{15}$, $\frac{236}{236}$.

II. *When the numerator is less than the denominator, the value of the fraction is less than 1;* because a less number of fractional units is expressed than equal an integral unit or 1. Thus, $\frac{3}{5}$, $\frac{7}{12}$, $\frac{15}{16}$, $\frac{158}{1728}$.

III. *When the numerator is greater than the denominator,* *the value of the fraction is more than 1;* because a greater number of fractional units is expressed than equal an integral unit or 1. Thus, $\frac{3}{2}$, $\frac{7}{4}$, $\frac{54}{13}$, $\frac{352}{67}$.



308. A ***Proper Fraction*** is a fraction whose value is less than 1; as $\frac{4}{9}$, $\frac{1}{10}$, $\frac{5}{6}$, $\frac{7}{16}$.

309. An ***Improper Fraction*** is a fraction whose value equals or exceeds 1; as $\frac{3}{3}$, $\frac{10}{10}$, $\frac{5}{3}$, $\frac{13}{10}$, $\frac{37}{15}$.

310. A ***Mixed Number*** is a number expressed by an integer and a decimal, or an integer and a fraction; as 3.7, 21.4, 9.35; $3\frac{5}{8}$, $11\frac{2}{3}$, $14\frac{20}{27}$.

NOTE.—In reading a mixed number, *and* belongs between the integer and the fraction or decimal.

311. ***Similar Fractions*** are fractions that have a common fractional unit; as $\frac{4}{7}$, $\frac{5}{7}$; $\frac{2}{9}$, $\frac{5}{9}$, $\frac{8}{9}$.

312. ***Dissimilar Fractions*** are fractions that have different fractional units; as $\frac{3}{5}$, $\frac{4}{7}$; $\frac{2}{7}$, $\frac{5}{8}$, $\frac{1}{2}$.

313. If we multiply the numerator of $\frac{1}{8}$ by 2, we obtain $\frac{2}{8}$. The fractional unit in $\frac{1}{8}$ and $\frac{2}{8}$ is the same. If we multiply the numerator of $\frac{1}{4}$ by 3, we obtain $\frac{3}{4}$,—a number of 3 times as many fractional units as $\frac{1}{4}$, each unit of both fractions being of the same value. That is, $\frac{1\times 2}{8}=\frac{2}{8}$, $\frac{1\times 2}{4}=\frac{2}{4}$, $\frac{1\times 3}{4}=\frac{3}{4}$, etc. Hence,

Multiplying the numerator multiplies the fraction. (See **272**.)

314. If we divide the numerator of $\frac{2}{8}$ by 2, we obtain $\frac{1}{8}$, a number of one half as many fractional units as $\frac{2}{8}$, each unit of both fractions being of the same value. So, also, if we divide the numerator of $\frac{6}{7}$ by 3, we obtain $\frac{2}{7}$. That is, $\frac{2\div2}{8}=\frac{1}{8}$, $\frac{2\div2}{4}=\frac{1}{4}$, $\frac{6\div3}{7}=\frac{2}{7}$, etc. Hence,

Dividing the numerator divides the fraction. (See **273.**)

315. If we multiply the denominator of $\frac{3}{4}$ by 2, we obtain $\frac{3}{8}$, a fraction of the same number of fractional units as $\frac{3}{4}$, each unit of the $\frac{3}{8}$ being one half the value of a unit of the $\frac{3}{4}$. So, also, if we multiply the denominator of $\frac{1}{2}$ by 2, we obtain $\frac{1}{4}$. That is, $\frac{1}{2\times2}=\frac{1}{4}$, $\frac{3}{4\times2}=\frac{3}{8}$, $\frac{2}{5\times3}=\frac{2}{15}$, etc. Hence,

Multiplying the denominator divides the fraction. (See **274.**)

316. If we divide the denominator of $\frac{3}{8}$ by 2, we obtain $\frac{3}{4}$, the number of fractional units in the $\frac{3}{4}$ being the same as in the $\frac{3}{8}$, while the value of each unit is 2 times as great. So, also, if we divide the denominator of $\frac{1}{6}$ by 3, we obtain $\frac{1}{2}$. That is, $\frac{3}{8\div2}=\frac{3}{4}$, $\frac{1}{4\div2}=\frac{1}{2}$, $\frac{3}{15\div3}=\frac{3}{5}$, etc. Hence,

Dividing the denominator multiplies the fraction. (See **275.**)

317. If we multiply both terms of $\frac{1}{4}$ by 2, we obtain $\frac{2}{8}$. The number of fractional units in $\frac{2}{8}$ is 2 times as many as in $\frac{1}{4}$, but the value of each unit is only one half as much. In other words, the fraction $\frac{1}{4}$ is multiplied by 2 by multiplying its numerator by 2, and the result ($\frac{2}{4}$) is divided by 2 by multiplying its denominator by 2. That is, $\frac{1\times2}{2\times2}=\frac{2}{4}$, $\frac{1\times2}{4\times2}=\frac{2}{8}$, $\frac{3\times4}{7\times4}=\frac{12}{28}$, etc. Hence,

Multiplying both terms of a fraction by the same number does not change its value. (See **276.**)

318. If we divide both terms of $\frac{4}{8}$ by 2, we obtain $\frac{2}{4}$. The number of fractional units in $\frac{2}{4}$ is one half as many as in $\frac{4}{8}$, but the value of each unit is 2 times as much. In other words, the fraction $\frac{4}{8}$ is divided by 2 by dividing its numerator by 2, and the result ($\frac{2}{8}$) is multiplied by 2 by dividing its denominator by 2. That is, $\frac{4\div2}{8\div2}=\frac{2}{4}$, $\frac{2\div2}{4\div2}=\frac{1}{2}$, $\frac{9\div3}{12\div3}=\frac{3}{4}$, etc. Hence, *Dividing both terms of a fraction by the same number does not change its value.* (See **277**.)

319. The deductions in the last six articles are the

General Principles of Fractions.

I. *A fraction is multiplied by multiplying its numerator or dividing its denominator.*

II. *A fraction is divided by dividing its numerator or multiplying its denominator.*

III. *The value of a fraction is not changed by either multiplying or dividing both terms by the same number.* See Manual.

320. In integers, decimals, and compound numbers the successive orders of units increase and decrease by fixed scales. In fractions the scales (that is, the number of fractional units required to equal an integral unit) vary with every change of the denominator. This feature of fractions gives rise to the principal difference between computations in fractions and integers, decimals, and compound numbers.

SECTION II.

REDUCTIONS.

CASE I.

Fractions to Lowest Terms.

321. A fraction is in its *Lowest Terms*, when its numerator and denominator are prime to each other; as $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{9}$, $\frac{23}{36}$.

When the terms of a fraction are not prime to each other, they have some common factor.

Ex. Reduce $\frac{18}{24}$ to its lowest terms.

EXPLANATION.—Since the value of a fraction is not changed by dividing both terms

FIRST SOLUTION.

$\frac{18}{24}=\frac{9}{12}=\frac{3}{4}$

by the same number (see **319**, III.), we reduce $\frac{18}{24}$ to lower terms, by dividing its terms by the common factor 2, ($\frac{18}{24}=\frac{9}{12}$); and the result, $\frac{9}{12}$, we reduce to still lower terms, by dividing its terms by the common factor 3, ($\frac{9}{12}=\frac{3}{4}$) as shown in the First Solution. Since the terms 3 and 4 are prime to each other, $\frac{3}{4}$ must be the lowest terms of the fraction $\frac{18}{24}$; and consequently $\frac{3}{4}$ is the result required. Or, we can reduce $\frac{18}{24}$ to its lowest terms at one operation, by dividing both terms by their greatest common divisor, 6, as shown in the Second Solution.

SECOND SOLUTION.

$\frac{18}{24}=\frac{3}{4}$

PROBLEMS.

1. Reduce the fraction $\frac{15}{20}$ to its lowest terms. *$\frac{3}{4}$.*
2. Reduce $\frac{18}{27}$ and $\frac{21}{35}$ to their lowest terms. *$\frac{2}{3}$, $\frac{3}{5}$.*
3. Reduce $\frac{15}{45}$, $\frac{48}{120}$, and $\frac{96}{112}$ to their lowest terms.
4. In what lower terms can the value of $\frac{32}{48}$ be expressed? *In four different fractions.*
5. What are the lowest terms of $\frac{25}{40}$ and $\frac{65}{85}$?
6. What are the lowest terms of the fractions $\frac{44}{55}$, $\frac{21}{77}$, $\frac{63}{117}$, and $\frac{108}{162}$? *$\frac{4}{5}$, $\frac{3}{11}$, $\frac{7}{13}$, and $\frac{2}{3}$.*

CASE II.

Fractions to Given Denominators.

322. Ex. Reduce $\frac{5}{7}$ to a fraction having 42 for a denominator.

EXPLANATION.—Since the value of a fraction is not changed by multiplying both terms by the same number (see **319**, III.), we multiply both terms of $\frac{5}{7}$ by an integer that will give 42 for a new denominator. We find this integer by dividing 42, the required denominator, by 7, the denominator of $\frac{5}{7}$. Then, multiplying both terms of $\frac{5}{7}$ by 6, the integer thus found, we have $\frac{30}{42}$, the fraction required.

SOLUTION.

$$\begin{array}{l|l} 42 & 7 \\ \hline 6 & \end{array}$$

$\frac{5\times 6}{7\times 6}=\frac{30}{42}$

Hence, $\frac{5}{7}=\frac{30}{42}$

PROBLEMS.

7. Reduce $\frac{3}{4}$ to sixteenths. $\frac{12}{16}$.

8. Reduce $\frac{3}{5}$ to tenths, and to twenty-fifths.

9. Reduce $\frac{4}{7}$ to a fraction having 63 for a denominator.

10. Reduce $\frac{7}{9}$ to 54ths, and $\frac{7}{12}$ to 84ths. $\frac{42}{54}$, $\frac{49}{84}$.

11. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{5}$ to sixtieths.

12. In $\frac{3}{13}$ are how many twenty-sixths, how many sixty-fifths and how many ninety-firsts? $\frac{6}{26}$, $\frac{15}{65}$, $\frac{21}{91}$.

CASE III.

Dissimilar Fractions to Similar Fractions.

323. Ex. 1. Reduce $\frac{1}{4}$ and $\frac{3}{5}$ to similar fractions.

EXPLANATION.—Fourths can not be reduced to fifths, nor fifths to fourths. But since 4 times 5 = 20, we reduce $\frac{1}{4}$ to twentieths by multiplying its terms by 5; and since 5 times 4 = 20, we reduce $\frac{3}{5}$ to twentieths by multiplying its terms by 4.

SOLUTION.

$$\frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

Hence, $\frac{1}{4}, \frac{3}{5} = \frac{5}{20}, \frac{12}{20}$.

Ex. 2. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{7}$ to similar fractions.

EXPLANATION.—Since the product of the denominators, 3 times 5 times 7, = 105, we may reduce these fractions to 105ths. This we do by multiplying the terms of the first fraction, $\frac{2}{3}$, by 5 and 7; the terms of the second, $\frac{4}{5}$, by 3 and 7; and the terms of the third, $\frac{6}{7}$, by 3 and 5. That is, we multiply the terms of each fraction by the denominators of the other fractions.

SOLUTION.

$$\frac{2 \times 5 \times 7}{3 \times 5 \times 7} = \frac{70}{105}$$

$$\frac{4 \times 3 \times 7}{5 \times 3 \times 7} = \frac{84}{105}$$

$$\frac{6 \times 3 \times 5}{7 \times 3 \times 5} = \frac{90}{105}$$

Hence, $\frac{2}{3}, \frac{4}{5}, \frac{6}{7} = \frac{70}{105}, \frac{84}{105}, \frac{90}{105}$.

From these examples it will be seen that

The denominator of the similar fractions is a common multiple of the denominators of all the given fractions.

NOTES.—1. Fractions having like denominators are said to have a *Common Denominator.*

2. Reducing dissimilar to similar fractions is sometimes called *reducing fractions to equivalent fractions having a common denominator.*

PROBLEMS.

13. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to similar fractions. $\frac{3}{6}$, $\frac{4}{6}$.

14. Reduce $\frac{5}{6}$ and $\frac{4}{5}$ to similar fractions.

15. What similar fractions are equal to $\frac{5}{9}$ and $\frac{7}{12}$?

16. What similar fractions are equal to $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$?

17. Reduce $\frac{1}{5}$ and $\frac{3}{97}$ to similar fractions. $\frac{97}{485}$, $\frac{15}{485}$.

18. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{5}{9}$ to similar fractions.

19. Reduce $\frac{9}{11}$, $\frac{5}{7}$, $\frac{2}{5}$, and $\frac{7}{12}$ to equivalent fractions having a common denominator. $\frac{3780}{4620}$, $\frac{3300}{4620}$, $\frac{1848}{4620}$, $\frac{2695}{4620}$.

20. Reduce $\frac{5}{8}$, $\frac{5}{9}$, and $\frac{5}{10}$ to equivalent fractions having a common denominator.

CASE IV.

Dissimilar Fractions to Least Similar Fractions.

324. Since, in reducing dissimilar to similar fractions, the common denominator must be a common multiple of the denominators of all the given fractions (see **323**), it follows that

The common denominator of least similar fractions must be the least common multiple of the denominators of all the given fractions.

325. Ex. Reduce $\frac{5}{6}$, $\frac{1}{4}$, and $\frac{4}{9}$ to least similar fractions.

Explanation.—We first find the least common multiple of all the given denominators 6, 4, 9, to be 36 (see **299**). Since 36 is the common denominator of the least similar fractions that are equal to the given fractions $\frac{5}{6}$, $\frac{1}{4}$, and $\frac{4}{9}$, we reduce each of these fractions to 36ths by Case II. (See **322**.)

SOLUTION.

$$6=2\times 3$$
$$4=2\times 2$$
$$9=3\times 3$$
$$3\times 3\times 2\times 2=36$$

$$\frac{36}{6}\lfloor 6 \qquad \frac{36}{9}\lfloor 4 \qquad \frac{36}{4}\lfloor 9$$

$$\frac{5\times 6}{6\times 6}=\frac{30}{36}$$
$$\frac{1\times 9}{4\times 9}=\frac{9}{36}$$
$$\frac{4\times 4}{9\times 4}=\frac{16}{36}$$

Hence, $\frac{5}{6}, \frac{1}{4}, \frac{4}{9}=\frac{30}{36}, \frac{9}{36}, \frac{16}{36}$.

NOTES.—1. The fractional units of dissimilar fractions are unlike, but the fractional unit of their equivalent similar fractions is common. (See 311.)

2. The fractional unit of least similar fractions is the greatest fractional unit common to the given dissimilar fractions. See Manual.

PROBLEMS.

21. Reduce $\frac{5}{9}$ and $\frac{5}{6}$ to least similar fractions. $\frac{10}{18}$, $\frac{15}{18}$.

22. Reduce $\frac{6}{15}$ and $\frac{7}{10}$ to least similar fractions. $\frac{4}{10}$, $\frac{7}{10}$.

23. What are the least similar fractions equal to $\frac{1}{6}$ and $\frac{3}{4}$?

24. What least similar fractions are equal to $\frac{5}{9}$, $\frac{1}{4}$, $\frac{7}{18}$, and $\frac{5}{6}$?

25. Reduce $\frac{5}{12}$, $\frac{1}{8}$, and $\frac{2}{3}$ to least similar fractions.

26. Reduce $\frac{11}{15}$, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{28}{45}$, and $\frac{1}{3}$ to least similar fractions.
$\frac{33}{45}$, $\frac{36}{45}$, $\frac{35}{45}$, $\frac{28}{45}$, $\frac{15}{45}$.

27. What is the fractional unit of the least similar fractions to which $\frac{8}{3}$, $\frac{3}{8}$, $\frac{5}{70}$, and $\frac{63}{108}$ can be reduced? $\frac{1}{168}$.

CASE V.

Improper Fractions to Integers or Mixed Numbers.

326. Ex. 1. How many ones in $\frac{21}{7}$?

SOLUTION.

$$\frac{21 \text{ sevenths.} \,\big\lfloor\, 7 \text{ sevenths.}}{3}$$

EXPLANATION.—Since every 7 sevenths are 1, 21 sevenths are as many 1's as the number of times 7 sevenths are contained in 21 sevenths, which is 3 times.

Ex. 2. Find the value of the improper fraction $\frac{17}{5}$.

SOLUTION.

$$\frac{17 \text{ fifths.} \,\big\lfloor\, 5 \text{ fifths.}}{3\frac{2}{5}}$$

EXPLANATION.—Since every 5 fifths are 1, 17 fifths are as many 1's as the number of times 5 fifths are contained in 17 fifths. The quotient figure is 3; and since the remainder is always like the dividend (see **109**, VIII.), and the dividend is fifths, the remainder 2, is 2 fifths or $\frac{2}{5}$. Writing the $\frac{2}{5}$ at the right of the quotient figure 3, we have $3\frac{2}{5}$, the value required.

Or, we may regard the numerator and denominator as dividend and divisor, and both concrete numbers (fifths).

Then, the quotient figure is 3, and the remainder, 2, is a less number to be divided by a greater, 5; and the result is $\frac{2}{5}$ (see **301**). Writing the $\frac{2}{5}$ at the right of the 3, we have the abstract number $3\frac{2}{5}$, as before. (See **109, III.**)

NOTE.—Any dividend may be written as the numerator, and the divisor as the denominator of a fraction.

PROBLEMS.

28. How many apples are $\frac{12}{4}$ apples?

29. In $\frac{56}{8}$ miles are how many miles?

30. Reduce the improper fraction $\frac{19}{4}$ to a mixed number. *$4\frac{3}{4}$.*

31. Reduce $\frac{143}{13}$ to an integer. *11.*

32. How many yards are $\frac{74}{16}$ yd.?

33. Find the integer or mixed number equal to each of the improper fractions $\$\frac{103}{8}$, $\frac{115}{24}$ da., $\frac{188}{19}$, $\frac{388}{12}$ ft., $\frac{2317}{27}$ cu. yd., $\frac{10000}{16}$ lb.

34. 29 quarter-dollars are how many dollars?

35. Reduce $\frac{56}{9}$, $\frac{31}{4}$, $\frac{63}{7}$, $\frac{47}{8}$, and $\frac{1596}{6}$ to integers or mixed numbers. *$6\frac{2}{9}$, $7\frac{3}{4}$, 9, $5\frac{7}{8}$, 266.*

CASE VI.

Integers or Mixed Numbers to Improper Fractions.

327. Ex. 1. Reduce the integer 8 to fifths.

SOLUTION.

5 fifths.
8
40 fifths.

EXPLANATION.—Since 1 is 5 fifths, 8 are 8 times 5 fifths, or 40 fifths.

Hence, $8 = \frac{40}{5}$.

Ex. 2. Reduce the mixed number $4\frac{3}{7}$ to an improper fraction.

FULL SOLUTION.	COMMON SOLUTION.
7 sevenths. *4* *28 sevenths.* *3 sevenths.* *31 sevenths.*	*$4\frac{3}{7}$* *7* *$28+3=31$*

EXPLANATION. — Since 1 is 7 sevenths, 4 are 4 times 7 sevenths, or 28 sevenths; and 28 sevenths + 3 sevenths are 31 sevenths.

Hence, $4\frac{3}{7} = \frac{31}{7}$.

The reduction of $4\frac{3}{7}$ to sevenths is similar to the reduction of a compound number of two denominations to the lower denomination. Thus, the 4 ones corresponds to the higher denomination, and the 3 sevenths to the lower. In the second or Common Solution we reduce the 4 ones to sevenths and add the given 3 sevenths, in the same manner as we would reduce 4 wk. 3 da. to days. (See **223.**)

PROBLEMS.

36. Reduce 12 to sevenths, and 13 to ninths. $\frac{84}{7}$, $\frac{117}{9}$.
37. In $5\frac{3}{8}$ are how many eighths? $\frac{43}{8}$.
38. Reduce $19\frac{3}{4}$ to an improper fraction.
39. Reduce $5\frac{2}{13}$ and $43\frac{1}{29}$ to improper fractions. $\frac{67}{13}$, $\frac{1248}{29}$.
40. What improper fractions are equal to $15\frac{5}{12}$ and $17\frac{4}{9}$?
41. Change $14\frac{13}{16}$, $12\frac{5}{33}$, and $11\frac{8}{9}$ to improper fractions.
42. In $365\frac{1}{4}$ days there are how many fourths of a day? $\frac{1461}{4}$ *da.*

328. Brief directions for performing the processes in the preceding six Cases form the

Rules for Reductions of Fractions.

I. A fraction to its lowest terms.

Cancel all the factors common to both terms.

II. Fractions to given denominators.

Divide the given denominator by the denominator of the fraction, and multiply both terms of the fraction by the quotient.

III. Dissimilar to similar fractions.

Multiply both terms of each fraction by the denominators of all the other fractions.

IV. Dissimilar to least similar fractions.

1. *For the least common denominator, find the least common multiple of all the denominators.*

2. *For each new numerator, divide the least common multiple by the denominator of each fraction, and multiply the numerator by the quotient.*

V. An improper fraction to an integer or a mixed number
Divide the numerator by the denominator.

VI. An integer or a mixed number to an improper fraction.

1. *Multiply the integer by the denominator, and if there be a numerator, add it to the product.*

2. *Write this result and the given denominator for the terms of the required fraction.*

PROBLEMS.

43. To what lower terms can $\frac{567}{1863}$ be reduced? $\frac{189}{621}$, $\frac{63}{207}$, $\frac{21}{69}$, $\frac{7}{23}$.

44. How many one hundred fifty-thirds are equal to eleven seventeenths? $\frac{99}{153}$.

45. Reduce $\frac{8}{15}$, $\frac{4}{21}$, and $\frac{8}{27}$ to equivalent fractions having a common denominator.

46. What similar fractions are equal to $\frac{7}{8}$, $\frac{7}{9}$, and $\frac{7}{10}$?

47. Reduce $\frac{5}{12}$, $\frac{13}{32}$, $\frac{4}{15}$, and $\frac{3}{10}$ to least similar fractions.

48. Reduce $\frac{61}{7}$, $\frac{213}{8}$, $\frac{126}{18}$, $\frac{151}{5}$, and $\frac{117}{13}$ to integers or mixed numbers.

49. Reduce 59 to a fraction having 59 for a denominator. Reduce it to 9ths. $\frac{3481}{59}$, $\frac{531}{9}$.

50. What least similar fractions are equal to $\frac{1}{5}$, $\frac{3}{7}$, $\frac{4}{15}$, $\frac{1}{21}$, and $\frac{11}{35}$?

51. What is the greatest common fractional unit of $\frac{17}{26}$, $\frac{20}{39}$, $\frac{7}{12}$, and $\frac{37}{52}$? $\frac{1}{156}$.

52. Find the lowest terms of $\frac{160}{1376}$, $\frac{291}{485}$, and $\frac{73}{2029}$.

53. Change $\frac{3}{19}$ to ninety-fifths, to one hundred seventy-firsts, and to two hundred ninths. $\frac{45}{95}$, $\frac{27}{171}$, $\frac{33}{209}$.

54. Reduce the fractions $\frac{11}{21}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{4}{7}$, and $\frac{15}{28}$ to eighty-fourths.

55. What are the lowest terms of $\frac{936}{1144}$, $\frac{297}{736}$, $\frac{3333}{14847}$, and $\frac{6778}{10167}$?

56. What similar fractions are equal to $\frac{5}{13}$ and $\frac{7}{16}$?

57. Reduce $\frac{1}{2}$, $\frac{3}{8}$, $\frac{8}{15}$, and $\frac{7}{15}$ to similar fractions.

58. Reduce $\frac{8}{9}$, $\frac{7}{8}$, $\frac{6}{7}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{2}{3}$ to fractions having a common fractional unit.

59. What least similar fractions are equal to $\frac{3}{10}$, $\frac{7}{8}$, $\frac{4}{15}$, and $\frac{1}{24}$?

60. Reduce $1000\frac{1}{1000}$ and $99\frac{1}{99}$ to improper fractions.

61. What improper fractions are equal to $67\frac{5}{13}$ and $123\frac{4}{7}$?

62. Reduce $\frac{18}{209}$, $\frac{1}{57}$, $\frac{5}{66}$, and $\frac{21}{38}$ to least similar fractions.
Their greatest common fractional unit is $\frac{1}{1254}$.

SECTION III.

ADDITION AND SUBTRACTION.

329. Since only like orders of units can be added one to another (see **39**, **II.**), or subtracted one from another (see **52**, II.), and dissimilar fractions are of unlike orders of units, it follows that they must be reduced to similar fractions, (that is, to the same fractional unit), before they can be added or subtracted.

CASE I.

All the Given Numbers Fractions.

330. Ex. 1. What is the sum of $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{6}$?

Explanation.—The given fractions being dissimilar, we first reduce them to the similar fractions $\frac{45}{60}$, $\frac{24}{60}$, and $\frac{10}{60}$. Since the parts of these similar fractions are all of the same kind or denomination (sixtieths), and since the numerators express the numbers of the parts, we add the similar fractions by adding their numerators, 45 + 24 + 10 = 79; and since the fractional unit of the parts is $\frac{1}{60}$, we write the denominator, 60, under the 79, making $\frac{79}{60}$. Then, reducing the $\frac{79}{60}$ to a mixed number, we have $1\frac{19}{60}$, the result required.

FIRST SOLUTION.

$$\frac{3}{4}+\frac{2}{5}+\frac{1}{6}=\frac{45}{60}+\frac{24}{60}+\frac{10}{60}=\frac{79}{60}=1\frac{19}{60}$$

SECOND SOLUTION.

$$\frac{3}{4}+\frac{2}{5}+\frac{1}{6}=\frac{45+24+10}{60}=\frac{79}{60}=1\frac{19}{60}$$

Ex. 2. Subtract $\frac{3}{5}$ from $\frac{7}{8}$.

Explanation.—The given fractions being dissimilar, we first reduce them to the similar fractions $\frac{35}{40}$ and $\frac{24}{40}$. Since all the parts of these similar fractions are of the same kind or denomination (fortieths), we subtract 24 fortieths from 35 fortieths, and the difference, $\frac{11}{40}$, is the result required.

FIRST SOLUTION.

$$\frac{7}{8}-\frac{3}{5}=\frac{35}{40}-\frac{24}{40}=\frac{11}{40}$$

SECOND SOLUTION.

$$\frac{7}{8}-\frac{3}{5}=\frac{35-24}{40}=\frac{11}{40}$$

In reducing dissimilar to similar fractions, the common denominator need be written but once, and the several numerators may be written above it, connected by the appropriate signs, as shown in the Second Solution of each of the two preceding examples.

From these examples we learn that

I. *The numerators of similar fractions only can be added or subtracted;* and

II. *The common denominator is written under the sum or difference.*

PROBLEMS.

1. What is the sum of $\frac{1}{2}$ and $\frac{2}{3}$? *$\frac{11}{12}$.* $\frac{7}{6}$

2. What is the sum of $\frac{1}{3}$ and $\frac{2}{5}$? *$\frac{11}{15}$.*

3. What is the difference between $\frac{5}{6}$ and $\frac{3}{4}$? *$\frac{1}{12}$.*

4. From $\frac{9}{10}$ subtract $\frac{4}{7}$. *$\frac{23}{70}$.*

5. William gathered $\frac{1}{2}$ bu. of butternuts one day, and $\frac{3}{5}$ bu. the next. How many did he gather in the two days?

6. From $\frac{7}{8}$ yd. of velvet a lady used $\frac{1}{4}$ yd. How much velvet had she left?

7. A Michigan farmer made $\frac{13}{20}$ T. of maple sugar, and sold $\frac{3}{8}$ T. How much sugar did he keep? *$\frac{11}{40}$ T.*

8. The tide rose $\frac{5}{6}$ ft. one hour, $\frac{13}{15}$ ft. the next hour, and $\frac{2}{3}$ ft. the third hour. How much did it rise in the three hours? *$2\frac{11}{30}$ ft.*

CASE II.

Any of the Given Numbers Mixed Numbers.

331. Ex. 1. What is the sum of $5\frac{2}{3}$, $\frac{4}{7}$, $6\frac{1}{2}$, and 11?

SOLUTION.

$$\begin{array}{rcr} 5\frac{2}{3} & = & 5\frac{28}{42} \\ \frac{4}{7} & = & \frac{24}{42} \\ 6\frac{1}{2} & = & 6\frac{21}{42} \\ 11 & = & 11 \\ \hline & & 23\frac{31}{42} \end{array}$$

EXPLANATION.—We write the given numbers in columns, integers under integers, and fractions under fractions. Reducing the fractional parts to similar fractions, we have $5\frac{2}{3} = 5\frac{28}{42}$, $\frac{4}{7} = \frac{24}{42}$, and $6\frac{1}{2} = 6\frac{21}{42}$. Adding the fractions, we have $\frac{73}{42}$ or $1\frac{31}{42}$. We write the $\frac{31}{42}$ in the result, and add the 1 with the given integers. 23, the sum of all the integers, written before the $\frac{31}{42}$, gives $23\frac{31}{42}$, the required sum.

Ex. 2. From $7\frac{1}{4}$ subtract $3\frac{5}{9}$.

SOLUTION.

$$\begin{aligned} 7\tfrac{1}{4} &= 7\tfrac{9}{36} \\ 3\tfrac{5}{9} &= 3\tfrac{20}{36} \\ \hline & 3\tfrac{25}{36} \end{aligned}$$

Explanation.—We write the subtrahend under the minuend, and reduce the fractional parts to similar fractions. Since $\frac{20}{36}$ can not be subtracted from $\frac{9}{36}$, and since the difference will not be affected by adding the same number to both minuend and subtrahend (see **52**, III.), we add $\frac{36}{36}$ (= 1) to the $\frac{9}{36}$ of the minuend, and 1 ($=\frac{36}{36}$) to the 3 of the subtrahend. We then subtract $\frac{20}{36}$ from $\frac{45}{36}$ $(=\frac{9+36}{36})$, and 4 from 7, writing the results, $\frac{25}{36}$ and 3, as the fractional and integral parts of the remainder. The result, $3\frac{25}{36}$, is the remainder required. See Manual.

When any of the given numbers are mixed numbers, we may

Regard the fractions as lower, and the integers as higher denominations, and add and subtract as in compound numbers.

PROBLEMS.

9. What is the sum of $4\frac{5}{6}$ and $3\frac{1}{4}$? *$8\frac{1}{12}$.*

10. From $6\frac{7}{10}$ subtract $2\frac{2}{5}$. *$4\frac{3}{10}$.*

11. A lady bought $15\frac{1}{2}$ yd. of delaine, $11\frac{3}{4}$ yd. of calico, and $4\frac{5}{8}$ yd. of merino. How many yards of dress goods did she buy?

12. I bought $13\frac{3}{8}$ cd. of wood, and at the end of a year, had $1\frac{7}{12}$ cd. left. How much had I used? *$11\frac{19}{24}$ cd.*

13. A mechanic spent $\$9\frac{7}{10}$ from his week's wages, and had $\$3\frac{4}{5}$ left. What was the amount of his wages?

14. A merchant sold a pair of fur gloves for $\$3\frac{1}{4}$, upon which his profit was $\$\frac{24}{25}$. What was the first cost of the gloves?

15. My farm consists of five fields that contain respectively $12\frac{7}{10}$ A., $15\frac{3}{8}$ A., $13\frac{5}{12}$ A., $11\frac{7}{20}$ A., and $14\frac{2}{3}$ A. How many acres in my farm? *$67\frac{61}{120}$.*

332. Upon the principles in **330, 331**, is based the

Rule for Addition and Subtraction of Fractions.

I. *Reduce dissimilar to similar fractions.*

II. For Addition. *Add the numerators.*
For Subtraction. *Subtract the numerator of the subtrahend from the numerator of the minuend.*

III. *Under the result write the common denominator.*

NOTES.—1. If the given fractions are reduced to least similar fractions, the numerators to be added or subtracted will be the smallest numbers possible.

2. In all final results reduce fractions to lowest terms, and improper fractions to integers or mixed numbers.

PROBLEMS.

16. What is the sum of $\frac{5}{7}$ and $\frac{1}{5}$? *$\frac{32}{35}$.*

17. From $\frac{2}{3}$ subtract $\frac{5}{16}$. *$\frac{17}{48}$.*

18. George paid \$$\frac{7}{8}$ for a pair of skates, and \$$\frac{13}{20}$ for straps. What was the whole cost?

19. From $7\frac{5}{12}$ subtract $6\frac{8}{9}$. *$\frac{19}{36}$.*

20. The parts are $4\frac{2}{3}$, $5\frac{1}{4}$, $3\frac{5}{6}$, $4\frac{7}{10}$, and $1\frac{11}{15}$. What is their sum?

21. Mary had \$$\frac{1}{2}$, but she spent \$$\frac{2}{5}$ for a ribbon. How much money has she left? *\$$\frac{1}{10}$.*

22. What is the sum of $\frac{5}{6}$, $\frac{2}{7}$, and $\frac{3}{8}$?

23. From $\frac{8}{9}$ subtract $\frac{11}{21}$. *$\frac{23}{63}$.*

24. A lady purchased a shawl for \$$8\frac{7}{8}$, and gave the merchant a 10-dollar bill. How much change should she receive?

25. If a family burn $\frac{3}{4}$ T. of coal in Dec., $\frac{7}{8}$ T. in Jan., and $\frac{13}{16}$ T. in Feb., how much do they burn in the three months?

26. How much greater is $\frac{1}{4}$ than $\frac{7}{34}$? *$\frac{3}{68}$.*

27. A merchant sold a lace collar for \$$\frac{15}{16}$, that had cost him \$$\frac{13}{20}$. How much was his profit? *\$$\frac{23}{80}$.*

28. A contractor having a contract to build $23\frac{17}{40}$ mi. of rail road, has completed $14\frac{7}{15}$ mi. How much has he yet to build?

29. From $32\frac{1}{17}$ subtract $\frac{11}{12}$. *$31\frac{29}{204}$.*

30. Add $\frac{25}{33}$, $\frac{1}{6}$, $\frac{7}{10}$, $\frac{1}{15}$, and $\frac{4}{11}$. *Sum, $2\frac{3}{55}$.*

31. A stone-mason in building a wall, used $\frac{3}{5}$ cd. of stone one day, $\frac{5}{12}$ cd. the second day, $\frac{3}{10}$ cd. the third day, and $\frac{4}{15}$ cd. the fourth day. How much stone did he use in the four days?

32. A founder used $\frac{14}{15}$ T. of iron in making $\frac{8}{9}$ T. of castings. How much was the waste? *$\frac{2}{45}$ T.*

33. If the less of two numbers is $7\frac{11}{18}$, and the greater is $27\frac{1}{10}$, what is the difference? *$19\frac{22}{45}$.*

34. Find the sum of $391\frac{4}{5}$, $19\frac{7}{51}$, $4\frac{7}{10}$, $57\frac{11}{34}$, and $\frac{8}{15}$. *$473\frac{42}{85}$.*

35. $\frac{5}{24}$ is how much greater than $\frac{7}{40}$?

36. What is the distance round a farm $\frac{7}{8}$ mi. long and $\frac{8}{15}$ mi. wide? *$2\frac{49}{60}$ mi.*

37. What is the sum of $\frac{47}{91}$ and $\frac{19}{26}$? What is their difference?

Sum, $1\frac{45}{182}$; difference, $\frac{3}{14}$.

38. The minuend is $11\frac{2}{15}$, and the subtrahend $5\frac{17}{25}$. What is the remainder?

39. A cake of ice $1\frac{11}{27}$ ft. thick floats with $\frac{19}{108}$ ft. of its thickness above the water. What thickness of the ice is under water?

$1\frac{25}{108}$ ft.

40. A farmer sold $13\frac{1}{8}$ T. of his hay crop, put $11\frac{4}{5}$ T. into his barn, and stacked $9\frac{7}{12}$ T. How much hay did he raise? *$34\frac{61}{120}$ T.*

SECTION IV.

MULTIPLICATION.

CASE I.

One Factor a Fraction.

333. Ex. 1. Multiply $\frac{8}{15}$ by 5.

FIRST SOLUTION.

$$\frac{8}{15}\times 5=\frac{40}{15}=2\frac{10}{15}=2\frac{2}{3}$$

SECOND SOLUTION.

$$\frac{8}{15\div 5}=\frac{8}{3}=2\frac{2}{3}$$

EXPLANATION.—In the First Solution we have multiplied 8, the numerator of the fraction, by 5, and in the Second Solution we have divided 15, the denominator, by 5 (see **319**, I.) The results in the two solutions are the same.

Ex. 2. Multiply 7 by $\frac{5}{14}$, or find $\frac{5}{14}$ of 7.

Explanation.—To multiply a number by 5 is to find 5 times the number; to multiply it by 1 is to find 1 time the number; to multiply it by $\frac{1}{14}$ is to find $\frac{1}{14}$ of it; and to multiply it by $\frac{5}{14}$ is to find 5 times $\frac{1}{14}$ or $\frac{5}{14}$ of it. In the First Solution we divide 7 by 14, and obtain $\frac{7}{14}$, which is $\frac{1}{14}$ of 7; and we then multiply this result by 5, and obtain $\frac{35}{14}$, or $2\frac{1}{2}$, which is $\frac{5}{14}$ of 7.

FIRST SOLUTION.

$$7 \div 14 = \frac{7}{14}$$

$$\frac{7}{14} \times 5 = \frac{35}{14} = 2\frac{7}{14} = 2\frac{1}{2}$$

SECOND SOLUTION.

$$7 \times \frac{5}{14} = \frac{35}{14} = 2\frac{7}{14} = 2\frac{1}{2}$$

Or

$$7 \times \frac{5}{14} = \frac{5}{2} = 2\frac{1}{2}$$

Since $7 \times \frac{5}{14} = \frac{5}{14} \times 7$ (see **80**, V.), in the Second Solution we multiply 7 and $\frac{5}{14}$ together, in the manner explained in Ex. 1.

PROBLEMS.

1. Multiply $\frac{3}{11}$ by 8, or find 8 times $\frac{3}{11}$. *$2\frac{2}{11}$.*
2. How much is 13 times $\frac{7}{9}$?
3. At \$$\frac{5}{8}$ a yard, how much will 7 yards of alpaca cost? *\$$4\frac{3}{8}$.*
4. Multiply 18 by $\frac{7}{12}$, or find $\frac{7}{12}$ of 18. *$10\frac{1}{2}$.*
5. How much is $\frac{5}{16}$ of 14 miles?
6. What is the product of 19 and $\frac{4}{17}$? Of 31 and $\frac{26}{41}$? Of $\frac{9}{29}$ and 34? *$4\frac{8}{17}$, $19\frac{27}{41}$, $10\frac{16}{29}$.*
7. How much will 12 gal. of kerosene cost, at \$$\frac{18}{20}$ per gal.?
8. A farmer bought $\frac{3}{4}$ bu. of grass seed @ \$5. How much did it cost him? *\$$3\frac{3}{4}$.*

CASE II.

Both Factors Fractions.

334. Ex. 1. Multiply $\frac{5}{6}$ by $\frac{3}{4}$, or find $\frac{3}{4}$ of $\frac{5}{6}$.

Explanation.—$\frac{3}{4}$ of any number is 3 times as much as $\frac{1}{4}$ of it, and $\frac{1}{4}$ of it is found by dividing it by 4. In the First Solution we multiply the denominator of $\frac{5}{6}$ by 4 to find $\frac{1}{4}$ of $\frac{5}{6}$ (see **319**, II.). We then multiply the numerator of the result, $\frac{5}{24}$, by 3, to find 3

FIRST SOLUTION.

$$\frac{5}{6 \times 4} = \frac{5}{24}$$

$$\frac{5^{\times 3}}{24} = \frac{15}{24} = \frac{5}{8}$$

SECOND SOLUTION.

$$\frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}$$

times $\frac{1}{4}$, or $\frac{3}{4}$, of $\frac{5}{6}$. This result, $\frac{15}{24}$, $=\frac{5}{8}$, the result required.

THIRD SOLUTION.

$$\frac{5}{\underset{2}{\cancel{6}}}\times\frac{\cancel{3}}{4}=\frac{5}{8}$$

In the First Solution we multiply the denominators 6 and 4 together for the denominator, 24, of the product; and the numerators 5 and 3 together for the numerator, 15, of the product. The Second Solution shows the work in the usual form.

Since the given numerators are factors of the numerator of the product, and the given denominators are factors of its denominator, we may cancel like factors from the numerators and denominators of the given fractions (see **328**, I.). The product will then be in its lowest terms, as shown in the Third Solution.

Ex. 2. Multiply $5\frac{2}{3}$ by $3\frac{7}{8}$.

SOLUTION.

$$5\tfrac{2}{3}\times 3\tfrac{7}{8}=\frac{17}{3}\times\frac{31}{8}=\frac{527}{24}=21\tfrac{23}{24}$$

EXPLANATION.—We first reduce the mixed numbers to improper fractions, and then multiply as in Ex. 1.

Ex. 3. What is the product of $\frac{2}{3}\times 4\frac{1}{2}\times 8$?

SOLUTION.

$$\frac{2}{3}\times 4\tfrac{1}{2}\times 8=\frac{\cancel{2}}{\cancel{3}}\times\frac{\overset{3}{\cancel{9}}}{\cancel{2}}\times\frac{8}{1}=24$$

EXPLANATION. — We reduce the mixed number $4\frac{1}{2}$ to an improper fraction, the integer 8 to the form of a fraction by writing 1 for its denominator, and then multiply as in Ex. 1.

NOTES.—1. The word *of* between fractions signifies multiplication. Thus, $\frac{4}{7}$ of $\frac{9}{10}=\frac{4}{7}\times\frac{9}{10}$ or $\frac{9}{10}\times\frac{4}{7}$; $\frac{2}{3}$ of $11=\frac{2}{3}\times 11$ or $11\times\frac{2}{3}$.

2. When a fraction is connected to any other number by *of*, the expression is commonly called a *Compound Fraction;* as $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{9}{20}$, $\frac{5}{6}$ of $12\frac{3}{4}$. $\frac{2}{3}$ of $\frac{5}{6}$ of 18.

PROBLEMS.

9. Multiply $\frac{5}{8}$ by $\frac{7}{10}$, and $\frac{4}{9}$ by $\frac{12}{17}$. *$\frac{7}{16}$, $\frac{16}{51}$.*
10. How much is $\frac{9}{10}$ of $\frac{15}{16}$ of a mile? *$\frac{27}{32}$ mi.*
11. Multiply $7\frac{4}{5}$ by $4\frac{4}{9}$. *$34\frac{2}{3}$.*
12. Multiply $4\frac{5}{19}$ by $6\frac{1}{11}$, $\frac{5}{21}$ by $3\frac{4}{11}$, and $18\frac{4}{13}$ by 9.
13. The factors are $\frac{13}{18}$ and $\frac{9}{17}$. What is the product?
14. What is the product of $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{4}{9}$? *$\frac{1}{9}$.*

335. From **333** and **334** we deduce the

Rule for Multiplication of Fractions.

I. *Reduce mixed numbers to improper fractions, and integers to the form of fractions.*

II. *Multiply all the numerators together for the numerator, and all the denominators for the denominator, of the product.*

PROBLEMS.

15. A fruit dealer put up 30 baskets of peaches, putting $\frac{5}{8}$ of a bushel in each basket. How many peaches did he put in all the baskets? *$18\frac{3}{4}$ bu.*

16. If a man earns \$78 in a month, how much will he earn in $\frac{3}{4}$ of a month? *\$$58\frac{1}{2}$.*

17. A man who owned $\frac{2}{3}$ of a vessel, sold $\frac{5}{8}$ of his share. What part of the vessel did he sell? *$\frac{5}{12}$.*

18. How much will $\frac{7}{8}$ of a yard of linen cost, at \$$\frac{17}{20}$ a yard?

19. Multiply $11\frac{1}{8}$ by $\frac{6}{7}$. *$9\frac{15}{28}$.*

20. How much is 8 times $9\frac{1}{7}$? *$73\frac{1}{7}$.*

21. John's kite string is 118 yards long, and Frank's is $\frac{17}{20}$ as long. What is the length of Frank's kite string?

22. What is the product of 43 multiplied by $\frac{5}{13}$? *$16\frac{7}{13}$.*

23. How many days' work can 54 men do in $\frac{7}{8}$ of a day?

24. What is the product of $\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}$; or what is the cube of $\frac{3}{4}$?

25. How much will $4\frac{3}{4}$ bu. of sweet potatoes cost, at \$$1\frac{4}{5}$ a bushel?

26. How many square rods are there in a lot $15\frac{4}{9}$ rd. long and $12\frac{6}{7}$ rd. wide? *$198\frac{4}{7}$.*

27. What will be the cost of $\frac{27}{32}$ A. of land, at \$156 an acre?

28. What is the product of $\frac{5}{12}$ of $\frac{6}{7}$ of $\frac{14}{15}$? *$\frac{1}{3}$.*

29. If it takes $1\frac{7}{8}$ bu. of wheat to seed 1 acre, how many bushels will it take to seed $17\frac{4}{5}$ acres? *$33\frac{3}{8}$.*

30. If in talking, a man speaks 75 words in a minute, how many words will he utter in $\frac{11}{15}$ of a minute?

31. Raise $\frac{1}{3}$ to the fourth power, $\frac{2}{5}$ to the sixth power, $\frac{5}{8}$ to the fifth power, and square $\frac{24}{25}$.

32. $\frac{3}{4}$ of $\frac{1}{6}$ of $\frac{3}{5}$ of a ream of paper is what part of a ream?

33. A railroad train ran at the rate of 22 miles an hour, for $5\frac{3}{10}$ hours. How far did it run? *$116\frac{3}{5}$ mi.*

34. If it takes a man $\frac{3}{4}$ of a day to mow an acre of grass, how long will it take him to mow $\frac{7}{10}$ of an acre?

35. Cube $6\frac{1}{4}$, and square $16\frac{2}{3}$. *$244\frac{9}{64}$, $277\frac{7}{9}$.*

36. What is the product of $\frac{4}{7}$ of $\frac{2}{3}$ of $\frac{1}{2}$ multiplied by $\frac{2}{3}$ of $\frac{7}{9}$ of $\frac{3}{8}$? *$\frac{1}{27}$.*

SECTION V.

DIVISION.

CASE I.

The Divisor an Integer.

336. Ex. Divide $\frac{6}{7}$ by 3.

FIRST SOLUTION.

${\frac{6}{7}}^{+3}=\frac{2}{7}$.

SECOND SOLUTION.

$\frac{6}{7_{\times 3}}=\frac{6}{21}=\frac{2}{7}$.

THIRD SOLUTION.

$\frac{6}{7}\div 3=\frac{1}{\cancel{3}}$ *of* $\frac{\overset{2}{\cancel{6}}}{7}=\frac{2}{7}$

EXPLANATION.—In the First Solution we divide the numerator, 6, of the dividend by the divisor, 3 (see **319**, II.); and in the Second Solution we multiply the denominator, 7, of the dividend by the divisor. The result in each solution is $\frac{2}{7}$, the required quotient. Since to divide a number by 3 is to find $\frac{1}{3}$ of it (see **334**), in the Third Solution we find $\frac{1}{3}$ of $\frac{6}{7}$ as in multiplication of fractions (see **335**), and the result is $\frac{2}{7}$, the same as before.

PROBLEMS.

1. Divide $\frac{7}{8}$ by 5, and $\frac{16}{21}$ by 12. *$\frac{7}{40}$, $\frac{4}{63}$.*

2. What is the quotient of $4\frac{4}{9}$ divided by 15 $(=\frac{40}{9})$?

3. If a family consume 5 bar. of flour in a year, in what time will they consume 1 bar.?

4. A dealer in real estate sold $\frac{33}{40}$ of an acre of land in 6 equal building lots. How much land did each lot contain? *$\frac{11}{80}$ A.*

5. If a carpenter can build $13\frac{1}{5}$ rd. of picket fence in 3 days, how many rods can he build in 1 day? *$4\frac{2}{5}$.*

CASE II.

The Divisor a Fraction.

FIRST METHOD.

337. Ex. 1. How many times is $\frac{8}{9}$ contained in 7?

EXPLANATION.—Since the quotient is not changed by multiplying both dividend and divisor by the same number (see **276**), we multiply them both by 9, and thus obtain 63 for a new dividend and 8 for a new divisor. Then, $63 \div 8 = \frac{63}{8} = 7\frac{7}{8}$, the required quotient. In the First Solution the numbers are written as in division of integers and decimals; but the common manner of writing the numbers is shown in the Second Solution.

FIRST SOLUTION.

$$\begin{array}{r|l} 7 & \frac{8}{9} \\ \underline{9} & \underline{9} \\ \underline{63} & 8 \\ 7\frac{7}{8} & \end{array}$$

SECOND SOLUTION.

$$7 \times 9 = 63, \text{ and } \frac{8}{9} \times 9 = 8$$
$$63 \div 8 = \frac{63}{8} = 7\frac{7}{8}$$
$$\text{Hence, } 7 \div \frac{8}{9} = 7\frac{7}{8}.$$

Ex. 2. Divide $\frac{7}{8}$ by $\frac{3}{4}$.

EXPLANATION. — We first multiply both dividend and divisor by 4, the denominator of the divisor, and then divide the new dividend, $\frac{28}{8}$, by the new divisor, 3, as in Case I. Hence,

To divide by a fraction consists of two operations,—a multiplication by the denominator, and a division by the numerator.

FIRST SOLUTION.

$$\begin{array}{r|l} \underline{\frac{7}{8} \times 4} & \underline{\frac{3}{4} \times 4} \\ \underline{\frac{28}{8}} & 3 \\ \multicolumn{2}{c}{\frac{28}{24} = 1\frac{4}{24} = 1\frac{1}{6}} \end{array}$$

SECOND SOLUTION.

$$\frac{7}{8} \times 4 = \frac{28}{8}$$
$$\frac{28}{8} \div 3 = \frac{28}{24} = 1\frac{4}{24} = 1\frac{1}{6}$$

SECOND METHOD.

338. We have seen (**97**, (1) and (1)) that when the divisor is a concrete number, the dividend must also be a concrete number. We have also seen (**303**) that the denominator of a fraction gives denomination or name to the fractional units. We may therefore regard any numerator as one or more concrete units. Hence,

When the divisor is a fraction, the dividend and divisor should be reduced to similar fractions, before dividing.

339. Ex. What is the quotient of $\frac{7}{8}$ divided by $\frac{2}{3}$?

EXPLANATION. — In the First Solution we reduce both dividend and divisor to similar fractions (twenty-fourths), and then divide 21 twenty-fourths by 16 twenty-fourths, in the same manner as we divide 21 by 16. The result, $1\frac{5}{16}$, is the quotient required.

FIRST SOLUTION.

$$\frac{7}{8} \div \frac{2}{3} = \frac{21}{24} \div \frac{16}{24} = \frac{21}{16} = 1\frac{5}{16}$$

SECOND SOLUTION.

$$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16} = 1\frac{5}{16}$$

If we change the places of the terms of the divisor, and multiply the dividend, $\frac{7}{8}$, by $\frac{3}{2}$, the fraction thus formed, we shall multiply the same numbers together as in the First Solution. This is shown in the Second Solution. That is,

To divide by a fraction, is the same as to change the places of the terms of the divisor, and multiply the dividend by the fraction thus formed.

NOTE 1.—When the places of the terms of a fraction are changed, as $\frac{4}{7}$, $\frac{7}{4}$, the fraction is said to be *inverted.*

PROBLEMS.

6. Divide 6 by $\frac{4}{7}$. *$10\frac{1}{2}$.*

7. What is the quotient of $\frac{5}{6}$ divided by $\frac{3}{7}$? *$1\frac{17}{18}$.*

8. How many times is $2\frac{1}{12}$ contained in $2\frac{1}{2}$? ($2\frac{1}{12}=\frac{25}{12}$, and $2\frac{1}{2}=\frac{5}{2}$.)

9. At \$$\frac{4}{5}$ per cwt., how many hundred-weight of feed can be bought for \$12?

10. How many quarts of chestnuts can be bought for \$$\frac{3}{5}$, at \$$\frac{2}{25}$ per quart?

11. If a man can plow $\frac{7}{8}$ A. of fallow in a day, how long will it take him to plow $5\frac{15}{32}$ A.?

12. What is the quotient of 1 divided by $\frac{2}{5}$; or, what is the reciprocal of the fraction $\frac{2}{5}$? *$\frac{5}{2}$.*

NOTES.—2. From this problem we see that the *reciprocal of a fraction* is 1 divided by the fraction; or, it is the fraction inverted.

3. Division of fractions is sometimes expressed, by writing the dividend above, and the divisor below a horizontal line. Thus, $\frac{\frac{1}{2}}{8}, \frac{5}{\frac{4}{15}}, \frac{6}{7\frac{1}{5}}, \frac{\frac{3}{4}\text{ of }2}{8\times3\frac{1}{4}}$. Such expresssions are often called *Complex Fractions.*

340. The processes developed in **337, 338, 339,** are all included in the following

Rule for Division of Fractions.

I. *Reduce mixed numbers to improper fractions, and integers to the form of fractions.*

II. *Multiply the dividend by the reciprocal of the divisor.*

PROBLEMS.

13. Divide $\frac{15}{16}$ by 10, and $\frac{20}{23}$ by 16. *$\frac{3}{32}$, $\frac{5}{92}$.*

14. How many yards of gingham @ $\$\frac{7}{24}$, can be bought for \$4 ?

15. What is the quotient of 26 divided by $\frac{39}{45}$? *30.*

16. If 12 tea-spoons weigh $\frac{9}{20}$ of a pound, how much does each spoon weigh ?

17. Divide $\frac{2}{9}$ by $\frac{11}{15}$, and $\frac{9}{10}$ by $\frac{3}{40}$. *$\frac{10}{33}$, 12.*

18. At $\$\frac{7}{8}$ a pound, how much tea can be bought for $\$\frac{21}{50}$? *$\frac{12}{25}$ lb.*

19. Divide $13\frac{4}{7}$ by 25, and $16\frac{11}{25}$ by 9. *$\frac{19}{35}$, $1\frac{62}{75}$.*

20. A locomotive ran $22\frac{1}{2}$ miles in 35 minutes. What was the rate per minute ? *$\frac{9}{14}$ mi.*

21. Divide $7\frac{1}{3}$ by $\frac{6}{7}$, and $1\frac{1}{15}$ by $\frac{4}{5}$.

22. What is the cost of a pair of skates, if $\frac{17}{35}$ of their cost is $\$\frac{17}{20}$?

23. How many times is 17 contained in $234\frac{3}{5}$? *$13\frac{4}{5}$.*

24. $\dfrac{\frac{12}{13}}{\frac{6}{65}}$, and $\dfrac{45\frac{5}{9}}{3\frac{1}{27}}$ equal what numbers ? *10, 15.*

25. If $\frac{8}{15}$ bu. of salt can be made from 48 gal. of salt water, how much salt can be made from 1 gal. ?

26. What is the quotient of $\frac{3}{4}$ divided by $3\frac{3}{8}$? *$\frac{2}{9}$.*

27. If $\frac{1}{12}$ bu. of mortar cover 1 sq. yd. of wall, how many square yards will $5\frac{1}{4}$ bu. cover ? *63.*

28. Divide $11\frac{2}{7}$ by $3\frac{3}{8}$, and $16\frac{1}{2}$ by $6\frac{1}{3}$.

29. If $12\frac{1}{2}$ lb. of rice cost $\$1\frac{9}{16}$, how much will 1 lb. cost ? *$\$\frac{1}{8}$.*

30. How many gallons of oysters, at $\$1\frac{2}{5}$ a gallon, can be bought for $\$11\frac{9}{10}$?

31. If $\frac{27}{32}$ oz. of gold be obtained from 18 cwt. of gold quartz, what is the yield from 1 cwt. ? *$\frac{3}{64}$ oz.*

32. A lawyer's clerk wrote 36 pages in $6\frac{2}{3}$ hours. How much did he write in 1 hour? *$5\frac{2}{5}$ pages.*

33. I bought $14\frac{2}{3}$ qt. of vinegar for $\$\frac{22}{25}$. What was the price per quart?

34. At $\$5\frac{1}{3}$ a bushel, how much clover seed can be bought for $\$\frac{2}{3}$?

35. If $8\frac{2}{3}$ qt. of strawberries can be bought for $\$\frac{39}{50}$, what is the price per quart? *$\$\frac{9}{100}$.*

36. If 1 rod of fence require $74\frac{1}{4}$ ft. of boards, how many rods will require $1811\frac{7}{10}$ ft.?

37. $\dfrac{\frac{7}{12} \text{ of } 2\frac{5}{8}}{\frac{5}{8} \text{ of } 8\frac{3}{4}}$ = what number? *$\frac{7}{25}$.*

38. A plank $18\frac{2}{3}$ ft. long and $\frac{1}{6}$ ft. thick, contains $2\frac{1}{3}$ cu. ft. What is its width? *$\frac{3}{4}$ ft.*

SECTION VI.

REVIEW PROBLEMS IN FRACTIONS.

1. If a ship sails 1 mi. in $\frac{1}{10}$ h., how far will she sail in 14 h.?

2. Add $3\frac{4}{7}$, $\frac{3}{11}$, $17\frac{1}{2}$, and $6\frac{2}{3}$. *$28\frac{5}{462}$.*

3. \$900 is $\frac{4}{15}$ of what I paid for my house and lot. How much did they cost me? *\$3,375.*

4. What is the difference between $\frac{7}{8}$ and $\frac{6}{7}$?

5. A miller paid $\$2{,}156\frac{7}{10}$ for $1{,}540\frac{1}{2}$ bu. of wheat. What was the price per bushel? *$\$1\frac{2}{5}$.*

6. A regiment, when it was mustered out of service, consisted of 305 men, which was $\frac{5}{17}$ of the original number. How many men belonged to the regiment at first? *1,037.*

7. Add $\frac{3}{10}$, $\frac{7}{15}$, and $\frac{5}{6}$.

8. A man having a lot containing $\frac{19}{20}$ A. of land, sold from it $\frac{9}{25}$ A. to one man, and $\frac{1}{8}$ A. to another. How much land had he left? *$\frac{93}{200}$ A.*

9. How long must I rent a house at $\$22\frac{1}{2}$ a month, to cancel a debt of \$423? *$18\frac{4}{5}$ mo.*

10. If $\frac{1}{4}$ rm. of letter paper cost $\$\frac{7}{8}$, what is the price per ream?

11. A jeweler melted together $\frac{4}{5}$ oz. of gold, $\frac{5}{8}$ oz. of silver, and $\frac{1}{4}$ oz. of copper. How much did the mixture weigh?

12. From $11\frac{1}{11}$ subtract $10\frac{1}{10}$. *$\frac{109}{110}$.*

13. $\frac{9}{10}$ of $\frac{5}{8}$ of $\frac{4}{7}$ of $15\frac{3}{4}$ = what number? *$5\frac{1}{16}$.*

14. A man bought a cow, paying $\$20\frac{1}{4}$ down, which was $\frac{9}{10}$ of the cost. How much did the cow cost? *$\$22\frac{1}{2}$.*

15. What is the sum of $3\frac{19}{20}$, $\frac{3}{4}$, $2\frac{9}{10}$, and $1\frac{4}{5}$? *$9\frac{2}{5}$.*

16. Divide $\frac{7}{12}$ of $2\frac{5}{8}$ by $\frac{5}{8}$ of $8\frac{3}{4}$. *$\frac{7}{25}$.*

17. A farmer has $\frac{13}{16}$ mi. of rail fence on his farm, $\frac{27}{40}$ mi. of stone fence, $\frac{9}{20}$ mi. of board fence, and $\frac{1}{5}$ mi. of picket fence. How many miles of fence has he on his farm?

18. The greater of two fractions is $\frac{4}{5}$ and the less is $\frac{37}{60}$. What is the difference?

19. At $\$1\frac{9}{20}$ a hundred-weight, how much will it cost to transport 15 hundred-weight from Buffalo to Boston?

20. The minuend is $1\frac{1}{50}$, and the subtrahend is $\frac{48}{49}$. What is the remainder? *$\frac{99}{2450}$.*

21. I sold a quantity of wool for $\$536\frac{3}{4}$, which was $1\frac{17}{21}$ times its cost. How much did it cost me? *$\$296\frac{5}{8}$.*

22. How much is $\frac{7}{15}$ of $\frac{4}{27}$ of $\frac{11}{14}$ of $3\frac{1}{3} \times 4\frac{2}{7}$? *$\frac{440}{567}$.*

23. Bell-metal is composed of $\frac{4}{5}$ copper and $\frac{1}{5}$ tin. How much of each of these metals is there in a church bell that weighs $\frac{87}{100}$ T.? *Copper, $\frac{87}{125}$ T.; Tin, $\frac{87}{500}$ T.*

24. Multiply $\frac{5}{17}$ by $\frac{8}{15}$; $\frac{18}{25}$ by $\frac{15}{22}$; $\frac{1}{51}$ by $\frac{11}{88}$; and $\frac{219}{427}$ by $\frac{123}{159}$.

25. How much will $19\frac{1}{2}$ bu. of apples cost, at the rate of $\$4\frac{7}{10}$ for $11\frac{3}{4}$ bu.? *$\$7\frac{4}{5}$.*

26. How much will 35 men earn in $19\frac{1}{2}$ days, at $\$1\frac{3}{10}$ a day?

27. How many loads of sand at $\$\frac{5}{8}$ a load, will pay for $290\frac{5}{8}$ yards of plastering at $\$\frac{1}{5}$ a yard? *93.*

28. How many yards of cloth $\frac{7}{8}$ yd. wide, will line $23\frac{1}{3}$ yd., $1\frac{1}{4}$ yd. wide? *$33\frac{1}{3}$.*

29. A seamstress bought a sewing-machine for \$56.50, paying \$25 down. How much must she save from her earnings each month, to pay for it in 6 months?

CHAPTER 6.

CONVERSE OPERATIONS.

SECTION I.

CONVERSE OPERATIONS IN THE DIFFERENT CLASSES OF NUMBERS.

341. Addition, subtraction, multiplication, and division are often called the *Fundamental Rules of Arithmetic.*

342. Addition is putting together, and subtraction is taking away, or taking apart; multiplication is repeated addition, and division repeated subtraction of the same number. Hence, addition and multiplication are the reverse of subtraction and division.

343. ***Converse Operations*** are those arithmetical processes which are the reverse of each other.

CASE I.

Converse Operations in the Fundamental Rules.

344. The sum of the parts 73 and 48 is 121; $73+48=121$. This sum minus either part $\left\{\begin{matrix}121-73=48\\121-48=73\end{matrix}\right\}$ equals the other part. Hence,

Addition and subtraction are converse operations.

345. The product of the factors 57 and 26 is 1482; $57\times26=1482$.
This product divided by either factor $\left\{\begin{matrix}1482\div57=26\\1482\div26=57\end{matrix}\right\}$ equals the other factor. Hence,

Multiplication and division are converse operations.

346. From **344, 345,** we learn that

I. *Either part is the difference between the sum and the other part.*

II. *The minuend is the sum of the subtrahend and remainder.*

III. *Either factor is the quotient of the product divided by the other factor.*

IV. *The dividend is the product of the divisor and quotient.*

NOTE 1.—Addition may be proved by subtraction, and subtraction by addition. So also multiplication may be proved by division, and division by multiplication.

PROBLEMS.

1. The sum of two parts is 219.5, and one of them is 96.875. What is the other? *122.625.*

2. The subtrahend is 27$\frac{3}{4}$, and the remainder 16$\frac{7}{15}$. What is the minuend? *44$\frac{13}{60}$.*

3. What number must I add to 4 rd. 7 ft., that the sum may be 1 mi.?

4. The sum of three parts is 298, and two of the parts are 47.5 and 5.95. What is the other part?

NOTE 2.—Any one of the parts is the difference between the sum and the sum of the other parts. *244.55.*

5. The sum of three parts is 43$\frac{5}{8}$, and two of them are 17$\frac{4}{5}$ and $\frac{3}{4}$. What is the other part?

6. The divisor is .25, and the quotient .344. What is the dividend?

7. The product of three numbers is 3402, and two of them are 9 and 27. What is the other number?

NOTE 3.—Any factor is the quotient of the product divided by the product of the other factors.

8. The product of three factors is 19$\frac{1}{2}$, and two of them are 1$\frac{4}{9}$ and 2$\frac{5}{8}$. What is the other? *5$\frac{1}{7}$.*

9. The sum of two numbers is 1,765, and their difference is 235. What is the greater number?

NOTE 4.—The sum of two numbers plus their difference equals two times the greater number. See Manual. *1000.*

10. The sum of two numbers is 71$\frac{29}{30}$, and their difference is 16$\frac{7}{15}$. *What are the numbers?* *44$\frac{13}{60}$, 27$\frac{3}{4}$.*

CASE II.

Multiplication and Division by Factors of Composite Numbers.

347. Ex. 1. Multiply 67 by 48.

EXPLANATION.—Since 48 = 6 times 8, 48 times 67 = 6 times 8 times 67, which is 3216.

SOLUTION.

$$\begin{array}{r} 48=6\times 8 \\ 67 \\ 6 \\ \hline 402 \\ 8 \\ \hline 3216 \end{array}$$

Ex. 2. Divide 3216 by 48.

EXPLANATION.—Since 48 is 6 times 8, $\frac{1}{48}$ of any number is $\frac{1}{6}$ of $\frac{1}{8}$ of the number. We find $\frac{1}{6}$ of $\frac{1}{8}$ of 3216 by dividing first by 8 and then by 6. Hence,

SOLUTION.

$$\begin{array}{r|l} 3216 & \underline{8} \\ \hline 402 & \underline{6} \\ \hline 67 & \end{array}$$

Rule for Multiplying or Dividing by a Composite Number.

Multiply or divide successively by any set of factors of the number.

PROBLEMS.

11. Multiply 293 by 24.

12. How many square rods are there in a field 41.25 rd. long and 35 rd. wide? *1443.75.*

13. How much will 4.5 bu. of wheat cost, at \$1.93¾ a bushel? (4.5=9×.5.) *\$8.71875.*

14. Divide 2124 by 72.

15. A peat company sold 54 tons of peat for \$202.50. What was the price per ton? *\$3.75.*

16. A farmer sowed 38 bu. 2 pk. of barley on 28 A. of land. How much did he sow to the acre? *1 bu. 1 pk. 4 qt.*

17. A man cleared 13¾ A. of woodland, cutting 49 cords of wood to the acre. How many cords did he cut?

18. If 6.4 tons of porcelain clay cost \$112, what is the cost of .81 of a ton? *\$14.17½.*

CASE III.

Multiplication and Division by Aliquot Parts.

348. An *Aliquot Part* of a number is any one of its exact divisors. Thus, 5 is $\frac{1}{2}$ of 10, 4 in. are $\frac{1}{3}$ ft., 6 h. are $\frac{1}{4}$ da., etc.

The aliquot parts of any number may be found by dividing that number successively by 2, 3, 4, 5, 6, etc.

349. *The Unit of an Aliquot Part* is that number which is divided to obtain the part.

350. TABLE OF ALIQUOT PARTS.

Aliquot Parts of	1	10	100 or \$1.00	1000	1 Ton of 2000 lb.	1 ft., or 1 doz.	1 lb. of 16 oz.	1 yd.	1 A.
One half is	$\frac{1}{2}$	5	50	500	1000	6	8 oz.	1 ft. 6 in.	80 sq. rd.
One third is	$\frac{1}{3}$	$3\frac{1}{3}$	$33\frac{1}{3}$	$333\frac{1}{3}$	$666\frac{2}{3}$	4			
One fourth is	$\frac{1}{4}$	$2\frac{1}{2}$	25	250	500	3	4 oz.	9 in.	40 sq. rd.
One fifth is	$\frac{1}{5}$	2	20	200	400				32 "
One sixth is	$\frac{1}{6}$	$1\frac{2}{3}$	$16\frac{2}{3}$	$166\frac{2}{3}$	$333\frac{1}{3}$	2			
One eighth is	$\frac{1}{8}$	$1\frac{1}{4}$	$12\frac{1}{2}$	125	250		2 oz.	$4\frac{1}{2}$ in.	20 sq. rd.
One tenth is	$\frac{1}{10}$	1	10	100	200				16 "
One twelfth is	$\frac{1}{12}$		$8\frac{1}{3}$	$83\frac{1}{3}$		1			
etc.									

351. Ex. 1. Multiply 937 by $166\frac{2}{3}$.

SOLUTION.

937000 ⌊*6*

$156166\frac{2}{3}$

EXPLANATION.—Since $166\frac{2}{3}$ is $\frac{1}{6}$ of 1000, $166\frac{2}{3}$ times any number is $\frac{1}{6}$ of 1000 times that number. We therefore multiply 937 by 1000, and divide the product, 937000, by 6.

Ex. 2. What will 40 sq. rd. of land cost, at \$275 per acre?

SOLUTION.

\$275 ⌊*4*

\$68.75

EXPLANATION.—Since \$275 is the price of 1 acre, 40 sq. rd. or $\frac{1}{4}$ A. will cost $\frac{1}{4}$ of \$275. We therefore divide \$275 by 4.

Ex. 3. Divide 2775 by $33\frac{1}{3}$.

SOLUTION.

27.75

3

83.25

EXPLANATION.—Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, $33\frac{1}{3}$ is contained in any number 3 times as many times as 100 is contained in that number. We therefore divide 2775 by *100, and* multiply the quotient, 27.75, by 3.

Ex 4. If 4 eggs cost \$.11, what is the price per dozen?

EXPLANATION.—4 eggs are ⅓ of a dozen, and 1 dozen eggs cost 3 times as much as ⅓ dozen. We therefore multiply \$.11, the price of ⅓ dozen, by 3.

SOLUTION.

\$.11
3
\$.33

352. These illustrations are sufficient to establish the following

Rules for Multiplying and Dividing by Aliquot Parts.

I. The multiplier an aliquot part.

1. When the unit of the aliquot part is any power of 10 :—*Multiply by the unit, and divide the product by the number of aliquot parts in the unit.*

2. When the unit of the aliquot part is 1 :—*Divide by the number of aliquot parts in the unit.*

II. The divisor an aliquot part.

1. When the unit of the aliquot part is any power of 10 :—*Divide by the unit, and multiply the quotient by the number of aliquot parts in the unit.*

2. When the unit of the aliquot part is 1 :—*Multiply by the number of aliquot parts in the unit.*

PROBLEMS.

19. Multiply 364 by 1¼.

20. Divide 455 by 1¼.

21. How much will 12½ bu. of millet cost, at \$3.42 a bushel?

22. If 12½ bu. of millet cost \$42.75, what is the price per bushel?

23. What is the product of 333⅓ times 198?

24. What is the quotient of 66000 divided by 333⅓?

25. How much will 83⅓ A. of land cost, at \$92 an acre?

26. If 83⅓ A. of land cost \$7666.66⅔. what is the price per acre?

27. Multiply 7.14 by 16⅔.

28. Divide 119 by 16⅔.

29. How many bushels of potatoes, at \$.33⅓ a bushel, can be bought for \$19.50? *58.5.*

30. At \$.06$\frac{1}{4}$ a dozen, how much will 144 dozen clothes-pins cost?

31. At \$.25 a yard, how much will 37.75 yards of shirting come to? *\$9.43$\frac{3}{4}$.*

32. What is the cost of 376 bushels of corn, at \$1.12$\frac{1}{2}$ per bushel?

33. How much will 625 bushels of potatoes come to, at \$.75 a bushel? (\$1$-$\$$\frac{1}{4}$$=$\$$\frac{3}{4}$$=$\$.75.)

34. How much will 250 lb. of iron cost, at \$65 a ton? *\$8.12$\frac{1}{2}$.*

35. If it costs \$483 to build 66$\frac{2}{3}$ rd. of Macadamized road, how much will it cost to build 83$\frac{1}{3}$ rd.? *\$603.75.*

36. A gardener raised 23 bu. of strawberries from a piece of land 8 rd. long and 4 rd. wide. What was the yield per acre? *115 bu.*

SECTION II.

CONVERSE REDUCTIONS.

CASE I.

Decimals to Fractions, and Fractions to Decimals.

353. All decimals may be written in two forms, the decimal and the fractional. Thus 7 tenths is .7 or $\frac{7}{10}$; 59 thousandths is .059 or $\frac{59}{1000}$; 3 ten-thousandths is .0003 or $\frac{3}{10000}$, etc. In the decimal form the denomination or unit is indicated by the position of the decimal point, and in the fractional form it is expressed by the denominator.

Ex. 1. Express .075 in the fractional form.

EXPLANATION.—We write the number without the decimal point, and express its denomination or unit by the known denominator, 1000.

SOLUTION.

$$.075 = \frac{75}{1000} = \frac{3}{40}$$

Ex. 2. Reduce .008$\frac{1}{3}$ to the fractional form.

SOLUTION.

$$.008\tfrac{1}{3} = \frac{8\frac{1}{3}}{1000} = \frac{25}{3000} = \frac{1}{120}$$

Ex. 3. Reduce $\frac{7}{8}$ to the decimal form.

SOLUTION.

$$\begin{array}{l|l} 7.000 & 8 \\ \hline .875 & \end{array}$$

EXPLANATION.—Since $\frac{7}{8}$ expresses the quotient of 7 divided by 8, we annex decimal ciphers to 7, and divide by 8, as in division of decimals. (See **152.**)

354. From these explanations we deduce the following

Rules for the Converse Reductions of Decimals and Fractions.

I. A decimal to a fraction.

Write the given number of decimal units, omit the decimal point, and express the denomination or fractional unit by a denominator.

II. A fraction to a decimal.

Annex a decimal cipher or ciphers to the numerator, and divide by the denominator.

PROBLEMS.

1. Reduce .375 to a fraction.
2. Reduce $\frac{3}{8}$ to a decimal.
3. What fraction equals $.16\frac{2}{3}$?
4. What decimal equals $\frac{1}{8}$?
5. Reduce 6.75 to a mixed fractional number.
6. Reduce $6\frac{3}{4}$ to a mixed decimal number.
7. .00004 of a mile = what fractional part of a mile?
8. $\frac{1}{25000}$ of a mile = what decimal part of a mile?
9. Reduce $\frac{3}{160}$ T. to the decimal of a ton. *.01875 T.*
10. What fractional part of a day = $.2\frac{3}{4}$ da.? *$\frac{11}{40}$ da.*
11. Reduce .06875 to the fractional form.
12. Reduce $7\frac{3}{40}$ to a mixed decimal number. *7.075.*
13. What fractional part of a cord equals .85 cd.?
14. Reduce $\frac{5}{13}$ to a decimal. *$.3\frac{11}{13}$ or $.38\frac{6}{13}$ or $.384\frac{8}{13}$.*

NOTE.—Sometimes the decimal is interminable. In such cases a fraction may be written after the decimal figures; thus, $.3\frac{1}{3}$, $.73\frac{8}{9}$; or the quotient may be carried to any desired number of decimal places, and the sign + placed after it to show that the division is incomplete, or that there was a remainder after the last decimal figure of the quotient was obtained. Thus, $\frac{2}{3} = .666+$; $\frac{3}{7} = .428571+$. See Manual.

CASE II.

Denominate Decimals to Compound Numbers, and Compound Numbers to Denominate Decimals.

355. Ex. 1. Reduce .75 rd. to a compound number.

EXPLANATION.—We reduce the .75 rd. to yards by multiplying by 5.5; the decimal part of this result, .125 yd., to feet by multiplying by 3; and this result, .375 ft., to inches by multiplying by 12, as in reduction of compound numbers (see **225**). The 4 rd. and 4.5 in. taken together form the required compound number, 4 rd. 4.5 in.

SOLUTION.

$$\begin{array}{r} .75\ \textit{rd.}\\ \underline{5.5}\\ 375\\ \underline{375\ \,}\\ 4.125\ \textit{yd.}\\ \underline{3}\\ .375\ \textit{ft.}\\ \underline{12}\\ 4.500\ \textit{in.} \end{array}$$

Hence, .75 *rd.* = 4 *yd.* 4.5 *in.*

Ex. 2. Reduce 2 pk. 3 qt. 1 pt. to the decimal of a bushel.

EXPLANATION.—We write the denominations in order in a column, with the lowest at the top. We reduce the 1 pt. to the decimal of a quart by dividing by 2, as in division of decimals, and annex the result to the quarts, making 3.5 qt. We reduce the 3.5 qt. to the decimal of a peck by dividing by 8, and annex the result to the pecks, making 2.4375 pk. We then reduce this result to the decimal of a bushel by dividing by 4, as in reduction of compound numbers. (See **225**.)

SOLUTION.

$$\begin{array}{l} \underline{1.0\ \textit{pt.}}\ \lfloor 2\\ \underline{3.5\ \textit{qt.}}\ \lfloor 8\\ \underline{2.4375\ \textit{pk.}}\ \lfloor 4\\ \ \ .609375\ \textit{bu.} \end{array}$$

Hence, *2 pk. 3qt. 1 pt.=.609375 bu.*

Ex. 3. Reduce 4 yd. 4.5 in. to the decimal of a rod.

EXPLANATION.—Since there are 0 ft. in the compound number, we write a cipher in the place of feet in the column, and then *proceed* as in Ex. 2.

SOLUTION.

$$\begin{array}{l} \underline{4.5\ \textit{in.}}\ \lfloor \underline{12}\\ \underline{0.375\ \textit{ft.}}\ \lfloor \underline{3}\\ 4.1'25\ \textit{yd.}\ \left\lfloor \begin{array}{l} \underline{5.5}\\ .75\ \textit{rd.} \end{array}\right.\\ \underline{385\ \ }\\ \ \ 275\\ \ \ \underline{275} \end{array}$$

356. From these explanations we deduce the following

Rules for the Converse Reductions of Denominate Decimals and Compound Numbers.

I. A denominate decimal to a compound number.

1. *Multiply the decimal by the number which it takes of the next lower denomination to equal one of the given denomination.*

2. *Treat the decimal part of the product thus obtained in the same manner, and also the decimal part of each succeeding product, until there is no decimal in it, or until the lowest denomination is reached.*

3. *Write the integral parts of the several results and the final result in order, for the required compound number.*

II. A compound number to a denominate decimal.

1. *Write the denominations of the compound number in a column, with the lowest at the top.*

2. *Divide the lowest denomination by the number which it takes of that denomination to equal one of the next higher, and annex the result to the given number of the next higher denomination.*

3. *Treat the result thus obtained, and each succeeding result, in the same manner, until the whole has been reduced to the required denomination.*

PROBLEMS.

15. In .8 lb. Troy there are how many ounces and pennyweights?

16. 9 oz. 12 pwt. are what part of a pound Troy?

17. Reduce .21675 of a ton to a compound number.

18. Reduce 4 cwt. 33 lb. 8 oz. to the decimal of a ton.

19. Reduce .26 of a bushel to a compound number.

20. Reduce 1 pk. .64 pt. to the decimal of a bushel.

21. How many days and hours in .75 of the year 1875?

22. What part of a diurnal revolution does the earth make in 15 h. 50 min. 24 sec.? *.66.*

23. How much wheat must be sowed upon .85 of an acre, at the rate of 1 bushel to the acre? *3 pk. 3 qt. .4 pt.*

24. What part of a rod = 2 yd. 2 ft. 3 in.? *.5.*

25. Reduce 4 da. 4 h. 48 min. to the decimal of a week.

26. Reduce .45 of a cord to a compound number. *3 cd.ft. 9.6 cu.ft.*

CASE III.

Denominate Fractions to Compound Numbers, and Compound Numbers to Denominate Fractions.

357. Ex. 1. Reduce $\frac{4}{11}$ sq. mi. to a compound number.

SOLUTION.

$\frac{4}{11}$ *sq. mi.* $\times 640 = \frac{2560}{11} = 232\frac{8}{11}$ *A.*

$\frac{8}{11}$ *A.* $\times 160 = \frac{1280}{11} = 116\frac{4}{11}$ *sq. rd.*

$\frac{4}{11}$ *sq. rd.* $\times 30\frac{1}{4} = \frac{\cancel{4}}{11} \times \frac{121}{\cancel{4}} = 11$ *sq. yd.*

Hence, $\frac{4}{11}$ *sq. mi. = 232 A. 116 sq. rd. 11 sq. yd.*

EXPLANATION.—We reduce the $\frac{4}{11}$ sq. mi. to acres, by multiplying by 640; the fractional part of this result, $\frac{8}{11}$ A., to square rods, by multiplying by 160; and the fractional part of this result, $\frac{4}{11}$ sq. rd., to square yards, by multiplying by $30\frac{1}{4}$ (= 30.25); as in reduction of compound numbers (see **225**, I.). The 232 A., 116 sq. rd., and 11 sq. yd., taken together, form the required compound number.

Ex. 2. Reduce 22 h. 13 min. 20 sec. to the fraction of a day.

SOLUTION.

20 *sec.* $\div 60 = \frac{20}{60} = \frac{1}{3}$ *min.*

13 *min.* $+ \frac{1}{3}$ *min.* $= 13\frac{1}{3}$ *min.* $= \frac{40}{3}$ *min.*

$\frac{40}{3}$ *min.* $\div 60 = \frac{40}{180} = \frac{2}{9}$ *h.*

22 *h.* $+ \frac{2}{9}$ *h.* $= 22\frac{2}{9}$ *h.* $= \frac{200}{9}$ *h.*

$\frac{200}{9}$ *h.* $\div 24 = \frac{200}{216} = \frac{25}{27}$ *da.*

Hence, *22 h. 13 min. 20 sec.* $= \frac{25}{27}$*da.*

EXPLANATION.—We reduce the 20 sec. to the fraction of a minute, by dividing by 60, and annex or add the result to the minutes, making $13\frac{1}{3}$ min. We next reduce the $13\frac{1}{3}$ min., $= \frac{40}{3}$ min., to the fraction of an hour, by dividing by 60, and add the result to the hours, making $22\frac{2}{9}$ h. We then reduce this result, $22\frac{2}{9}$ h. $= \frac{200}{9}$ h., to the fraction of a day, by dividing by 24, as in reduction of compound numbers (see **225**, II.). The final result, $\frac{25}{27}$ da., is the denominate fraction required.

358. From these examples we deduce the following

Rules for the Converse Reductions of Denominate Fractions and Compound Numbers.

I. A denominate fraction to a compound number.

1. *Multiply the fraction by the number which it takes of the next lower denomination to equal one of the given denomination.*

2. *Treat the fractional part of the product thus obtained in the same manner, and also the fractional part of each succeeding product, until there is no fraction in it, or until the lowest denomination is reached.*

3. *Write the integral parts of the several results and the final result in order, for the required compound number.*

II. A compound number to a denominate fraction.

1. *Divide the lowest denomination by the number which it takes of that denomination to equal one of the next higher, express the result in a fraction, and annex it to the given number of the next higher denomination.*

2. *Treat the result thus obtained, and each succeeding result, in the same manner, until the whole has been reduced to the required denomination.*

PROBLEMS.

27. In \$$\frac{5}{16}$ how many cents and mills?

28. In 31 cents 2.5 mills how many dollars?

29. Reduce $\frac{8}{15}$ of a ream to a compound number.

30. Reduce 10 quires 16 sheets to the fraction of a ream.

31. Reduce £$\frac{17}{32}$ to a compound number.

32. Reduce 10 s. 7 d. 2 far. to the fraction of a pound.

33. Reduce $\frac{2}{3}$ of a square mile to a compound number.

34. Reduce 426 A. 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in. to tho fraction of a square mile.

35. What part of a bushel is 3 pk. ½ pt.? *$\frac{97}{128}$.*

36. A tobacco grower had $\frac{11}{12}$ of an acre of tobacco, which yielded at the rate of a ton to the acre. How much tobacco was in the crop? *18 cwt. 33⅓ lb., or 1833⅓ lb.*

37. If 11 silver forks weigh 1 pound of silver, how much will 1 set weigh? *6 oz. 10 pwt. 21$\frac{9}{11}$ gr.*

38. What part of a hogshead is 60 gal. 2 gi.?

39. What part of a bissextile year is 219 da. 14 h. 24 min.? *⅗.*

40. How many powders of 12 grains each will ½ ounce of quinine make?

SECTION III.

PRICE, QUANTITY, AND COST.

359. In all transactions of purchase and sale, and of labor and wages, four elements are considered, viz., Price, the Unit of Price, Quantity, and Cost.

360. *Price* is the sum paid or allowed for a unit, or a fixed number of units of the commodity; as one, a dozen, a hundred.

361. The *Unit of Price* is the number of units of the commodity upon which the price is based.

362. *Quantity* is the number of units or parts of a unit of the commodity.

363. *Cost* is the whole sum paid or allowed for the entire quantity.

CASE I.

Price and Quantity given, to find Cost.

364. Ex. 1. At $3.50 a day, what sum can a mechanic earn in 17¾ days?

EXPLANATION.—In this example 1 day is the unit of price. In 17¾ days a man can earn 17¾ times as much as he can in 1 day, or 17¾ times $3.50, which is $62.12½.

SOLUTION.

$$\begin{array}{r} \$3.50 \\ \underline{17\frac{3}{4}} \\ 875 \\ \underline{3} \\ 2625 \\ 2450 \\ \underline{350} \\ \$62.125 \end{array}$$

Ex. 2. How much will 760 strawberry plants cost, at $1.75 a hundred?

EXPLANATION.—Since in this example 1 hundred is the unit of price, we reduce the 760 to hundreds, which we do by dividing by 100. Since 1 hundred plants cost $1.75, 7.60 hundred plants will cost 7.60 times $1.75, or $13.30.

SOLUTION.

$$\begin{array}{r} 760 = 7.60 \text{ hundreds} \\ \$1.75 \\ \underline{7.60} \\ 10500 \\ \underline{1225} \\ \$13.30 \end{array}$$

Ex. 3. How much must I pay for 1968 hop poles, at $43.75 per thousand?

EXPLANATION.—Since 1 thousand is the unit of price, we first reduce 1968 to thousands, and then proceed as in Ex. 2.

SOLUTION.

$$\begin{array}{r} 1968 \div 1000 = 1.968 \\ \$43.75 \\ \underline{1.968} \\ 35000 \\ 26250 \\ 39375 \\ \underline{4375} \\ \$86.10 \end{array}$$

Ex. 4. How much will 3754 pounds of flax cost, at $40 per ton?

SOLUTION.

$$\begin{array}{r} 3754 \div 2000 = 3.754 \div 2 = 1.877 \\ 1.877 \\ \underline{\$40} \\ \$75.080 \end{array}$$

EXPLANATION.—Since 1 ton (=2000 pounds) is the unit of price, we first reduce 3754 pounds to tons, which we do by dividing by the factors 1000 and 2 (see **347**). We then multiply $40, the price of 1 ton, by 1.877, the number of tons, as in Ex. 1. Hence,

The product of the price multiplied by the number of units of price equals the cost.

PROBLEMS.

1. How much will 183.76 A. of land cost, at $56.25 per acre?

2. A lady bought ⅝ yd. of velvet, at $4.50 a yard. How much did it cost her? *$2.81¼.*

3. How much will 385 lb. of beef cost, at $11.50 per hundred weight? *$44.27½.*

4. A builder bought 15,650 brick, at $9.50 per thousand. What was the cost? *$148.67½.*

5. At $3.50 a ton, how much will 4,680 lb. of plaster cost?

365. A written statement containing a list of goods sold and their prices and cost, with the names of buyer and seller, and the date of the transaction, is a *Bill*. Finding the cost of each *item* or article, is *Extending the Item;* and the total cost of the items is the *Footing.*

Extend the items, and find the footings in the following bills:

6. Cleveland, *Jan. 2, 1869.*

Mr. *Henry Williams*

Bought of **James Parsons.**

4 lb.	*Java Coffee,*	*@ $ .45*	*$1.80*
½ ,,	*Blk. Tea,*	*,, 1.50*	*.75*
15 ,,	*Coffee Sugar,*	*,, .16*	*2.40*
2 ,,	*Cut Loaf,,*	*,, .18*	*.36*
4 oz.	*Nutmegs,*	*,, .12*	*.48*
2½ doz.	*Eggs,*	*,, .22*	*.55*
1	*Broom,*		*.31*
	Rec'd Payment,		*$6.65*

James Parsons.

7. Springfield, *Nov. 6, 1869.*

Mr. *Thos. Baker*

Bought of **Brown & Thomas.**

1 ps.	*Sheeting, Atlantic Mills, 40¼ yd.,*	*@ $ .16*
13 yd.	*Merrimac Prints,*	*,, .15*
2¼ ,,	*Broadcloth,*	*,, 5.25*
9 ,,	*Merino, Royal Purple,*	*,, 1.44*

1 8/12 doz. Buttons, @ 40¢; Twist, 5¢; Basting Cotton, 10¢.

$33.98

CASE II.

Price and Cost given, to find Quantity.

366. Ex. 1. How many pounds of wool, at $.56 a pound, can be bought for $122.92?

SOLUTION.

```
$122.92 | $.56
 112    |------
------  | 219½
  109
   56
  ---
  532
  504
  ---
  28/56 = ½
```

EXPLANATION.—Since $.56 will buy 1 pound, $122.92 will buy as many pounds as the number of times $.56 are contained in $122.92, which is 219½ times.

Ex. 2. How many hop poles can be bought for $86.10, at $43.75 per thousand?

SOLUTION.

```
$86.10 | $43.75
 4375  |-------
 -----    1.968
 42350        1000
 39375     --------
 ------    1968.000
  29750
  26250
  -----
   35000
   35000
   -----
```

EXPLANATION.—Since $43.75 will buy 1 thousand poles, $86.10 will buy as many thousand poles as the number of times $43.75 are contained in $86.10, which is 1.968 times. 1.968 thousand = 1.968 × 1000 = 1968.

Ex. 3. At $95 a ton, how many pounds of iron can be bought for $83.60?

SOLUTION.

```
$83.60 | $95
  760  |-----
 ----- |  .88
  760  |  2000
  760  |-------
  ---   1760.00
```

EXPLANATION.—Since $95 will buy 1 ton, $83.60 will buy as many tons as the number of times $95 are contained in $83.60, which is .88 of 1 time, or .88 T. We reduce the .88 T. to pounds, by multiplying it by 2000, and obtain 1760, the required number of pounds. Hence,

The quotient of the cost divided by the price of a unit equals the number of units of price.

PROBLEMS.

8. At $56.25 per acre, how many acres of land can be bought for $10336.50 ? *183.76.*

9. How many gallons of molasses, at $.87½ a gallon, can be bought for $27.56¼ ?

10. My winter's supply of coal cost me $48.47, at $9.25 a ton. How much coal did I buy ? *5 T. 480 lb.*

11. A butcher received $44.27½ for 385 lb. of beef. How much did he receive per hundred-weight ? *$11.50.*

12. A brick-maker sold brick at $9.50 per thousand, and received $148.67½. How many brick did he sell ? *15,650.*

13. How many pounds of plaster, at $3.50 per ton, can be bought for $8.19 ? *4,680.*

CASE III.

Quantity and Cost given, to find Price.

367. Ex. 1. If 65 army wagons cost $23725, what is the price of one wagon ?

SOLUTION.

$$\begin{array}{r|l}
\$23725 & 65 \\
\underline{195} & \overline{\$365} \\
422 & \\
\underline{390} & \\
325 & \\
\underline{325} &
\end{array}$$

EXPLANATION. — One wagon will cost $\frac{1}{65}$ as much as 65 wagons. Since 65 wagons cost $23725, 1 wagon will cost $\frac{1}{65}$ of $23725, which is $365.

Ex. 2. If the transportation of 425 lb. of freight cost $11.22, what is the price per cwt. ?

SOLUTION.

$$425 \text{ lb.} = 4.25 \text{ cwt.}$$

$$\begin{array}{r|l}
\$11.22 & 4.25 \\
\underline{8\,50} & \overline{\$2.64} \\
2\,72'0 & \\
\underline{2\,55\,0} & \\
1700 & \\
\underline{1700} &
\end{array}$$

EXPLANATION.—Since 1 cwt. (= 100 lb.) is the unit of price, we first reduce the 425 lb. to cwt. Since the transportation of 4.25 cwt. costs $11.22, the transportation of 1 cwt. will cost as many dollars as the number of times 4.25 is contained in 11.22, which is 2.64 times, or $2.64.

Ex. 3. What is the price per ton for hay, when 1680 lb. cost $12.18?

SOLUTION.

1680 lb.÷2000=.84 T.

$$\begin{array}{r|l} \$12.18 & .84 \\ \underline{84} & \overline{\$14.50} \\ 378 & \\ \underline{336} & \\ 420 & \\ \underline{420} & \end{array}$$

Explanation.—Since 1 ton (= 2000 lb.) is the unit of price, we first reduce 1680 lb. to tons. We now have $12.18 the cost of .88 T., and we find the cost of a ton, as in Ex. I., by dividing the cost by the quantity. Hence,

The quotient of the cost divided by the quantity expressed in units of price, equals the price.

PROBLEMS.

14. What price per acre must be paid for 183.76 acres of land, to have it cost $10336.50? *$56.25.*

15. If ⅝ yd. of velvet cost $2.81¼, what is the price per yard?

16. At $11.50 per cwt., how much beef can be bought for $44.27½? *385 lb.*

17. If I pay $148.67½ for 15650 brick, what is the price per thousand? *$9.50.*

18. What is the price per ton for plaster, when 4680 lb. cost $8.19? *$3.50.*

368. Upon the principles deduced in **364**, **366**, **367**, are based the

Rules for the Converse Operations in Price, Quantity, and Cost.

I. Price and quantity given, to find cost.

Reduce the quantity to units of price, and multiply the price by this result.

II. Price and cost given, to find quantity.

Divide the cost by the price of a unit.

III. Quantity and cost given, to find price.

Reduce the quantity to units of price, and divide the cost by this result.

Note 1.—In business, the abbreviation C. is often used for 100, and M. for 1000. See Manual.

PROBLEMS.

19. How much will 3 lb. 8 oz. of opium cost, at $4.75 a pound?

NOTE 2.—Since 1 pound is the unit of price, we reduce the ounces to the decimal or fraction of a pound.

20. $16.62½ will buy how many pounds of opium, at $4.75 a pound?

21. An apothecary paid $16.62½ for 3 lb. 8 oz. of opium. What was the price per pound?

22. I bought 765 pickets for my door-yard fence, at $1.12½ per C. How much did they cost me? *$8.60⅝.*

23. At $4.50 a yard, how much velvet can be bought for $2.81¼?

24. How much will a cigar maker receive for making 13,450 cigars, at $7.50 per M.? *$100.87½.*

25. A farmer sold 3,575 pounds of hay, at $12.50 a ton. How much did he receive for it?

26. A hotel keeper paid $22.34⅜ for hay, at $12.50 per ton. How many pounds did he buy?

27. A teamster paid $22.34⅜ for 3,575 pounds of hay. What was the price per ton?

28. A man dug a cellar 28 ft. long, 24 ft. wide, and 8 ft. deep, at $.66⅔ a cubic yard. How much did the job amount to?

29. When wood is $3.75 per cord, how much can be bought for $2.81¼? *¾ cd.*

30. A hardware merchant paid $61.68¾ for $27\frac{5}{12}$ gross of wardrobe hooks. What was the price per gross? *$2.25.*

31. A farmer paid $196.42 for 10,675 black-ash rails. What was the price per C.? *$1.84.*

32. A potter bought 6,720 lb. of porcelain clay, at $18 a ton. How much did it cost him? *$60.48.*

33. A plank-road 5 mi. 235.2 rd. long was built, at a cost of $12473.62½. What was the cost per mile? *$2,175.*

34. A merchant paid $250.04 for the gas burned in his store in one year, at $4.75 a thousand feet. How much did he burn?

35. A paper manufacturer paid $46.25 for 1,480 pounds of rags. What was the price per ton? *$62.50.*

36. At 10 s. 4 d. sterling per bushel, how much will 59½ bu. of wheat cost? *£30 14 s. 10 d.*

37. If it costs $170.10 to stereotype a book of 252 pages of 1,080 ems each, how much is that per 1,000 ems? *$.62½.*

38. How much lumber, at $24 per M., can be bought for $258? *10.75 M.*

SECTION IV.

ANALYSIS.

369. The method of stating, in order, the reasons for all the different steps in the solution of problems, is often called *Solving Problems by Analysis.* See Manual.

Ex. If 3 barrels of flour cost $34.50, how much will 8 barrels cost?

EXPLANATION.—1 barrel will cost ⅓ as much as 3 barrels, and 8 barrels will cost 8 times as much as 1 barrel. ⅓ of $34.50, the cost of 3 barrels, is $11.50, the price of 1 barrel; and 8 times $11.50, the price of 1 barrel, is $92, the cost of 8 barrels. Hence,

SOLUTION.

$$\begin{array}{r|l} \$34.50 & 3 \\ \hline \$11.50 & \\ 8 & \\ \hline \$92.00 & \end{array}$$

370. ***Rule for Solving Problems by Analysis.***

I. *From the number and value of the things given, find the value of a unit of the thing required.*

II. *From this value, find the value of the entire number of units of the thing required.*

PROBLEMS.

1. If 6 men lay 21 rods of stone-wall in a day, how many rods can 9 men lay?

2. If 9 men lay 31.5 rods of stone-wall in a day, how many rods can 6 men lay?

3. If 6 men lay 21 rods of stone-wall in a day, how many men will be required to lay 31.5 rods?

4. If 9 men lay $31\frac{1}{2}$ rods of stone-wall in a day, how many men will be required to lay 21 rods?

5. When $\frac{3}{4}$ yd. of velvet costs \$5, how much will $\frac{7}{8}$ yd. cost?

6. If $\frac{5}{8}$ T. of hay costs \$13.75, what will 1,745 lb. cost?

7. If 20 men can do a piece of work in 12 days, how many days will it take 15 men to do $3\frac{1}{3}$ times as much work?

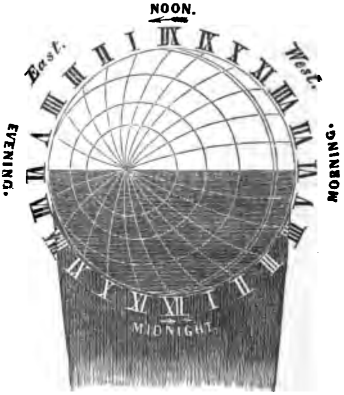
8. If 2 lb. 10 oz. of wool make $2\frac{1}{8}$ yd. of cloth $1\frac{1}{2}$ yd. wide, how much wool will it take to make 150 yd. $1\frac{3}{4}$ yd. wide?

NOTE.—More practice can be had, by solving the converse of each of the last four problems.

SECTION V.

LONGITUDE AND TIME.

371. The circumference of any circle may be divided into 360 equal parts, called degrees. (See **242**).



The equator of the earth may be divided into 24 equal parts of 15° each ($360° \div 24 = 15°$), as shown in the cut. Since the earth revolves on its axis from west to east once in 24 h. (=1 da), the sun appears to pass round the earth from east to west in the same time. See Manual.

Since the sun appears to pass round the earth (360°) in 24 h., it appears to pass over 15° ($=\frac{1}{24}$ of 360) in 1 h., $15'$ ($=\frac{1}{60}$ of 15°) in 1 min., and $15''$ ($=\frac{1}{60}$ of $15'$) in 1 sec. Consequently, all places on the earth change their relative position to the sun 15° in 1 h., $15'$ in 1 min., and $15''$ in 1 sec.; and the relative position of any place to the sun *determines* the time at that place.

372. TABLE OF LONGITUDE AND TIME.

15°	difference	in	longitude	make	1 h.	difference	in	time.
15′	"	"	"	"	1 min.	"	"	"
15″	"	"	"	"	1 sec.	"	"	"

373. Ex. 1. The difference in time between Washington and London is 5 h. 7 min. 46 sec. What is the difference n longitude.

SOLUTION.

5 h. 7 min. 46 sec.
15
76° 56′ 30″

EXPLANATION.—Since every second of difference in time makes 15″ of difference in longitude; every minute of difference in time, 15′ of difference in longitude; and every hour of difference in time, 15° of difference in longitude; and since either factor may be used as the multiplier, we multiply 5 h. 7 min. 46 sec. by 15. The result, 76° 56′ 30″, is the required difference in longitude.

Ex. 2. The difference in longitude between Washington and London is 76° 56′ 30″. What is the difference in time?

SOLUTION.

76° 56′ 30″ ⌊15
5 h. 7 min. 46 sec.

EXPLANATION. — Since every 15° of difference in longitude makes 1 h. of difference in time; every 15′ of difference in longitude, 1 min. of difference in time; and every 15″ of difference in longitude, 1 sec. of difference in time; we divide 76° 56′ 30″ by 15. The result, 5 h. 7 min. 46 sec., is the required difference in time.

374. *Rules for the Converse Reductions of difference in Longitude and Time.*

I. Difference in Time to Difference in Longitude.

Multiply the time by 15; observing that when seconds, minutes, and hours of time are multiplied, the respective products are seconds, minutes, and degrees of longitude.

II. Difference in Longitude to Difference in Time.

Divide the longitude by 15; observing that when degrees, *minutes, and seconds of longitude are divided, the respective quotients are hours, minutes, and seconds of time.*

NOTE.—The time is later at the easterly, and earlier at the westerly of any two given places.

PROBLEMS.

1. The difference in time between Chicago and New York is 55 min. 44 sec. What is the difference in longitude?

2. The difference in longitude between Chicago and New York is 13° 56′. What is the difference in time?

3. When it is 12 o'clock M. at St. Louis, it is 1 h. 20 min. 24 sec. P.M. at Portland, Me. What is the difference in longitude?

4. St. Louis is 90° 25′ west longitude, and Portland is 70° 19′ west. What is the difference in time?

5. It is 1 h. 2 min. 52 sec. P.M. at Richmond, Va., 77° 27′ W., when it is 12 o'clock M. at St. Paul, Minn. What is the longitude of St. Paul?

6. When it is 12 o'clock M. at St. Paul, 93° 10′ W., what is the time at Richmond, 77° 27′ W.?

7. When it is 12 o'clock M. at the Island of St. Helena, 5° 54′ W. longitude, what is the time at Washington, 77° 3′ 30″ W. longitude?

8. The time at Quito, 78° 50′ W., is 1 o'clock P. M., when it is 10 h. 7 min. 20 sec. A. M. at Sacramento City. What is the longitude of Sacramento City? *122° W.*

SECTION VI.

REVIEW PROBLEMS IN CONVERSE OPERATIONS.

1. The product is $55\frac{4}{5}$, and the multiplier $4\frac{3}{7}$. What is the multiplicand? *$12\frac{3}{5}$.*

2. A farmer sheared 259 lb. of wool from 56 sheep. What was the average weight of the fleeces?

3. How much carpeting will be required for a flight of stairs of *17 steps*, each 10 in. wide and 8 in. high? *$8\frac{1}{2}$ yd.*

4. Reduce .06875 to the fractional form. *$\frac{11}{160}$.*

5. Reduce $18\frac{2}{5}$ sq. rd. to the decimal of an acre. *.115 A.*

6. How many pounds of potash, at $85 a ton, can be bought for $$37.18\frac{3}{4}$? *875.*

7. How many inches are there in $.00\frac{1}{3}$ of a mile ? *211.2.*

8. A farmer raised 23 bu. 2 pk. 5 qt. of clover seed, and sold it at $6.50 a bushel. How much did he receive for it ? *$153.77.*

9. Four men paid $575 for a thrashing-machine. A paid $175, B, $125, and C, $140. How much did D pay ?

10. It cost $4,312 to dig a sewer $1\frac{3}{4}$ mi. long, 6 ft. wide, and 10 ft. deep. What was the price per cubic yard ? *$.21.*

11. A butcher bought three beeves on foot, weighing 1,463 lb., 1,521 lb., and 1,584 lb., at $5.75 per cwt. How much did they cost him ?

12. I paid an ice dealer $8.10 for supplying me with 15 lb. of ice daily, Sundays excepted, for 24 weeks. What was the price per C. ? *$$.37\frac{1}{2}$.*

13. The product of five factors is 13, and four of them are $4\frac{2}{3}$, 1.25, $\frac{1}{6}$, and 2. What is the other factor ?

14. A blacksmith paid $170.45 for 3,896 pounds of bar-iron. What was the price per ton ? *$87.50.*

15. When it is 20 min. past 3 o'clock P. M. at Albany, N. Y., 73° 42′ W., what is the time at Athens, Greece, 23° 44′ E. ?

16. A rectangular-shaped farm of 72.4 acres, is 90.5 rods wide. What is its length ? *128 rd.*

17. What is the rate of speed of a railroad train that runs 117 mi. in 5 h. 12 min. ? *22.5 mi. per hour.*

18. A dealer bought 417 T. of coal by the long ton, at $4.65 a ton, and sold it at $5.75 per short ton. How much did he gain by the transaction ? *$746.43.*

19. A cabinet maker paid $112.50 for cherry lumber, at $60 per M. How much did he buy ? *1,875 ft.*

20. Reduce $\frac{9}{16}$ to a decimal.

21. From Dayton, Ohio, due south to St. Marks, Fla., is $.024\frac{1}{6}$ of the earth's circumference. How many miles is it ? *601.692.*

22. Reduce 17 cwt. 44 lb. 11 oz. to the fraction of a ton.

23. A gardener bought 45 bushels of potatoes when they were worth \$.56¼ a bushel, agreeing to pay in kind, bushel for bushel, the next year. At the time of making payment, potatoes were worth \$.87½ a bushel. How much did he lose by the transaction ?

24. My parlor is 18 ft. × 19 ft. 6 in., and I wish to carpet it with Brussels carpeting, which is 26 inches wide. How much will my carpet cost, at \$1.87½ a yard, running measure ?

25. How much gold can be obtained from a ton of quartz rock, if it yields $\frac{1}{625}$ of its weight in gold ? *3 lb. 10 oz. 13 pwt. 8 gr.*

26. Albany, *May 19, 1869.*

Mr. *John Taylor*

Bought of **Chas. P. Easton & Co.**

2750	*ft.*	*Hemlock Timber,*	@	*\$22*	*pr. M.*
1325	"	*Roof Boards,*	"	*18*	"
5450	"	*Pine Lumber (Clear),*	"	*50*	"
1645	"	*Flooring,*	"	*36*	"
987	"	*Siding,*	"	*30*	"

\$

27. A merchant leaves New Orleans, 89° 45′ W., for Augusta, Ga., 81° 51′ W. Does he find his watcn too slow, or too fast, on arriving at Augusta, and how much ?

28. A dry-goods merchant bought 9 pieces of French calico, averaging 36 yd. each, at \$.16⅔ a yard. How much did his purchase amount to ? *\$54.*

29. What is the value of a hide that weighs 112 lb., at \$.08½ per pound ?

30. A farmer drew five loads of hay to market. The loads with the wagon weighed, respectively, 3,180 lb., 3,314 lb., 3,097 lb., 2,967 lb., and 3,234 lb., and the wagon weighed 1,142 lb. How much did the hay amount to, at \$16.50 a ton ? *\$83.18.*

31. A miner obtained \$85.78 in silver, from a quartz rock, the yield being at the rate of \$127.65 per ton. What was the weight of the rock ? *1344 lb.*

32. What must be the width of a bin 9 ft. long and 5½ ft. high, *to contain* 1½ times as much as a bin 8 × 5 × 6 ft ? *$7\frac{3}{11}$ ft..*

CHAPTER 7.

PERCENTAGE.

SECTION I.

DEFINITIONS AND NOTATION.

375. The term ***Per Cent*** in business transactions signifies hundredths of any thing or number. Thus, 17 per cent is 17 hundredths or 17 of every 100, 29 per cent is 29 hundredths, 66⅔ per cent is 66⅔ hundredths, etc.

376. Per cent may be applied to any number, great or small, concrete or abstract. Thus,

40 per cent of 1 bushel =.40 bu.;
88 " " 27 miles =.88 of 27 mi.;
14⅓ " " 395 days =.14⅓ of 395 da.;
7 " " $85.42 =.07 of $85.42;
65 " " 93¼ =.65 of 93¼.

377. ***Rate,*** or ***Rate Per Cent,*** is the number which expresses the per cent or number of hundredths.

378. The term ***Percentage*** has two significations:

1st. It is the process of finding any per cent of a number; and

2d. It is the name of the result of the computation.

379. The ***Base*** is the number on which the percentage is computed.

380. The ***Amount*** is the base plus the percentage.

381. The ***Difference*** is the base minus the percentage.

Example.—24 per cent of 50 cords of wood is .24 of 50 cords, or 12 cords.—In this example, 24 per cent is the *rate;* 50 cords, the *base*; 12 cords, the *percentage;* 62 cords (=50 + 12), the *amount;* and 38 cords (= 50 – 12), the *difference.*

382. The *Commercial Sign*, %, when written after a number, signifies per cent.

383. In computations, any per cent less than 100 is expressed by a decimal or a fraction; and 100 per cent or more, by an integer, a mixed number, or an improper fraction. Thus,

15	per cent	or	15%	is expressed	.15 or $\frac{15}{100} = \frac{3}{20}$;
6	"	"	6%	"	.06 or $\frac{6}{100}$;
$4\frac{1}{2}$	"	"	$4\frac{1}{2}$%	"	.045, $.04\frac{1}{2}$ or $\frac{9}{200}$;
$16\frac{2}{3}$	"	"	$16\frac{2}{3}$%	"	$.16\frac{2}{3}$ or $\frac{50}{300} = \frac{1}{6}$;
$\frac{1}{4}$	"	"	$\frac{1}{4}$%	"	.0025, $.00\frac{1}{4}$ or $\frac{1}{400}$;
100	"	"	100%	"	1 or $\frac{100}{100}$;
300	"	"	300%	"	3 or $\frac{300}{100}$;
125	"	"	125%	"	1.25 or $\frac{125}{100} = \frac{5}{4}$;
$233\frac{1}{3}$	"	"	$233\frac{1}{3}$%	"	$2.33\frac{1}{3}$ or $\frac{700}{300} = \frac{7}{3}$.

Hence, to express per cent decimally,

I. *Two decimal figures are always required.*

II. *Parts of 1 per cent require decimal figures or fractions at the right of hundredths.*

III. *100 per cent or more requires an integer or a mixed decimal number.*

EXERCISES.

1. Read 6%, 17%, 39%, 112%.

2. Read 21%, $12\frac{1}{2}$%, $6\frac{1}{4}$%, $\frac{1}{8}$%.

3. Read $7\frac{5}{8}$%, $31\frac{2}{3}$%, $\frac{5}{9}$%, $\frac{2}{3}$%, $\frac{7}{10}$%.

4. Write in both forms 7%, 19%, 84%, 48%, and 92%.

5. Express 22%, $56\frac{1}{2}$%, 2%, $5\frac{3}{4}$%, and $10\frac{5}{8}$% in both the decimal and the fractional form.

6. Write in both decimal and fractional forms $36\frac{4}{5}$ per cent, 125 per cent, $\frac{5}{8}$ per cent, $1\frac{1}{16}$ per cent, and $312\frac{1}{2}$ per cent.

7. Express decimally the amount and the difference of 1, at 6%, and at 7%.

8. Write the amount and the difference of 1, at 25%, in both the decimal and the fractional form.

SECTION II.

THE FIVE GENERAL CASES OF PERCENTAGE.

CASE I.

Base and Rate given, to find Percentage.

384. Ex. How much is 25% of 256?

EXPLANATION.—Since 25% of any number is .25, or $\frac{1}{4}$, of the number, we find 25% of 256 by multiplying it by .25, as shown in the First Solution; or, by multiplying it by $\frac{1}{4}$, as shown in the Second Solution. Hence,

FIRST SOLUTION.

$$\begin{array}{r} 256 \\ .25 \\ \hline 1280 \\ 512 \\ \hline 64.00 \end{array}$$

SECOND SOLUTION.

$256 \times \frac{1}{4} = \frac{1}{4}$ of $256 = 64$

The percentage is the product of the base and rate.

PROBLEMS.

1. How much is 20% of 960 bushels of corn? *192 bu.*

2. Find 12½% of 2,548 feet of lumber. *318.5 ft.*

3. What is 33⅓% of 12,837?

4. The silver used in coinage contains 10% of alloy. How much alloy is there in 7.5 ounces of silver coin? *.75 oz.*

5. A builder bought 8 boxes of glass, each containing 45 panes; but upon opening them, he found 7½% of the glass broken. How many panes were broken?

6. A farmer harvested 540 bushels of oats from one field, and 105% of that amount from another. How many bushels did he harvest from the second field? *567.*

7. From a hogshead that contained 125 gallons of molasses, a grocer lost 2% by leakage. How much molasses did he lose?

8. What is $\frac{4}{7}$% of 5,000 cords of wood? *28$\frac{4}{7}$ cd.*

9. Flaxseed contains 11% of oil. How much linseed-oil is there in 275 pounds of flaxseed?

CASE II.

Base and Percentage given, to find Rate.

385. Ex. The base is 275, and the percentage is 66. What is the rate?

FIRST SOLUTION.

$$\begin{array}{l|l} 66.00 & 275 \\ \underline{550} & \overline{.24} = 24\% \\ 1100 & \\ \underline{1100} & \end{array}$$

SECOND SOLUTION.

$$\tfrac{66}{275} = \tfrac{6}{25} = .24 = 24\%$$

Explanation.—The percentage on 1 is $\frac{1}{275}$ of the percentage on 275. Since 66 is the percentage on 275, we divide it by 275, and obtain .24, the percentage on 1, or the rate, as shown in the First Solution. Or, $\frac{1}{275}$ of 66 is $\frac{66}{275}$; and reducing this fraction to a decimal, we obtain .24, the required rate, as shown in the Second Solution. Hence,

The rate is the quotient of the percentage divided by the base.

PROBLEMS.

10. What % of 5,000 bushels are 50 bushels? *1%.*

11. 17 is what % of 51?

12. What % of 5,725 is 2,290? *40%.*

13. My income last year was $1,500, and my expenses were $1,275. What % of my income did I expend?

14. Of 8,900 soldiers who went into battle, 1,157 were either killed or wounded. What % of the army was lost? *13%.*

15. If 2,500 pounds of bell-metal are used to make a bell that weighs 2,450 pounds, what % of the bell-metal is waste?

16. A grocer sells tea that cost him $1.20 a pound, @ $1.50. At what % of the cost does he sell it? *125%.*

17. What % of 1 oz. Troy is 1 oz. avoirdupois? *$91\frac{7}{48}$%.*

CASE III.

Rate and Percentage given, to find Base.

386. Ex. 119 is 35 % of what number?

SOLUTION.

$$\begin{array}{r|l} 119.00 & .35 \\ \underline{105} & \overline{340} \\ 140 & \\ \underline{140} & \\ 0 & \end{array}$$

Explanation.—35% of any number is .35 of the number. Since 119, the percentage of some number, is .35 times *the number*—or .35 of the number—we divide 119 by .35, and obtain 340, the required number or base. Hence,

The base is the quotient of the percentage divided by the rate.

PROBLEMS.

18. 465 miles are 15% of how many miles? *3,100.*

19 32.12 days are 8⅘% of what number of days? *365.*

20. My orchard of 7.5 acres is 6% of my whole farm. How much land is there in my farm?

21. The 350 girls in a certain village school are 56% of the whole number of pupils. How many pupils in the school? *625.*

22. 24 is ⅔% of what number? *3,600.*

23. A shoemaker lost 39% of his property by a fire, and his loss was $936. How much was he worth before the fire?

24. William is 16 years old, and 37½% of William's age is 40% of Richard's age. How old is Richard? *15 years.*

CASE IV.

Base and Rate given, to find either Amount or Difference.

387. Ex. If the base is 375, and the rate 32%, what is the amount? What is the difference?

EXPLANATION.—The amount or the difference of 375 at any rate per cent, is 375 times the amount or the difference of 1 at the same rate. Since the amount of 1 at 32% is 1 + .32 = 1.32, the amount of 375 at the same rate is 375 × 1.32, or 495, as shown in Solution 1. And

Since the difference of 1 at 32% is 1 − .32 = .68, the difference of 375 at the same rate is 375 × .68, or 255, as shown in Solution 2. Hence,

I. *The amount is the product of the base multiplied by 1 plus the rate;* and

II. *The difference is the product of the base multiplied by 1 minus the rate.*

SOLUTION 1.

$$\begin{array}{r} 375 \\ \underline{1.32} \\ 750 \\ 1125 \\ \underline{375} \\ 495.00 \end{array}$$

SOLUTION 2.

$$\begin{array}{r} 375 \\ \underline{.68} \\ 3000 \\ \underline{2250} \\ 255.00 \end{array}$$

PROBLEMS.

25. If the base is 125, and the rate 25%, what is the difference?

26. The base is 63, and the rate $5\frac{5}{8}\%$. What is the amount?

27. An army of 5,800 men was re-inforced by a detachment of 39% of that number. How many were then in the army?

28. My farm contains 118.9 A., and 45% of it is woodland. How many acres of it are cleared land? *65.395.*

29. A farmer raised 625 bushels of wheat one year, and 88% of the same quantity the next year. How much wheat did he raise in the two years? *1,175 bu.*

30. I paid $2,400 for a house, and 6% of that sum for repairs upon it. How much did the house cost me? *$2,544.*

31. A mechanic who had $147 deposited in a savings-bank, drew out 75% of it. How much remained on deposit?

32. Last year the circulation of a weekly newspaper was 15% less than it is this year, and this year its circulation is 14,260 copies. How large was its circulation last year? *12,121 copies.*

CASE V.

Amount or Difference and Rate given, to find Base.

388. Ex. 1. The amount of a certain base, at 18%, is 508.58. What is the base?

Explanation.—Any given amount at any rate per cent, is as many times the amount of 1 at the same rate, as the number of times 1 plus the rate is contained in the amount. We therefore divide 508.58, the given amount, by 1.18, the amount of 1 at 18%, and obtain 431, the required base.

SOLUTION.

$$\begin{array}{l|l} 508.58 & \underline{1.18} \\ \underline{472} & 431 \\ 365 & \\ \underline{354} & \\ 118 & \\ \underline{118} & \end{array}$$

Ex. 2. The difference is 64.4, and the rate is $12\frac{1}{2}\%$. What is the base?

Explanation.—Any given difference at any rate per cent, is as many times the difference of 1 at the same rate, as the number of times 1 minus the rate is contained in the difference. We therefore

SOLUTION.

$$\begin{array}{l|l} 64.400 & \underline{.875} \\ \underline{6125} & 73.6 \\ 3150 & \\ \underline{2625} & \\ 525{}^{\vee}0 & \\ \underline{5250} & \end{array}$$

divide 64.4, the given difference, by .875, the difference of 1 at 12½%, and obtain 73.6, the required base.

From these examples we learn that

The base equals the quotient of the amount divided by 1 plus the rate, or the quotient of the difference divided by 1 minus the rate.

PROBLEMS.

33. What number increased by 7% of itself is equal to 267.5?

34. A horse-dealer sold a span of matched horses for $1,155, which was 16% less than they cost him. What did they cost him?

35. This year a clergyman's salary is $2,500, which is 25% more than it was last year. What salary did he receive last year?

36. The difference is 8,466, and the rate is 15%. What is the base? *9,960.*

37. In Dec. a manufacturer made 4,865 yards of cassimere, which was 12½% more than he made in Nov. How much did he make in Nov.?

38. This year a man's house rent is $325, which is 18¾% less than it was last year. What rent did he pay last year? *$400.*

39. A house painter painted three houses, using 23½ pounds of white lead for the first house, which was 20% less than he used for the second, and 17½% more than for the third. How much white lead did he use for the second house? How much for the third? *29 lb. 6 oz.; 20 lb.*

389. Upon the principles deduced in **384–388** are based the

Rules for Computations in Percentage.

I. Base and rate given, to find percentage.

Multiply the base by the rate.

II. Base and percentage given, to find rate.

Divide the percentage by the base.

III. Rate and percentage given, to find base.

Divide the percentage by the rate.

IV. Base and rate given, to find either amount or difference.

Multiply the base by 1 plus the rate, for the amount; and by 1 minus the rate, for the difference.

V. Amount or difference and rate given, to find base.

Divide the amount by 1 plus the rate; and the difference by 1 minus the rate.

NOTE.—Rules II. and III. are the converse of Rule I., and IV. and V. are the converse of each other.

PROBLEMS.

40. If wheat yields 72% of its weight in flour, how much flour can be made from 245 bushels of wheat? *54 bbl.*

41. After drawing 9 gallons from a cask of oil, the amount drawn was 40% of the amount remaining in the cask. How many gallons were in the cask at first? *31½.*

42. I paid a tax of $61.40 on my farm, and with it a collector's fee of 5%. What was the whole amount paid?

43. What % of 423 is 75⅕? *$17\frac{7}{9}$%.*

44. If Indian corn contains 73% of starch, how much starch is there in 1,192 pounds of corn?

45. If the ashes obtained from burning 2,275 pounds of coal, weigh 68¼ pounds, what % of the coal remains in the ashes? *3%.*

46. A merchant sold 51 yards from a roll of carpeting, and the amount sold was 37½% of the whole number of yards in the roll. How many yards were in the roll? *136.*

47. A miller bought 2,175 bushels of wheat, 76% of which was winter wheat. How much of it was spring wheat?

48. 63 is 64% of what number? *98.4375.*

49. The number of children of school age in a certain county is 11,275, and 3,157 children attend school. What % of the whole number attend school?

50. A merchant's sales for the year were 1124% of his sales for January, and his sales in January were $1,256. How much were his sales for the year? *$14117.44.*

51. I bought a house and lot for \$2,750, paying \$935 down, and the balance in 6 equal annual payments. What % of the purchase price did I pay down, and what % at each annual payment?

34%; 11%.

52. The length of the shadow cast by a tree is 32% greater than the height of the tree, and the tree is 45 feet high. How long is the shadow?

53. In a battle, 256 soldiers were killed. The number killed was 20% of the number wounded, and the number wounded was 16% of the number uninjured. How many men were in the army before the battle? *9,536.*

54. Find $\frac{16}{25}\%$ of 15 miles.

55. The difference between 24% and 55% of a number is 60.45. What is the number? *195.*

56. A grocer bought a hogshead that contained $110\frac{1}{2}$ gallons of N. O. molasses. $3\frac{11}{13}\%$ of it leaked out, and he sold 28% of the remainder. How many gallons had he left? *$76\frac{1}{2}$.*

57. 25% of 40% of a number is what part of the number?

10%, or $\frac{1}{10}$ of it.

58. A farmer sold 28% of his land, and afterward bought 35% of as much as he had left. He then had $5\frac{1}{4}$ acres less than at first. How many acres had he at first? *$187\frac{1}{2}$.*

59. What % of $27\frac{1}{3}$ is $4\frac{5}{6}$?

60. One year a farmer raised 560 bushels of wheat, and sold it at \$1.80 a bushel. The next year he raised 25% less, and sold it at 25% more per bushel. In which year did he realize the greater sum for his wheat? *The first year, \$63 more.*

61. Of a regiment of soldiers, 4% deserted, and $6\frac{1}{4}\%$ of the remainder were killed. Of those then left, $16\frac{2}{3}\%$ were taken prisoners, and 12% of the balance were discharged. There were then 660 men in the regiment. How many men were there at first? *1,000.*

62. A wood dealer contracted to deliver 8,100 cords of wood at a R.R. station, in 90 working days. When 70% of the time had passed, he had delivered but 65% of the wood. How many cords must he deliver each day for the balance of the time, to fulfill the contract? *105.*

63. 7,465 is 33⅓% of what number? *22,395.*

64. 15% of 484 is 33% of what number?

65. A manufacturer increased his capital by 24% the first year, and that capital by 25% the second year. The third year he lost 16% of his capital, and he then had $16,217 left. How much capital had he at first? *$12,455.45.*

SECTION III.

390. ***Insurance*** is a security against loss or damage within a given time, guarantied to one party by another, for a specified consideration.

391. ***Fire-Insurance*** is a security against loss by fire.

392. ***Marine Insurance*** is a security against loss at sea.

393. ***Health*** and ***Accident Insurance*** are securities against loss by sickness or accident.

394. ***Life-Insurance*** is a security guarantying to the parties interested in the life of a person, a specified sum at his death, if it occurs within a specified time.

395. ***Valuation*** is the sum for which property, life, or health is insured.

396. ***Premium*** is the sum paid for the insurance.

397. The ***Policy*** is the contract between the insurer and the insured.

NOTES.—1. The business of insuring is commonly carried on by corporations called *Insurance Companies.*

2. A corporation whose members have paid in money or capital, to secure the payment of losses, and among whom the profits are divided, is a *Stock Insurance Company.*

3. A corporation of which every person insured is a member, sharing in the profits and losses, is a *Mutual Insurance Company.*

4. In order that owners of property insured may not be tempted to destroy it, property is never insured for its full value.

COMPUTATIONS IN INSURANCE.

398. Valuation is the base;
Premium is the percentage; and
Rate % is the rate. Hence,

I. *Valuation and rate given, to find premium,* is *Base and rate given, to find percentage.*

II. *Valuation and premium given, to find rate,* is *Base and percentage given, to find rate.*

III. *Rate and premium given, to find valuation,* is *Rate and percentage given, to find base.*

PROBLEMS.

1. What premium must a merchant pay for an insurance of $7,250 on his stock of goods, at 1¼%? *$90.62½.*

2. A school-house was insured for $2,800, at ⅞%. What was the premium?

3. A physician gets a policy of insurance on his house for $1,500, his household furniture for $650, and his library for $475. How much does it cost him, at ¾%? *$19.68¾.*

4. If it costs $521.25 for an insurance of $27,800 on a merchant vessel, for a trip from New York to Havana, what is the rate?

5. It costs $172.50 to insure a steam planing-mill for $3,450. What is the rate? *5%.*

6. The premium paid for insuring a paper-mill for $15,500, was $116.25. What was the rate?

7. A shoemaker paid $9.37½ for having his shop and stock insured, at 1¼%. What amount did his policy cover? *$750.*

8. If I pay $20 for having my house insured, at ⅝%, what amount do I get it insured for? *$3,200.*

9. At 1½%, what amount must be covered by a policy that costs $141.75?

10. A grain dealer had a cargo of wheat insured from Milwaukee to Buffalo for $8,750, at ⅜%. What premium did he pay?

11. The premium paid for insuring a church for $12,750, was $63.75. What was the rate? *½%.*

12. At 1½%, how much will it cost to insure a hotel for $125,000, and the furniture for $40,000?

13. The premium for insuring a lake propeller for the season, at 1¾%, was $312.37½. For what amount was she insured? *$17,850.*

14. A policy of $2,675 on a sash and blind factory, cost $53.50. What was the rate? *2%.*

15. What will it cost per annum for a life-insurance policy for $5,000, on the life of a man 35 years old, at $27.50 per $1,000?

NOTE 5.—The Tables of Rates of most life-insurance companies are at certain sums per $1,000 of insurance, the rate per $1,000 increasing according to the age at which the insurance is made.

16. A man 30 years old has his life insured for $3,000, at an annual premium of $24.75 per $1,000. If he dies at the age of 50, how much more do his heirs receive upon his life-insurance than he has paid on it? *$1,515.*

17. A man 45 years old obtains a policy of insurance for $7,000, in a Mutual Endowment Insurance Co., the policy to be paid at 60, paying at the rate of $26.84 on every $1,000 semi-annually. What are the annual payments? *$375.76.*

SECTION IV.

399. An *Agent, Commission-Merchant, Factor,* or *Broker* is a person who, by authority, buys and sells goods, or transacts other financial business for another.

NOTE.—A person to whom property is delivered in trust, for sale, is a *Consignee;* and the person delivering the property is a *Consignor.*

400. *Commission* is the sum paid an agent or commission-merchant for transacting business.

COMPUTATIONS IN COMMISSION.

401. Commission is commonly computed at some % on the sum of money received or paid out by the agent in the transaction.

402. The sum on which commission is computed is the base;

Commission is the percentage;

Rate % is the rate; and

The sum on which commission is computed plus the commission is the amount. Hence,

I. *The sum on which commission is computed and the rate given, to find the commission,* is *Base and rate given, to find percentage.*

II. *The sum on which commission is computed and the commission given, to find the rate,* is *Base and percentage given, to find rate.*

III. *The rate and the commission given, to find the sum on which commission is computed,* is *Rate and percentage given, to find base.*

IV. *The amount and rate given, to find the sum on which commission is computed,* is *Amount and rate given, to find base.*

PROBLEMS.

1. In one month an insurance agent receives $1,328 for premiums, and his commission is 5%. How much do his fees amount to?

2. A commission-merchant bought for a provision dealer 420 barrels of pork, @ $21.50, at 1¼% commission. How much was his commission? *$112.87½.*

3. A real estate agent sold a farm of 119½ acres, at $96 per acre. What was his commission, at ⅝%? *$71.70.*

4. A miller paid a grain buyer $64.12½, for buying 15,000 bushels of corn, at $.57 per bushel. What rate of commission did he pay?

5. An auctioneer sold a lot of crockery for $416, and received $18.72 commission. What was his % for selling? *4½%.*

6. A merchant paid an attorney $56.70, for collecting bills to the amount of $945. What % were his fees for collecting? *6%.*

7. A wool buyer bought wool at $.44 per pound, and received a commission of $187.11, at 1½%. How much wool did he buy?

8. If I pay an agent $192.75 for purchasing goods, at 3% commission, what is the cost of the goods purchased? *$6,425.*

9. A commission-merchant received $157.75 for selling flour, commission 2½%. How much did the flour sell for? *$6,310.*

10. How much land, at $35 an acre, can an agent buy with $3126.20, after deducting his commission of 1½%? *88 A.*

11. An agent receives $901.25 with which to purchase hides, after deducting his commission of 3%. What sum will he invest in hides? *$875.*

12. A cotton factor received $4,076.80 to be invested in cotton at $.28 a pound, after deducting 4% for his fees. How many pounds did he buy? *14,000.*

13. A tax collector had a warrant for $37,600, upon which he collected $18,228, at 1%, and the balance at 5%. What was the amount of his fees? *$1,150.88.*

14. A fruit buyer received $7,315 with which to buy apples, after taking out his commission of 4½%. How much did he use in buying apples? *$7,000.*

15. A buyer of live stock receives $484.50 with which to buy sheep, after deducting his commission of 2%. What sum does he expend?

16. A produce commission house in Detroit received $4,725 from an eastern miller, to be invested in wheat, less a commission of 2¼%. How much was the commission?

17. A commission-merchant who buys produce at 2¾% commission, receives $1350.20 with which to purchase beef. How much is his commission? *$36.14.*

SECTION V.

PROFIT AND LOSS.

403. When goods are sold for more than cost, the excess is *Profit*, or *an Advance;* and when they are sold for less than cost, the deficiency is *Loss*, or a *Discount*. Hence,

404. ***Profit***, in business, is the sum above cost for which goods are sold, or the excess of receipts over expenditures; and

405. ***Loss*** is the sum below cost for which goods are sold, or the excess of expenditures over receipts.

COMPUTATIONS IN PROFIT AND LOSS.

406. Profit and loss are commonly computed at a % on the cost.

407. The cost is the base;
The profit or loss is the percentage;
The rate % is the rate; and
The selling price is the amount or difference. Hence,

I. *Cost and rate given, to find gain or loss,* is *Base and rate given, to find percentage.*

II. *Cost, and gain or loss given, to find rate,* is *Base and percentage given, to find rate.*

III. *Gain or loss, and rate given, to find cost,* is *Percentage and rate given, to find base.*

IV. *Cost and rate given, to find selling price,* is *Base and rate given, to find amount or difference.*

V. *Selling price and rate given, to find cost,* is *Amount or difference, and rate given, to find base.*

PROBLEMS.

1. A man bought a house and lot for $1,875, and sold it at a loss of 4%. How much did he lose? *$75.*

2. If a butcher buys beef at $.08 per pound, how must he sell it to gain 37½%? *At $.11 per pound.*

3. I bought a cow in the spring for $62.50, and sold her in the fall for $45. What % of the cost did I lose?

4. A grocer pays $12 a barrel for mackerel, and retails them at $.10 a pound. What % does he gain? *66⅔%.*

5. A grocer by selling butter at a profit of 20%, made $.05 on a pound. What did the butter cost him per pound?

6. A lumber dealer loses $10.50 per M. by selling a quantity of lumber at 37½% below cost. What does the lumber cost him per M.? *$28.*

7. A hardware merchant by selling a stove at 32% above cost, makes $6. What was the cost of the stove? *$18.75.*

8. At what price must a grocer sell cheese that cost him $.15 per pound, to gain 33⅓%? *$.20.*

9. I sold a watch that cost me $75, at a loss of 8%. For what price did I sell it? *$69.*

10. A carpenter built a house at a cost of $1,280, and sold it at a gain of 12½%. For how much did he sell it?

11. If I gain 30% by selling sheep at $4.87½ a head, how much did they cost me? *$3.75.*

12. A merchant loses 12% by selling damaged delaines at $.33 a yard. How much did they cost him? *$.37½ a yd.*

13. A builder erected a church by contract, for $15,300, and lost 15% upon its cost. How much did it cost him to build it?

14. How shall I mark calico that costs $.16 a yard, to gain 25%?

15. A merchant sells sugar at $.15 per pound, that cost him $.12½. What % does he gain? *20%.*

16. A wagon maker sold a lumber wagon that cost him $96, at 25% profit. At what price did he sell it? *$120.*

17. A dealer in musical instruments sold a piano for $540, and his profit was 20%. How much did the piano cost him?

18. A furrier sold a set of ladies' mink furs at 15% less than cost, and lost $10¼ on them. How much did he get for them?

SECTION VI.

STOCKS.

408. A *Corporation* is a company established by law, having power to transact business as an individual.

409. *Stock* is the property invested in the business of a corporation.

NOTE.—Stock is often called *Capital*, or *Capital Stock.*

410. A *Share* is one of the equal parts into which the stock of a corporation is divided. It is usually $100.

411. A *Certificate of Stock* states the number of shares of stock owned by the holder of the certificate, and also the par value of a share.

412. The ***Par Value*** of stock is the sum stated in the scrip or certificate ; and

413. The ***Market Value*** is the sum for which the stock will sell.

414. Stock is ***At Par*** when its market value is its par value, or 100% ;

415. It is ***Above Par*** when its market value is above its par value, or more than 100% ; and

416. It is ***Below Par*** when its market value is below its par value, or less than 100%.

417. ***Premium*** is the excess over 100% in the value of stock that is above par ; and

418. ***Discount*** is the deficiency under 100% in the value of stock that is below par.

419. ***Stock Quotations*** are published statements giving the market value of stocks. Thus, if stock is 8% above par, it is quoted at 108 ; and if it is 8% below par, it is quoted at 92.

420. A ***Stock Broker*** or ***Stock Jobber*** is a person who deals in stocks.

421. ***Brokerage*** is the commission paid to stock brokers for buying and selling stocks for others.

NOTE.—The rate of commission established by the N. Y. Board of Brokers is ¼% on the par value of the stock.

COMPUTATIONS IN STOCKS.

422. In stock transactions the computations are made on the par value of the stock.

423. The par value is the base ;
The premium or the discount is the percentage ;
The rate % is the rate ; and
The market value is the amount or difference. Hence,

I. *Par value and rate given, to find premium or discount,* is *Base and rate given, to find percentage.*

II. *Par value and rate given, to find market value,* is *Base and rate given, to find amount or difference.*

III. *Market value and rate given, to find par value,* is *Amount or difference, and rate given, to find base.*

These three cases cover the ordinary transactions in stocks.

PROBLEMS.

1. If I buy 17 shares of bank stock at par, and sell it at 5½% premium, how much do I gain? *$93.50.*

2. A man bought 38 shares of the stock of an express company at par, and sold it at 11% discount. How much did he lose?

3. How much will I receive for 192 100-dollar shares of insurance stock, if I sell it at 24¾% above par?

4. Mr. Clark took 7 1000-dollar shares in the stock of a woolen factory, at 13% below par. How much did it cost him? *$6,090.*

5. If I exchange 65 shares of bank stock at 26% premium, for R.R. stock at 9% discount, how many shares will I receive? *90.*

6. When State stocks are quoted at 82, what is the par value of the stock that can be purchased for $2,460?

7. When Panama R.R. stock is quoted at 123, how many shares can be bought for $6,642? *54.*

8. A stock jobber bought 50 shares of the stock of a coal company at 114½, and sold it at 135. How much did he gain?

9. How many 100-dollar Pacific R.R. bonds can be bought for $5694, at 9½% premium?

10. Bought 96 shares of the stock of an iron mill at 2% discount, and sold it at 9½% discount. How much did I lose? *$720.*

11. A broker bought 76 shares of mining stock at 4½% discount, and sold it at 7% premium. How much was his gain? *$874.*

12. A broker bought 84 shares of R.R. stock at 19% discount. He sold 35 shares at 27½% discount, and the balance at 8% discount. Did he gain or lose, and how much? *$241.50.*

13. Will I gain or lose, if I buy 112 shares of the stock of a transportation company at 17% premium, and after receiving a dividend of 9%, sell it at 8% less than it cost me?

SECTION VII.

TAXES AND DUTIES.

424. ***Revenue*** is the annual income which Government collects and receives into the treasury, for public use.

I. GENERAL TAXES.

425. ***Taxes*** or ***Duties*** are sums of money assessed upon persons and property, to meet public expenses.

NOTES.—1. A *Poll-Tax* is a tax upon the person; and a *Property Tax* is a tax upon the assessed value of property.

2. Property is of two kinds:—*Real Estate*, or houses and lands; and *Personal Property*, or movable property.

3. The general taxes levied or assessed are Road, School, Village, City, Town, County, and State taxes; and Special Property Taxes for local improvements.

COMPUTATIONS IN GENERAL TAXES.

426. In the assessment of taxes, assessors must first find the rate, and then the tax.

427. The valuation of property is the base;
The rate % is the rate; and
The tax is the percentage. Hence,

I. *Valuation and tax given, to find rate,*	is	*Base and percentage given, to find rate.*
II. *Valuation and rate given, to find tax,*	is	*Base and rate given, to find percentage.*
III. *Tax and rate given, to find valuation,*	is	*Percentage and rate given, to find base.*

These three cases cover the ordinary computations in general taxes.

PROBLEMS.

1. A school tax of $433.50 is levied in a district, and the property is assessed at $69,360. What is the rate? *.6¼%*, or *$.006¼ on a dollar.*

2. A tax of $95,935 is levied on a city, the assessed valuation of which is $7,674,800. What is the rate?

3. If I am assessed at $1,250 on a house and lot, $300 on a vacant lot, and $3,000 personal property, how much will my tax be, the rate being $.0097 on a dollar? *$44.13½.*

4. The assessed valuation of a village is $294,500, and a tax of $1,145 is to be laid. What must be the rate? *.00388*

5. A tax of $928.80 for building a bridge, is levied on a town, the assessed valuation of which is $967,500. What is the tax on property assessed at $1,250? *$1.20.*

6. A physician whose property was assessed at $2,750, paid a school tax of $23.37½. What was the rate of taxation? *.0085*

7. One year, a man whose property was assessed at $1,350, paid .35% village tax, .47% school tax, 1.05% county tax, and $1.00 poll tax. What was the amount of his taxes? *$26.24½.*

8. If the rate is $.001¼ on a dollar, and the tax is $1178.85, what is the valuation?

9. If a tax of $473.40 is levied on property assessed at $39,450, what is the assessed valuation of property that pays a tax of $29.70? *$2,475.*

II. INTERNAL REVENUE.

428. *Internal Revenue* is the income which Government receives from home business, products, and manufactures.

429. *Income Tax* is a tax levied upon income.

430. A *License Fee* is a tax levied for a license or permit to carry on any branch of business.

431. A *Tax on Manufactures* is a tax levied upon the value of home manufactures.

COMPUTATIONS IN INTERNAL REVENUE.

432. Income taxes are computed at some legal rate upon the income minus the exemptions; and

Taxes on manufactures, at some legal rate upon the value of the manufactured goods.

License fees are fixed sums established by law.

433. The assessed income (*i. e.* income minus exemptions), or the value of the manufactured goods, is the base;

The rate % is the rate; and

The tax is the percentage. Hence,

Assessed income, or value of manufactures, and rate given, to find tax, is *Base and rate given, to find percentage.*

PROBLEMS.

10. A lawyer's income for the year 1868 was $3,284, and his exemptions were $350 for house rent, and $1,000 for living expenses. How much income tax did he pay, the rate being 5%? *$96.70.*

11. A manufacturer's sales for the year amounted to $58,750, upon which he paid a government tax of .3%. What was the amount of the tax?

12. A milliner pays a license of $10, and her assessed income is $825, on which the tax is 5%. How much revenue does she pay? *$51.25.*

III. CUSTOMS.

434. ***Customs*** are duties paid to Government on imported goods and other property.

NOTE.—The office at which customs or duties are collected is a *Custom-House;* and a seaport town in which a custom-house is situated is a *Port of Entry.*

435. An ***Invoice*** is a written account containing a list of merchandise sent to a purchaser, with prices and charges annexed.

In custom-house transactions, certain deductions are made on some kinds of goods, before the duties are computed. These are tare, leakage, and breakage.

436. ***Tare*** is a deduction made from the weight of goods sold in chests, boxes, cases, casks, bags, or other envelope or covering, on account of the weight of such covering.

437. ***Leakage*** is a deduction made from the quantity of liquors imported in casks.

438. ***Breakage*** is a deduction made from the quantity of liquors imported in bottles.

439. ***Gross Weight*** is the entire weight of goods and case or covering.

440. ***Net Weight*** is the gross weight minus the tare.

441. ***Net Value*** is the value of goods at the original invoice price, after all deductions have been made.

442. ***Specific Duty*** is duty on the number or quantity.

443. ***Ad Valorem Duty*** is duty on the net value.

NOTE.—A list of rates of duties established by Government is called a *Tariff.*

COMPUTATIONS IN DUTIES.

444. In ad valorem duties,
The net value is the base ;
The rate % of duty is the rate ; and
The duty is the percentage. Hence,

I. *Net value and rate of duty given, to find duty,* } is { *Base and rate given, to find percentage.*

II. *Specific duties are found by multiplying the duty on one by the net number.*

PROBLEMS.

13. The gross weight of 175 boxes of raisins is 33⅓ lb. per box, and the tare is 25%. What is the total net weight ?

14. What are the duties, at $.25 per pound, on 150 chests of tea, invoiced at 62 lb. per chest ?

15. The duty on opium is 100%. What are the custom-house charges on 125 lb., invoiced at $5.37½ per lb. ?

16. A sugar refiner imports 72 hhd. W. I. sugar, weighing 475 lb. each, and 50 hhd. molasses containing 126 gal. each. What are the duties, sugar paying $.03 per lb., and molasses $.08 per gal., and the tare on the sugar being 12½% ? *$1,401.75.*

PROBLEMS IN TAXES AND DUTIES.

17. The valuation of the property of a certain county is $11,847,500, upon which a tax of $146,909 is levied. How much of this tax will be paid by the owner of a foundery which is assessed at $14,550? *$180.42.*

18. A collector's fees for collecting a town tax were $197.72, and the whole tax was $14,829. What rate % did the collector receive? *1⅓%.*

19. In 1865, a 5% tax was required upon the first $5,000 of a man's income, and 10% upon all above $5,000, the exemptions being $600 for living expenses, $200 for house rent, and the amount paid for taxes. How much tax did a man pay, whose income was $17,675, and who had paid $453 for taxes? *$1,392.20.*

20. A man pays a state tax of 2%, and a town and county of 3%, on property assessed at $10,000. What is the amount of his taxes?

21. The duty on tobacco being $.35 per lb.; and on segars $3 per lb. specific, and 50% ad valorem; what are the duties on 50 cases of tobacco invoiced at 65 lb. each, and 175,000 Havana segars, weighing 2,625 lb., and invoiced at $45 per M.? *$12,950.*

22. Upon the property of a city assessed at $3,824,600, a tax of $72,667.40 is levied. Make a table, embracing the tax on $1 to $10, from which the tax list can be computed. See Manual.

SECTION VIII.

INTEREST.

445. If a person hires a house or a farm, he pays the owner for the use of it. If a person hires or borrows money, when he pays the debt, he also pays an additional sum for the use of the money. And when a person pays a debt after it is due, he pays an additional sum for the credit; *i. e.*, for the use of the money after the debt is due.

446. *Interest* is the sum paid for the use of money.

447. *Principal* is the sum for the use of which interest *is paid.*

448. ***Amount*** is the sum of principal and interest.

449. ***Rate per Cent per Annum*** is the interest on $1 for 1 year.

450. ***Simple Interest*** is the sum paid for the use of the principal.

451. ***Compound Interest*** is the sum paid for the use of interest.

452. A ***Partial Payment*** is a payment of a part of an obligation due, or that is drawing interest.

453. ***Legal Rate*** is the rate of interest allowed by law.

NOTE.—Any rate of interest greater than the legal rate is *Usury.*

454. TABLE OF LEGAL RATES OF INTEREST.

WHEN NO RATE IS NAMED.		RATES ALLOWED BY SPECIAL CONTRACT.	
5%	La.	Not exceeding 8%	Fla. and La.
7%	N.Y., Conn., Mich., Wis., Minn., S. C., Geo., Utah, and Hudson Co., N. J.	10%	Ohio, Iowa, Miss., Ark., Utah, and for borrowed money in Mich. and Ill.
8%	Ala. and Tex.	12%	Wis., Tex., and on judgments in Minn.
10%	Cal., Or., Kan., Neb., W. T., Nev., and Col.	15%	Neb.
6%	All the other States, D. C., and bank interest in La. and Kan.	20%	Kan.
		Any rate per cent agreed upon.	R. I., Minn., Cal., W. T., Nev., and Col.

CASE I.

Computations of Simple Interest.

455. In all the previous Sections of this Chapter, rate % is a fixed sum without regard to time. But in interest, the entire rate per cent paid on $1, depends upon the time. Thus, if the rate is 6% per annum, the rate % on $1 for 1 year is .06; for 3 years it is 3 times .06, or .18; for 6 months or ½ year is ½ of .06, or .03, etc. Hence,

The rate, or the percentage on $1, is the product of the rate per cent per annum and the time expressed in years.

I. GENERAL METHOD.

456. Ex. What is the interest of $287.50 for 3 years, at 7%?

SOLUTION.

$$\begin{array}{r l}
\$287.50 & \text{Principal.} \\
.07 & \text{Rate.} \\
\hline
\$20.125 & \text{Int. for 1 yr.} \\
3 & \\
\hline
\$60.375 & \text{Int. for 3 yr.}
\end{array}$$

Explanation.—Since the rate is 7% per annum, the interest for 1 year is .07 times the principal, and the interest for 3 years is 3 times the interest for 1 year. We therefore multiply $287.50 by .07, and the product, $20.125, by 3. The final result, $60.37½, is the required interest. Hence,

Interest for years is the product of principal, rate, and time.

PROBLEMS.

1. What is the interest of $515.50 for 1 year, at 6%? *$30.93.*

2. What is the amount of $325 for 1 year, at 6%? (See **448**.)

3. What is the interest of $117.25 for 2 years, at 5%? *$11.72½.*

4. If I borrow $390 for 4 years, at 7%, what amount will be due at the expiration of the time? *$499.20.*

5. What is the interest, and what the amount, of $1,068.50 for 1 year, at 8%? *Amount, $1,153.98.*

457. Ex. What is the interest of $654.75 for 1 yr. 5 mo., at 6%?

SOLUTION.

$$\begin{array}{r l}
\$654.75 & \text{Principal.} \\
.06 & \text{Rate.} \\
\hline
\$39.2850 & \text{Int. for 1 yr.} \\
17 & \\
\hline
274995 & \\
39285 & \\
\hline
\$667.845 & \lfloor 12 \\
\hline
\$55.65+ & \left\{ \begin{array}{l} \text{Int. for } \frac{17}{12} \text{ yr.,} \\ \text{or 1 yr. 5 mo.} \end{array} \right.
\end{array}$$

Explanation.—Since 1 yr. 5 mo., or 17 mo., is $\frac{17}{12}$ yr., we first find the interest for 1 yr., as in (**456**); and then multiply this interest, $39.285, by $\frac{17}{12}$, the required time. That is, when there are months in the given time, we

Multiply the interest for 1 year by the number of months, and divide the product by 12.

NOTES.—1. In computations, the partial results should be carried to four decimal places.

2. In final results, if the mills are 5 or more, it is customary to call them 1 cent, and if they are less than 5 to reject them. (See **163.**)

PROBLEMS.

6. What is the interest of $2,160 for 1 year 3 months, at 7%?

7. What is the interest of $39.25 for 2 yr. 8 mo., at 5%?

8. What is the amount of $1,278 for 11 mo., at 7%? *$1,360.01.*

9. Find the interest of $9,500 for 3 yr. 1 mo., at 4%.

10. How much interest, at 8%, must I pay, for the use of $2,575 from May 11, 1868, to Sept. 11, 1869? *$274.67.*

458. Ex. What is the interest of $761.25 for 2 yr. 5 mo. 16 da., at 8%?

EXPLANATION. — Since 30 days are 1 month, every 3 days are 1 tenth of a month; 16 days are $\frac{16}{3}$ tenths or $5\frac{1}{3}$ tenths of a month; and 2 yr. 5 mo. 16 da. are $29.5\frac{1}{3}$ mo. or $\frac{29.5\frac{1}{3}}{12}$ yr. We therefore multiply the interest for 1 year by the number of months, and divide the product by 12, as in **457.**

SOLUTION.

$$\begin{array}{r l}
\multicolumn{2}{c}{2\ \text{yr. } 5\ \text{mo. } 16\ \text{da.} = 29.5\tfrac{1}{3}\ \text{mo.}}\\
\$761.25 & \text{Principal.}\\
.08 & \text{Rate.}\\ \hline
\$60.9000 & \\
29.5\tfrac{1}{3} & \\ \hline
203 & \\
3045 & \\
5481 & \\
1218 & \\ \hline
\$1798.58 & \lfloor 12 \\ \hline
\$149.88\tfrac{1}{6} & \left\{\begin{array}{l}\text{Int. for 2 yr.}\\ \text{5 mo. 16 da.}\end{array}\right.
\end{array}$$

PROBLEMS.

11. What is the interest of $198.50 for 4 mo. 9 da., at 4%?

12. What is the interest of $10,796 for 2 yr. 1 mo. 24 da., at 7%?

13. Find the amount of $18,450 for 1 mo. 15 da., at 5½%.

14. How much interest, at 6%, has accrued on a note for $94.75, that has been due 3 yr. 2 mo. 6 da.? *$18.10.*

15. What is the amount of $978.18 from Sept. 24, 1867, to Oct. 25, 1869, at 8%? *$1,141.43.*

II. SIX PER CENT METHOD.

459. The interest of $2 for 1 year, or 12 months, at 6%, or of $1 for the same time, at 12%, is $.12. Hence,

I. *The interest of any sum at 6% is the same as the interest of one half that sum at 12%.*

II. *At 12% per annum, the rate is 1% per month.*

Ex. Find the interest, at 6%, of $52.69 for 2 yr. 3 mo. 18 da.

SOLUTION.

2 yr. 3 mo. 18 da. = 27.6 mo.
$52.69 | 2
$26.345 ½ *the Prin.*
.276 Rate at 1% per mo.
158070
184415
52690
$7.271220 Interest.

EXPLANATION.—We first divide the principal, $52.69, by 2, to find the sum on which to compute interest at 12% (I.). We then multiply this result, $26.345, by the rate at 1% per month, which is .01 of the time expressed in months (II.). The final result, $7.27, is the required interest.

PROBLEMS.

16. How much interest, at 6%, will be due in 5 yr. on a loan of $5,790? *$1,737.*

17. What is the interest of $728.18 for 1 yr. 11 mo., at 6%?

18. If $2,765 be placed at interest at 6%, Mar. 14, 1869, what will be due Dec. 13, 1870? *$3,054.86.*

19. What is the interest of $20 for 12 yr., at 5%?
(6% — ⅙ of itself = 5%.) *$12.*

20. Find the amount of $417.61 for 3 yr. 7 mo., at 8%.
(6% + ⅓ of itself = 8%.) *$537.32.*

III. SEVEN PER CENT METHOD, FOR DAYS.

460. Computing interest on the basis of 30 days to a month, gives 360 days to a year. Hence,

I. *The product of any principal multiplied by any given number of days expressed as hundredths, is the interest at 360% per annum, or 1% per day.*

II. *The product of any principal multiplied by any given number of days expressed as thousandths, is the interest at 36% per annum, or .1% per day.*

III. *The interest at 36% divided by 6 gives the interest at 6%.*

IV *Interest at 6% plus interest at 1% equals interest at 7%.*

Ex. What is the interest of $125 for 63 days, at 7%?

Explanation.—We first multiply the principal, $125, by .063, and obtain $7.875, the interest at 36% (II.). We divide this result by 6, and obtain $1.31¼, the interest at 6% (III.). We then divide this result by 6, and obtain the interest at 1%; and adding the last two results, we have $1.53⅛, the required interest (IV.).

SOLUTION.

```
 $ 1 2 5  Prin.
   .0 6 3
 ---------
   3 7 5
 7 5 0
 ---------
$ 7.8 7 5 | 6
---------
$ 1.3 1 2 5 | 6
   .2 1 8 7 5
 -----------
$ 1.5 3 1 2 5  Int.
```

PROBLEMS.

21. What is the interest of $735 for 27 days, at 7%? *$3.86.*

22. What is the amount of $250 from Jan. 18 to March 30, 1868, at 7%? *$253.50.*

23. Dec. 24, 1868, I borrowed $25.50, and paid it June 1, 1869, with 7% interest. What amount was due?

24. What is the interest of $45.75 for 90 da., at 7%? *$.80.*

25. What is the amount of $1,250 for 63 da., at 7%? *$1,265.31.*

461. *Rules for Computing Interest.*

I. General Method.

1. For 1 year, *Multiply the principal by the rate.*

2. For 2 or more years, *Multiply the interest for 1 year by the number of years.*

3. For any other time, *Multiply the interest for 1 year by the time expressed in months and tenths of a month, and divide the product by 12.*

II. Six Per Cent Method.

Divide the principal by 2, and multiply the quotient by .01 of the time expressed in months.

III. Seven Per Cent Method, for Days.

1. *Multiply the principal by .001 of the number of days.*

2. *Divide the product by 6, and to the quotient add $\frac{1}{6}$ of itself.*

NOTES.—1. 360 days + $\frac{1}{72}$ of 360 days (5 days) = 365 days. Hence, if interest for days is required at 365 days to a year, subtract from itself $\frac{1}{73}$ of the interest found by the 7% method.

2. To find the amount, we may first find the amount of $1 at the given rate for the given time, by any one of the above rules, and then multiply the principal by this amount.

3. The months and days may be reduced to the decimal of a year (see **356**, II.), and the interest for 1 year, at the given rate %, may then be multiplied by the time expressed in years and decimals of a year.

PROBLEMS.

26. What interest must I pay for the use of $278.64 for 8 years, at 7%?

27. What is the amount of $473 for 7 yr. 7 mo., at 5%?

28. What is the interest of $419.84 for 1 yr. 11 mo. 18 da., at 5%? *$41.28.*

29. A debt of $1,560 was contracted May 23, 1868. How much was due June 18, 1869, interest at 6%? *$1,660.10.*

30. The balance due on a mortgage, Nov. 20, 1868, was $3,750. What was the amount due Aug. 20, 1869, interest 7%?

31. What is the amount of $75 for 8 yr., at 10%? *$135.*

32. A note for $1,116, bearing date Albany, N.Y., Oct. 9, 1866, was paid Oct. 9, 1869, with interest. What amount was paid?

33. What amount was due Aug. 5, 1869, on a note for $1,650, dated Philadelphia, Dec. 5, 1867? *$1,815.*

34. I bought a house and lot in Cleveland, for $4,750, paying $2,000 down, and giving a mortgage for the balance, due in 3 years. What was the amount of the mortgage when due? *$3,245.*

35. How much was due, May 3, 1869, on a note for $2,860, dated *San Francisco*, July 3, 1867?

36. What is the amount of $743.18 for 1 yr. 10 mo. 12 da., at 8%?

37. If I borrow $12,500 in New Haven, Conn., and loan it in New York, how much do I gain in 1 yr. 7 mo.? *$197.92.*

38. If I loan $1,500, at 7%, Aug. 17, 1869, how much will be due June 30, 1871? *$1,696.29.*

39. What is the interest of $10 for 15 yr. 4 mo., at 6%? 9.20

40. A note for $293, dated Detroit, Apr. 26, 1867, was paid Jan. 26, 1869. What was the amount paid? *$328.89.*

41. At 7%, what is the amount of $73.49 from Nov. 27, 1867, to Feb. 7, 1870? *$84.78.*

42. A man bought a farm in Minnesota for $2,280, paying $1,000 down, and the balance in 10 months, with interest. How much was the last payment? *$1,354.67.*

43. Find the amount of $856.75 for 2 years, at 5%. *$942.42½.*

44. How much interest will I have to pay on a loan of $7,650 for 20 days, at 7%? *$29.75.*

45. What is the amount of $25,390 for 7 months, at 10%?

46. Jan. 10, 1869, I borrowed $1,280 in Hartford, Conn., and paid it Aug. 7, 1869, with interest. How much did I pay?

47. What is the interest of $1,310 for 1 yr. 1 mo., in Va.?

48. How much will be due June 19, 1871, on a note for $1,750, dated Boston, June 19, 1869, with interest? *$1,960.*

49. Find the amount due Mar. 17, 1870, on a note for $217.85, dated St. Louis, Sept. 17, 1867, with interest.

50. A man who is paying $375 a year for house rent, borrows $5,000, at 6%, with which he buys the house. Does he gain or lose by the transaction? *He gains $75 per annum.*

CASE II.

Compound Interest.

462. In computing compound interest,

I. *The amount of the principal for 1 year is the principal for the second year, the amount of this principal for 1 year is the principal for the third year, and so on.*

II. *The final amount minus the principal is the interest.*

Ex. What is the amount of $127.50 at compound interest for 2 yr., at 6%? What is the interest?

Explanation. — Since the amount is the product of the principal multiplied by 1 plus the rate (see **461**, Note 2), we multiply the principal, $127.50, by 1.06, and obtain $135.15, the amount for 1 year. We multiply this amount by 1.06, as before, and obtain $143.259, the required amount for 2 years. Then, subtracting the principal, $127.50, from this amount, we have $15.76, the required interest.

SOLUTION.

```
  $127.50   Prin.
    1.06    1 + rate.
  -------
    7650
  1275
  -------
  $135.15   { Amt. for 1 yr., or
            { Prin. for 2d yr.
    1.06    1 + rate.
  -------
   81090
 13515
  -------
 $143.259   Amt. for 2 yr.
  127.50    Prin.
  -------
  $15.759   Int.
```

Note.—When interest is due semi-annually, quarterly, or monthly, the amount of the principal for the fixed period of time is the principal for the next period.

PROBLEMS.

51. What is the amount of $721.45 for 3 years, at 6% compound interest? *$859.26.*

52. What is the compound interest of $75.50 for 2½ years, at 6%, payable semi-annually? *$12.03.*

53. The principal is $25.75, the time 4 years, and the rate 7% compound interest. What is the amount?

54. How much will $525 amount to in 1½ years, at 7% compound interest, payable semi-annually? *$582.08.*

55. What is the compound interest of $437.50 for 1 yr. 3 mo., at 6%, payable quarterly?

56. At 24½%, interest payable monthly, what is the compound interest of $575 for 2½ yr.?

57. What is the difference between the simple and the compound interest of $5,425 for 4 years, at 6%? *$121.94.*

CASE III.

Partial Payments.

463. When partial payments are made upon notes, bonds, mortgages, or other obligations bearing interest, the U. S. Courts have established the following principles :

I. *Payments must be applied in the first place to the discharge of interest due, and the balance toward the discharge of the principal.*

II. *Interest must not be added to the principal so as to draw interest.*

III. *The principal must remain unaltered, when a payment is less than interest due.*

Ex. A note for $960, at 6% interest, was given Apr. 10, 1868. A payment of $225 was made Jan. 19, 1869, and another of $25, Nov. 3, 1869. What amount was due Jan. 5, 1870?

SOLUTION.

$960	Prin. ⌊2
$480	
.093	
144	
432	
$44.64	Int. to Jan. 19, 1869.
960	Prin.
$1004.64	Amt. due Jan. 19, 1869.
225	Payment, " "
$779.64	New Prin. ⌊2
$389.82	
.115⅓	
12994	
194910	
38982	
38982	
$44.95924	Int. to Jan. 5, 1870.
779.64	Prin.
$824.59924	Amt. to Jan. 5, 1870.
25	Payment, Nov. 3, 1869.
$799.60	Amt. due Jan. 5, 1870.

EXPLANATION.—We first find the amount of the principal from the date of the note to Jan. 19, 1869, the time of the first payment (see **461**, II.), to be $1,004.64. We next subtract the payment, $225, from this amount (I.), and have a remainder of $779.64 for a new principal. Since the payment, $25, made Nov. 3, 1869, did not exceed the interest due (III.), we find the amount of $779.64 from Jan. 19, 1869, to Jan. 5, 1870. Then, subtracting from this amount, $824.59924, the payment of $25, made Nov. 3, 1869, we have $799.60, the required amount due Jan. 5, 1870. Hence,

464. *Rule for Computing Interest in Partial Payments.*

I. *From the amount of the principal computed to the time when the payment or sum of the payments equals or exceeds the interest due, subtract the payment, or sum of the payments.*

II. *The remainder is a new principal, with which proceed as before.*

PROBLEMS.

58. On Nov. 5, 1867, the face of a mortgage on a farm in Mich. was $2,875, and $1,000 was paid Aug. 23, 1868. What amount was due May 17, 1869? *$2,140.51.*

59. A mechanic bought a house and lot in Salem, Mass., for $1,750, paying $500 down. One year afterward he paid $387.50. How much then remained due? *$937.50.*

60. Upon a note for $765, dated Buffalo, N. Y., Mar. 14, 1867, there was paid, Oct. 31, 1868, $50; and June 11, 1869, $285. The note was taken up, Sept. 25, 1869. How much was then due?

61. May 7, 1867, a capitalist loaned $10,000, at 5½%. Dec. 28, 1867, $4,800 was paid; and July 14, 1868, $3,750. What sum was due Jan. 3, 1869? *$2,020.04.*

VT., N. H., AND CONN. RULES.

465. When a note or other written obligation contains the words "with interest annually," or "with annual interest," the interest on the principal is due and payable at the end of each year. In some States the law allows to the holders of such obligations, simple interest on the unpaid yearly interests until they are paid. See Manual.

The differences between the U. S. Court Rule for computing interest in partial payments, and the rules in Vt., N. H., and Conn., are as follows:

466. *Vermont.*

I. *Unpaid yearly interests draw simple interest from the time they become due until they are paid.*

II. *All payments draw interest from the time they are made to the end of the year, or to the time of final settlement.*

III. *Interest on unpaid yearly interests does not draw interest.*

467. *New Hampshire.*

I. *Payments made expressly on account of interest accruing but not yet due, do not draw interest. At the end of the year they must be applied to the payment of the interest then accrued.*

II. *In all other respects the rule is the same as in Vermont.*

NOTES.—1. In computing annual interest in these States, the computations must be made for intervals of 1 year, or to the time of final settlement, when that occurs within a year.

2. The interest in partial payments is computed by the U. S. Court Rule, unless annual interest is stipulated in the note or agreement.

468. *Connecticut.*

I. *Interest is allowed on payments to the end of a year, when they exceed the interest due, at the time they are made; or to the time of settlement, when that occurs within a year.*

II. *When a year or more passes from the date of any computation, without payments being made, interest is computed for the whole time, by the U. S. Court Rule.*

PROBLEMS.

62–65. Find the amount due Oct. 1, 1869, on the following note, computing the interest by the U.S., the Vt., the N. H., and the Conn. Rule.

$1,850. ----------June 7, 1867.

On demand, I promise to pay the bearer Eighteen Hundred Fifty Dollars, for value received.

Wm. Wilder.

Indorsements on the back of this note: Oct. 17, 1867, $250; Feb. 23, 1868, $100; Dec. 30, 1868, $50; July 17, 1869, $225.

U.S. Rule, $1,449.44; Vt. and N.H., $1,447.82; Conn., $1,448.99.

CASE IV.

Converse Operations in Interest.

469. In computations in interest,

Principal is the base;

The product of rate % per annum and time expressed in years, is the rate;

The interest is the percentage; and

Interest is the product of the three factors, principal, rate, and time. Hence,

I. *Principal, rate and time given, to find interest,* is *Base and rate given, to find percentage.*

II. *Interest, rate and time given, to find principal,* is *Percentage and rate given, to find base.*

III. *Principal, rate and time given, to find amount,* is *Base and rate given, to find amount.*

IV. *Amount, rate and time given, to find principal,* is *Amount and rate given, to find base.*

V. *Principal, interest and time given, to find rate,* and

VI. *Principal, interest and rate given, to find time,* are *Base, percentage, and one of the two factors of rate given, to find the other factor.*

NOTE.—In V. and VI., the product of three factors (interest), and two of the factors (principal and rate, or principal and time) are given, to find the third factor.

PROBLEMS.

66. What is the interest of $580 for 11 mo. 27 da., at 6%?

67. What is the interest of $119.50 for 1 yr. 7 mo. 18 da., at $7\frac{3}{10}\%$?

68. The interest is $56, the rate .06, and the time 2 mo. 10 da. What is the principal? *$4,800.*

69. I loaned a certain sum of money, at 7%, and received $98.28 as the interest for 1 yr. 8 mo. 24 da. What was the sum loaned?

70. What is the amount of $7,365 for 2 yr. 4 mo., at 5%?

71. What is the amount of $390 for 7 mo. 10 da., at 9%?

72. What principal put at interest at 6%, will amount to $1,073.10 in 1 yr. 7 mo.? *$980.*

73. What principal put at interest at 4%, will amount to $182.20 in 5 yr. 4 mo. 12 da.?

74. The interest is $118.30, the principal $2,800, and the time 5 mo. 19 da. What is the rate?

75. At what rate will $508.50 earn $89.609 in 2 yr. 2 mo. 13 da.? *8%.*

76. In what time will $475 earn $71.25, at 6%?

77. The principal is $684, the interest $103.68, and the rate 10%. What is the time?

78. What is the interest of $3,750 for 1 yr. 1 mo. 13 da., at 7%?

79. What principal, at 5%, will produce $19.09 interest in 2 yr. 3 mo. 18 da.? *$166.*

80. The interest of $6,000 is $805, for 1 yr. 11 mo. What is the rate? *7%.*

81. I paid $165.37½ for the use of $1,350, at 7%. What was the time? *1 yr. 9 mo.*

82. The principal is $288, the rate 7%, and the time 3 yr. 4 mo. 10 da. What is the amount? *$355.76.*

83. The principal is $19,600, the amount $21,047.95, and the rate 4½%. What is the time? *1 yr. 7 mo. 21 da.*

84. New York, Jan. 25, 1869, I paid $169.85, the amount due on a note given Sept. 15, 1865. What was the face of the note?

85. At what rate will $560 amount to $659.40, in 2 yr. 11 mo. 15 da.? *6%.*

SECTION IX.

DISCOUNT.

470. ***Discount*** is a sum deducted for the payment, before it becomes due, of a note or other debt not drawing interest.

471. The ***Face*** of an obligation is the sum to be paid when the obligation is due, or *At Maturity*.

472. ***Present Worth,*** or ***Proceeds,*** is the face of an obligation minus the discount.

473. ***Commercial Discount,*** or ***Per Cent Off,*** is a deduction, from the face of the obligation, of some % agreed upon, without regard to time.

NOTE.—When the sum deducted depends upon both rate and time, it is sometimes called *True Discount.*

COMPUTATIONS IN DISCOUNT.

474. In Computations in Discount,

Present worth or proceeds is the base;

The product of the rate per cent per annum and the time expressed in years, is the rate;

The face of the obligation is the amount; and

The discount is the percentage. And

475. In Commercial Discount, or Per Cent Off,

Invoice price, or the face of an obligation, is the base;

Rate % off is the rate; and

The commercial discount is the percentage. Hence,

I. *Invoice price or face, and rate % off given, to find commercial discount,*	is	*Base and rate given, to find percentage.*
II. *Face, rate % per annum and time given, to find proceeds,*	is	*Amount and rate given, to find base.*

PROBLEMS.

1. What is the commercial discount on a bill of goods invoiced at $375.75, sold on 3 months' time, at 2½% off for cash? *$9.39.*

2. If I buy a bill of goods amounting to $237.50, on 30 days, 3% off for cash, what is the commercial discount?

3. A merchant buys a bill of goods amounting to $1,302.40, on 3 months' time, and is allowed 5% off for cash. What sum does he pay? *$1,237.28.*

4. When money is worth 6% per annum, how much must be discounted for the present payment of a note for $375.70, due in 8 months? *$14.45.*

5. What is the present worth of a note for $304.50, due in 1 yr. 3 mo., when money is worth 7%? *$280.*

6. Find the present worth of, and the discount on, $56, due in 7 mo. 6 da., in New York.

7. A merchant buys a bill of goods, invoiced at $975, on 60 days. If 5% off is allowed for cash, how much will he gain by borrowing the money at 7%, and cashing the bill? *$37.944.*

8. A farmer bought a horse for $140, giving his note due in 1 year. 4 mo. 24 da. afterward he paid the note, the holder allowing discount at 7%. How much did he pay?

9. Which is the greater, the interest or the discount of $1,712 for 1 year, at 7%, and how much? *The interest, $7.84.*

10. Sold a bill of goods amounting to $1,260, on 4 months, and made the buyer the customary discount of 5% off for 30 days, and a further discount of 10% off for cash. What were the cash proceeds of the sale? *$1,077.30.*

11. An invoice of books, at retail prices, amounts to $920, the commercial discount is 25% off, and 2½% off for cash. What are the net proceeds of the invoice? *$672.75.*

12. What is the difference between discounting a bill of goods at 20% and 5% off, and discounting the same bill at 5% and 20% off?

SECTION X.

GOVERNMENT SECURITIES.

476. A ***Bond*** is a written obligation from one person or party, securing to another the payment of a given sum at a specified time.

477. Whenever the U. S. Government borrows money, it gives to the lender a bond for the sum borrowed, with interest payable semi-annually, or annually.

478. ***Government Securities*** are bonds, or certificates of indebtedness, of the Government, to the holder of the same.

479. The principal U. S. Securities are 7-30's, 5-20's, 10-40's, 5's of '71, 5's of '74, 6's of '81, and the U. S. Pacific R. R. Currency 6's of '95, '96, and subsequent dates.

The rates of interest on these securities are, on

The 7-30's, $7\frac{30}{100}$%, payable in legal-tender notes;

The 5-20's, and *The 6's of '81,* 6%, payable in gold;

The 10-40's, 5%, payable in gold;

The U.S. Pacific R.R. Currency 6's, 6%, payable in currency.

NOTES.—1. The interest on Government bonds for small denominations, as $50 and $100, is payable annually, as was also the interest on the 7-30's.

2. The 5-20's can be paid at any time from 5 to 20 years from date, at the option of the Government. They were issued in 1862, '64, '65, '67, and '68.

3. The 5-20's of '67, '68, were issued in those years, to take up the 7-30's.

4. The 10-40's can be paid at any time from ten to forty years from date, at the option of the Government. They were issued in 1864.

5. The U. S. Pacific R.R. Currency 6's were issued in 1865 to 1869 inclusive, and are due in from 1895 to 1899 inclusive, according to date of issue.

COMPUTATIONS IN GOVERNMENT SECURITIES.

480. The par value or face of the bond is the base;
The rate % of premium or discount is the rate; and
The market value is the amount or difference. Hence,

I. *Face of bond and rate given, to find market value,*	is	*Base and rate given, to find amount or difference.*
II. *Market value and rate given, to find face of bond,*	is	*Amount or difference and rate given, to find base.*

PROBLEMS.

1. How much will 3 1000-dollar 7-30 bonds cost, at $3\frac{1}{4}$% premium?

2. If I sell $21,500 of Missouri 6's, at 32% discount, how much shall I realize? *$14,620.*

3. When gold is worth 124, what amount of currency can be bought for $5,400 in gold?

4. A capitalist invested $10,176 in New York City bonds, at 4% *discount.* What amount in bonds did he receive? *$10,600.*

76·

5. What is the gold value of $5,485 of currency, when gold is $137\frac{1}{8}$?

6. When gold is 140, which is the better investment, 7-30's at 102, or 6% 5-20's at 108?

7. A broker bought 5 300-dollar 6% county bonds, at $3\frac{1}{2}$% discount, and afterward sold them at 5% discount. How much did he lose by the transaction? *$22.50.*

8. If I invest $5,400 in Pacific R.R. 6's, at 28% below par, what will be the annual interest due me? The interest will be what rate per cent upon the investment? *Int. $450; rate, $8\frac{1}{3}$%.*

9. A capitalist invests $20,500 in U. S. 10-40's, at $2\frac{1}{2}$% premium. If gold is worth 135, what % in currency does he receive upon his investment? *$6\frac{24}{41}$%.*

SECTION XI.

481. A ***Promissory Note*** is a written promise to pay a certain sum of money, at a specified time, for value received.

COMMON FORM OF A PROMISSORY NOTE.

$213 $\frac{50}{100}$. *Cincinnati, July 3, 1869.*

Three months after date, I promise to pay to Richard Carter or order, Two Hundred Thirteen and $\frac{50}{100}$ Dollars, with interest, for value received. *Henry Seward.*

482. The *Maker* of a promissory note is the person who makes or signs the note ; and

483. The *Payee* is the person to whom, or to whose order, the note is to be paid.

484. An *Indorser* is a person who signs his name upon the back of a note, as security for its payment.

485. A *Negotiable Note* is one that may be bought or sold.

NOTES.—1. A note payable *to the bearer*, or *to A. B. or bearer*, is negotiable without indorsement. Other notes, payable *to A B. or order*, or *to the order of A. B.*, are not negotiable without the indorsement of *A. B.*

2. The sum for which a note is given should be written in words in the body of the note. The note should also contain the words "*value received.*"

3. If no mention of interest is made in a note, it draws interest from the day it is due.

486. *Days of Grace* are three days allowed after the time specified in a note has expired, before the note is legally due.

487. The *Maturity* of a note is the termination of the period of time it has to run. It is the last day of grace.

NOTE.—If the third day of grace falls on Sunday or a legal holiday, the note matures on the second day of grace.

488. A *Protest* is a written notice, in due form, to the indorser of a note, that the note has been presented to the maker, at maturity, for payment, and has not been paid by him ; and that the holder looks to the indorser for payment of the same.

NOTES.—1. A protest must be served on an indorser of a note on the last day of grace, to hold him responsible.

2. Protests are usually made out and served, by an officer called a *Notary Public.*

489. A *Bank* is an institution which receives deposits, loans money, and issues drafts, bills of exchange, and bank bills that circulate as money.

490. A *Savings-Bank* only receives money on deposit, *paying interest* on the sums deposited.

491. A ***Bank-Note*** is a note payable at a bank. Bank bills are also called bank-notes. (See **214.**)

COMMON FORM OF A BANK-NOTE.

$1,000. *New York, Jan. 16, 1869.*

Four months after date, I promise to pay to the order of David Barnum, at the Ninth National Bank of the City of New York, One Thousand Dollars, for value received.

Ezra P. Prentice.

492. ***Bank Discount*** is interest paid in advance, to a bank, for the loan of money on a note.

493. The ***Face*** of a note is the sum due at maturity. If the note is on interest, the face is the amount of principal and interest.

494. The ***Proceeds*** of a note is the face of the note minus the interest.

COMPUTATIONS IN BANKING.

495. In computing bank discount,

The face of the note to be discounted is the principal, or base;

The interest on the face of the note for the given time, at the given rate, is the bank discount, or percentage;

The proceeds of the note is the difference; and

The product of the rate % per annum and the time expressed in years, is the rate. Hence,

I. *Face of note, rate and time given, to find bank discount,*	is	*Principal, rate, and time given, to find interest.*
II. *Face of note, rate and time given, to find proceeds,*	is	*Base and rate given, to find difference.*
III. *Proceeds, rate and time given, to find face of note,*	is	*Difference and rate given, to find base.*

PROBLEMS.

1. What is the bank discount upon a note for $2,500, due in 4 months, at 6%? *$51.25.*

2. What are the proceeds of a note for $650, due in 90 days, if discounted at a N. Y. bank? *$638.25.*

3. A note for $7,350, due Aug. 12, 1868, was discounted at a bank in Charleston, S. C., May 20, 1868. What were the proceeds?

4. What sum can be realized at a bank, upon a note for $11,500, due in 30 days, at 6% discount?

5. A bank loans $4,500 on a note payable in 4 months, discounting it at 8%. What is the face of the note? *$4,626.46.*

6. I obtained $237, at a bank in New Orleans, on a note due in 3 months. What was the face of the note?

7. A merchant wishes to borrow $2,000 at a bank. For what amount must he make his note, due in 60 days, if he gets it discounted at 6%?

8. What will be the proceeds of a note for $4,500, due in 6 months, discounted at a bank in San Francisco? *$4,271.25.*

9. A manufacturer in St. Louis wishes to borrow $2,000 with which to pay his men. If he makes a bank-note, due in 30 days, what will be the face of the note? *$2,011.06.*

10. A merchant buys a bill of goods in Milwaukee, amounting to $3,700, on 3 months' time, or 5% off for cash. If he borrows the money at a Milwaukee bank, will he make or lose, and how much?

11. I had a note for $460, dated Detroit, Jan. 9, 1869, and due in 8 months, with interest. March 19, 1869, I had it discounted at a bank, at 8%. How much did I realize from the note?

12. For what sum must a bank-note, due in 5 months, be made, to have it produce $1,856 when discounted at the First National Bank of Boston? *$1,904.57.*

13. Find the face of a note, due in 6 months, on which I can borrow $875 at a Chicago bank. *$921.86.*

14. In payment of a debt, I took Chas. Marshall's note for $1,600, payable at the Sixth National Bank of Philadelphia, in 6 months, with interest. Four months afterward, I had it discounted at the First National Bank of Harrisburg. What were the proceeds?

SECTION XII.

EXCHANGE.

496. *Exchange* is a commercial transaction, in which a party in one place pays money to a second party in another place, by means of an order upon a third party, and without the transmission of money.

497. A *Draft,* or *Bill of Exchange,* is a written order for money, drawn in one place and payable in another.

Example.—A Chicago merchant, wishing to pay a debt in New York, buys at a bank in Chicago, a draft on a New York bank, payable to the order of the party in New York. The Chicago merchant sends this draft to his creditor in New York, and the latter indorses it, presents it to the New York bank, and receives the face of the draft in money.

Notes.—1. Any party may give a draft, or "draw" on another party, if the second party is debtor to the first.

2. A draft payable in the same country in which it is drawn, is an *Inland Bill of Exchange;* and one drawn in one country, and payable in another, is a *Foreign Bill of Exchange.*

498. A *Sight Draft* is a draft payable "at sight," *i. e.,* when it is presented; and

499. A *Time Draft* is a draft payable at a future time named in it.

Note.—Grace is allowed on time drafts, but not on sight drafts.

500. There are usually four parties to a transaction in exchange, viz.,

The *Drawer* or *Maker* of the draft;

The *Buyer* or *Remitter,* or the party who purchases the draft;

The *Drawee,* or the party on whom the draft is drawn;

The *Payee,* or the party to whose order the draft is made payable.

Note.—The maker and remitter of a draft may be the same party, in which case there will be but three parties to the transaction.

COMMON FORM OF DRAFT.

STAMP.

$1,500. Chicago, Jan. 19, 1869.

Ten days after sight, pay to the order of

Harper & Brothers

Fifteen hundred Dollars,

Value received, and charge to account of

To the Park Bank, S. C. Griggs & Co.

New York City.

NOTE.—The words *At Sight*, in place of "Ten days after sight" would make the above a sight draft.

COMPUTATIONS IN EXCHANGE.

501. Drafts or bills of exchange are bought at par, at a premium, or at a discount.

The face of the draft is the base;

The rate % of exchange is the rate;

The premium or discount is the percentage; and

The proceeds of the draft is the amount or difference. Hence,

I. *Face of draft and rate % of exchange given, to find premium or discount,*	is	*Base and rate given, to find percentage.*
II. *Proceeds and rate % of exchange given, to find face of draft,*	is	*Amount or difference and rate given, to find base.*

NOTES.—1. The face of a draft plus the premium, or minus the discount, is the proceeds.

2. The subject of Foreign Exchange is not considered in this book. A full presentation of it will be found in the Academic Arithmetic of this Series.

PROBLEMS.

1. At 2% premium, how much will it cost me to remit $2,700 by draft from Grand Rapids, Mich., to Philadelphia?

2. How much will it cost me to make a remittance of $280 from New York to Baltimore, exchange on Baltimore being at ½% discount? *$278.60.*

3. If I buy a draft for $1,285 in Pittsburg, to send to Louisville, at ½% discount, how much does it cost me? *$1,278.58.*

4. How much will a man in Galena have to pay for a draft on Boston for $532, exchange being at 3¼% premium?

5. A man at Springfield, Ill., wishing to remit to a creditor in Harrisburg, Penn., buys a draft, at 2½% premium, and pays $246 for it. What is the face of the draft? *$240.*

6. A man in Brooklyn, having $145.50 belonging to a man in St. Paul, Minn., purchases a draft with it on a St. Paul banker, at 3% discount. What is the face of the draft?

7. What must I pay in Wheeling, W. Va., for a draft on N. Y. for $1,200, payable 30 days after sight, exchange being at 1¾% premium?

NOTE.—On a time draft, both the discount and the rate of exchange must be computed on the face of the draft. *$1,214.40.*

8. A St. Louis banker discounts a draft on Baltimore for $860, payable 90 days after sight, exchange on Baltimore being at 4½% premium. What does he pay for the draft? *$885.37.*

9. A grocer in Rock Island paid $611.70 for a draft on New York, payable 60 days after date, when exchange on New York was at 3% premium. What was the face of the draft?

10. A broker in Columbus, Ohio, pays $352.62 for a draft, payable at Knoxville, Tenn., 30 days after sight, at 1½% discount. What is the face of the draft? *$360.*

SECTION XIII.

AVERAGE OF PAYMENTS.

502. *Average of Payments* is the process of finding the time for paying several sums due at different times, so that no loss shall be sustained by either party.

503. The *Term of Credit* is the time during which no interest is paid.

504. The *Average Term of Credit* is the average time during which no interest is paid on different debts due at different times.

505. The *Equated Time* is the date on which several debts due at different times may all be paid, without loss to either creditor or debtor.

506. *Averaging an Account* is the process of finding the mean or equated time for the payment of the balance of the account.

Equation or average of payments is of two kinds, Simple and Compound.

507. *Simple Average* is one in which the sums are either all debits or all credits.

508. *Compound Average* is one in which some of the sums are debits, and some are credits.

509. A *Focal Date* is any date taken as a standard of reference, and with which each given date is compared.

CASE I.

The terms of Credit beginning at the same Date.

510. Ex. July 1, I purchase a saw-mill for $1,200, paying $400 down, and agreeing to pay $200 in 3 months, $400 in 6 months, and $200 in 9 months, without interest. What is the average time for the payment of the whole amount of the purchase money?

SOLUTION.

$400 Cash payment.
200 for 3 mo. = $ 600 for 1 mo.
400 " 6 " = 2400 " 1 "
200 " 9 " = 1800 " 1 "
$1200 " ? " = $4800 " 1 "

$4800 | $1200
4 = 4 mo., average term of credit.
July 1 + 4 mo. = Nov. 1.

EXPLANATION. — The cash payment of $400 has no term of credit, and only forms a part of the purchase money. The use of $200 for 3 months is the same as that of 3 times $200, or $600, for 1 month: the use of $400 for 6 months is the same as that of $2,400 for 1 month; and the use of $200 for 9 months is the same as that of $1,800 for

1 month. The sum of the payments, due at different times, is $1,200, and of the equivalent sums for 1 month is $4,800. The use of $4,800 for 1 month is the same as the use of $1,200 for as many months as the number of times $1,200 are contained in $4,800. Dividing $4,800 by $1,200, we obtain 4 = 4 months, the average term of credit; and July 1+4 months = Nov. 1, the time required. Hence,

The product of the sum of all the payments multiplied by the average term of credit, equals the sum of the products of all the payments multiplied by their respective terms of credit.

511. On this principle is based the

Rule for finding the average time of payment, when the terms of credit all begin at the same date.

I. *Multiply each payment by its term of credit, and add all the payments, and also all the products.*

II. *For the average term of credit, divide the sum of the products by the sum of the payments.*

III. *For the average time of payment, add the term of credit to the given date.*

PROBLEMS.

1. March 8, I bought a building lot for $800, paying $200 down, and agreeing to pay $200 in 4 months, $200 in 8 months, and $200 in 12 months. Had I given my note for the payment of the whole amount at once, at what date should it have been made payable? *Sept. 8.*

2. May 29, a merchant bought bills of goods as follows: $825 on a credit of 3 months, $675 on 4 months, $450 on 2 months, and $800 on 1 month. What is the average time for the payment of the whole amount? *Aug. 14.*

3. On the first day of May, D hired a house at $300 per annum, agreeing to pay the rent quarterly. What would be the equated time for the payment of the whole? *Dec. 16.*

4. To-day I owe $150 due in 30 days, $200 due in 60 days, and $250 due in 90 days. If I give my note for the whole amount, made payable at the average time, when will the note be due?

5. What is the average time for the payment of 5 notes, all bearing date June 17; one for \$300 due in 3 months, one for \$500 due in 5 months, one for \$150 due in 7 months, one for \$350 due in 9 months, and one for \$200 due in 1 year?

CASE II.

The terms of Credit beginning at different Dates.

512. Ex. March 21, I bought a horse for \$175 on a credit of 4 months; June 5, a harness for \$55 on 3 months, and a top carriage for \$225 on 4 months. What is the average time for the payment of the three debts?

SOLUTION.

Mar. 21 + 4 mo. = July 21
June 5 + 3 " = Sept. 5
" 5 + 4 " = Oct. 5

\$175 cash, July 21.
55 for 46 da. = \$ 2530 for 1 da.
225 " 76 " = 17100 " 1 "
\$455 " ? " = \$19630 " 1 "

$$\begin{array}{r|l} \$19630 & \$455 \\ \underline{1820} & \overline{43} = 43 \text{ da.} \left\{ \begin{array}{c}\text{average term}\\ \text{of credit.}\end{array}\right. \\ 1430 & \\ \underline{1365} & \\ 65 & \end{array}$$

65 = $\frac{65}{455}$*, less than ½ da., dropped.*

July 21 + 43 da. = Sept. 2.

EXPLANATION.—To find the dates on which the several payments fall due, we add to each date the term of credit. Since the first payment falls due July 21, we take that date as a focal date. Comparing this focal date with the dates on which the several payments mature, we have \$175 due at the focal date, \$55 in 46 days, and \$225 in 76 days. Proceeding as in Case I., we find the average term of credit to be 43 days after July 21, or Sept. 2, the average time required. Hence,

When the terms of Credit begin on different dates,

The earliest date on which any one of the payments matures, may be taken as the focal date; and the time between this date and the date on which each of the other payments matures, may be taken as the term of credit of that payment.

513. On this principle is based the

Rule for finding the average time of payment, when the terms of credit begin at different dates.

I. *Find the time on which each payment becomes due, by adding the term of credit to the date of the transaction.*

II. *For the focal date, take the earliest date on which any one of the payments matures; and for the term of credit of each payment, take the time between the focal date and the time on which the payment matures.*

III. *For the average time of payment, find the average term of credit, and add it to the focal date.*

PROBLEMS.

6. Find the average time for the payment of the following bills:

Jan. 10, $415 on 3 months;
Feb. 25, 175 " 4 "
Apr. 5, 350 " 2 "

35 days from Apr. 10, or May 15.

7. What is the equated time for the payment of three notes, one for $650, dated July 12, and due in 90 days; one for $555, dated July 25, and due in 60 days; and one for $445, dated Aug. 14, and due in 30 days? *Sept. 27.*

8. George Adams bought provisions as follows:

Mar. 14, 40 bar. beef, @ $17.50, at 3 months.
May 1, 60 " pork, @ 24 "
June 10, 150 " flour, @ 8 Terms cash.

What is the average time for the payment of the whole?

9. I make the following advances of money for a friend: May 19, $107; May 28, $35; June 27, $130; July 3, $70; Aug. 24, $80, and Sept. 11, $175. If I take his note for the whole amount, dated at the equated time, what will be the date of the note? *July 19.*

10. B works for C 6 months from May 15, at $60 per month, his wages to be paid one half monthly, and the other half in 3 months. In lieu of receiving his pay according to contract, he takes C's note for the whole amount, bearing date from the average time, with interest. What is the date of the note? *Oct. 15.*

CASE III.

Accounts containing both Debits and Credits.

514. Computations in compound average are based upon the following equitable principles :

I. *All payments made before the average term of credit expires, should draw interest from the time they are made ;* and

II. *All debits not paid till after the average term of credit expires, should draw interest from the expiration of the average term of credit.*

515. Ex. Find the average time for the payment of the balance of the following %. :

Dr. ROBERT LANSING. *Cr.*

1869					1869				
Jan.	4	To Mdse.@4 mo.	325	50	Feb.	15	By Cash,	200	00
"	16	" do. 4 "	37	50	Apr.	17	" do.	75	00
Mar.	1	" do. 4 "	162	50					
"	26	" do. 4 "	250	00					

SOLUTION.

1st.—Averaging the Debits.

Jan. 4+4 mo. = May 4.
" 16+4 " = " 16.
Mar. 1+4 " = July 1.
" 26+4 " = " 26.
Focal date, May 4.

$325.50 cash payment, May 4.
37.50 for 12 da. = $ 450 for 1 da.
162.50 " 58 " = 9425 " 1 "
250 " 83 " = 20750 " 1 "
$775.50 " ? " = $30625 " 1 "

$30625.0 | $775.5
23265 | 39.4 = 39 da. { Average term of credit.
73600
69795
38050
31020
7030

May 4+39 da. = June 12 { Average time for payment of debits.

EXPLANATION.—*1st,* We find the average time of the debits to be June 12 ; and, *2d,* The average time of the credits to be Mar. 4 ; both by Case II.

3d, Since $275 was paid 100 days before it was due (*i.e.* before the average time at which *the debits* were due), interest must be allowed on \$775.50 − \$275 = \$500.50, the balance, for 100 days (**514**, I.); or, which is the same thing, the term of credit for \$500.50, the balance, must be extended a sufficient time to average the use of the \$275 paid 100 days before it was due. We find this additional term of credit for the balance of the account, by multiplying \$275, the sum of the credits, by 100, the number of days before the maturity of the debits, and dividing \$27,500, the product, by \$500.50, the balance of the %., as in **511**. The

2d.—Averaging the Credits.

Focal date, Feb. 15.

\$200 cash payment, Feb. 15.
75 for 61 da. = \$4575 for 1 da.
\$275 " ? " = \$4575 " 1 "

\$4575 | \$275
275 | 16.6 = 17 da. { ***Average term of payments.***
1825
1650
175'0
1650
100

Feb. 15 + 17 da. = Mar. 4 { ***Average time of payments.***

3d.—Averaging Debits and Credits.

Focal date, June 12.
From Mar. 4 to June 12 = 100 da.
\$775.50 − \$275 = \$500.50.
\$275 for 100 da. = \$27500 for 1 da.
500.50 " ? " = 27500 " 1 "

\$27500.0 | \$500.5
25025 | 54.9 = 55 da. { ***Average time of bal. of %.***
24750
20020
4730'0
45045
2255

June 12 + 55 da. = Aug. 6 { ***Date of payment of bal. of %.***

result, 55 days, added to June 12, the date of the maturity of the debits, gives Aug. 6, average time required.

Had the date of the average term of the credits been later than that of the average term of the debits, *i. e.*, after June 12, we should have dated back, or subtracted, the average time of the bal. of the %. from June 12. Hence,

516. ***Rule for Computing Compound Average.***

I. *Find the average term of the debits and credits separately.*

II. *For the average term of credit of the balance of the account, take the average date of the larger of the two sides of the account for a focal date, multiply the smaller side by the difference in time between its date and this focal date, and divide the product by the balance of the account.*

III. *For the date of the average time, count forward from the last focal date, if the larger side of the account falls due later; and backward, if it falls due earlier.*

PROBLEMS.

11. Balance the following %., and find when it is due:

Dr. E. M. Daniels & Co. *Cr.*

1869					1869				
June	14	To Mdse. @ 3 mo.	450	00	Sept.	3	By Cash,	400	00
Aug.	25	" " 3 "	175	00	Nov.	2	" do.	150	00
Oct.	11	" " 3 "	425	00	"	30	" do.	225	00

Feb. 17, 1870.

12. What is the balance of the following %., and when is it due?

Dr. John G. Anderson. *Cr.*

1869					1869				
Oct.	9	To Mdse. @ 3 mo.	300	00	Nov.	24	By Cash,	25	00
Nov.	18	" do. 3 "	329	00	Dec.	4	" do.	500	00
"	27	" do. 3 "	142	00	"	30	" Note,	150	00
Dec.	19	" do. 3 "	256	00					

$352; June 25, 1870.

13. If a note, drawing interest, be given to balance the following %., for what sum will it be drawn, and what will be its date?

Dr. AMES & POTTER. *Cr.*

1869					1869				
Mar.	17	To Mdse. @ 2 mo.	325	00	July	25	By Cash,	125	00
Apr.	20	" do. 3 "	108	00	Aug.	17	" do.	300	00
July	18	" do. Cash,	264	00	Oct.	24	Draft on N.Y.,	350	00
Aug.	11	" do. @ 4 mo.	50	00					
Sept.	25	" do. 2 "	125	00					

$97; May 25, 1868.

SECTION XIV.

REVIEW PROBLEMS IN PERCENTAGE.

1. What is 13⅔% of 837 bushels of wheat? *114.39 bu.*

2. If I sell a sewing-machine for $50 that cost me $56, what % do I lose? *10 5/7%.*

3. How much will 100 shares of N. Y. C. R.R. stock cost me, at 154½, brokerage ¼%?

4. Of every 1,375 persons 25 years old, 1,265 will live to the age of 26. What % of persons 25 years old die annually? *8%.*

5. A real estate agent receives $35 for selling a farm, his rates of commission being 1% from the buyer, and 1½% from the seller. What is the price obtained for the farm? *$1,400.*

6. A note for $36.50, dated June 27 of last year, was paid April 4 of this year, with interest at 10%. What was the amount paid?

7. A commission-merchant receives $820 with which to buy goods, after deducting his commission of 2½%. How much does he expend for goods? *$800.*

8. I borrow $4,000 on my note in Portland, Me., Feb. 21, and loan the money in Syracuse, N. Y., Feb. 23. If the money is paid to me, Nov. 12, and I pay my note Nov. 13, how much do I gain? *$26.78.*

9. Three men own a mill. C's share, which is $2,550, is 60% of B's, and B's share is 85% of A's. What is the value of the mill? *$11,800.*

10. A merchant paid a premium of $363.12½ for a policy of insurance covering $8,000 on his store, and $12,750 on his goods. What was the rate? 1¾%.

11. If a clergyman's salary is $1,500, and he pays $175 for house rent, what is his income tax?

12. If the annual rate on a life at 50 years is $4.439 per $100, payable semi-annually, what will be each payment on a policy for $2,500?

13. What is the premium, at ⅝%, for insuring a farm-house for $800, a barn for $750, and the hay and grain for $1,200?

14. One morning, five canal boats were weighed at the weigh-lock in Utica. The cargo of the first weighed 64 tons, and the cargoes of the others weighed, respectively, 85%, 67½%, 120%, and 56¼% of that amount. What was the total weight of the five cargoes? *274.4 tons.*

15. I buy a draft in Portsmouth, N. H., for $250, payable in Providence, R. I., 60 days after sight, exchange ½% premium. How much does the draft cost me? *$248.62½.*

16. In a village school of four departments, 33 pupils are in the first department, 44 in the second, 54 in the third, and 60 in the fourth. What % of all the pupils attend each department?

17. A note for $360, drawing 6% interest, was dated April 10, 1868, and $225 was paid on it, Jan. 19, 1869. What amount was due Nov. 3, 1869?

18. A stock jobber bought 100 shares of Pacific R.R. stock, at 117. He sold 55 shares at 104½, and exchanged the balance at 108 for Ocean Bank stock at 135. He afterward sold the bank-stock at 144½. Did he gain or lose? *He lost $750.50.*

19. A Va. planter took up a note for $843, Oct. 31, 1869, that was dated May 29, 1867. What interest had accrued? *$122.52.*

20. A box of soap, marked 60 lb., loses 15% by drying. What is its actual weight?

21. If a man buys U. S. 5-20 bonds at 106½, and sells the gold interest at 127½, what % in currency does his investment pay him?

22. A fruit dealer bought quinces at $1.60 per bushel. After assorting them, he sold the best at 35% profit, and the others at 15% loss. What were his selling prices?

23. In building a house which cost $1,480, 43% of the cost was for labor, and the balance for materials. What was the cost of the materials? *$843.60.*

24. How shall a merchant mark carpeting that cost him $1.242 per yard, so that he can fall 8% from the marked price, and still make 25%? *At $1.68¾ per yd.*

25. A mechanic buys a city lot for $600, payments $250 cash, and the balance in 1 year without interest. In 6 months he pays the balance, less the discount, at 6%. How much does he pay?

26. A field of 11 acres yielded 16.5 bushels of wheat to the acre; the cost of seed and labor was $193.60, and the wheat brought $1.60 per bushel. What % profit did the crop pay? *50%.*

27. What is the face of a sight draft that costs $360, exchange being at 1⅛% premium?

28. An Oswego miller buys a draft for $2,500 on Chicago, at ¾% discount. He remits the draft to a grain buyer in Chicago, with instructions to invest the proceeds, less his commission of 1%, in wheat. Is the miller's gain on the exchange more or less than the grain buyer's commission? *$6.*

29. What is the face of a note due in 2 mo. 15 da., the proceeds of which, discounted at the First National Bank of Burlington, Vt., are $370.12½? *$375.*

30. A drover paid $4,325 for cattle, and $1,498 for marketing them, and they sold for $6,375 at 60 days. What were his net cash profits, money being worth 8%? *$468.12.*

31. Memorandum:—Face of mortgage, April 23, 1866, $3,275.
Indorsements,—Sept. 4, 1866, $845; Feb. 27, 1868, $150; Aug. 19, 1868, $75; Jan. 7, 1869, $1,250.
What was due July 1, 1869, interest at 6%? *$1,417.92.*

32. Find the time from which interest should be reckoned on the sum of the debts in the margin, if all of them are paid when the last one is due. *Aug. 1.*

$ 400	due	Mar. 3.
325	„	May 19.
1,000	„	„ 25.
625	„	Sept. 4.
1,275	„	Nov. 12.

33. If you borrow $620 for 1 yr., at 8% interest, and 5 mo. afterward you pay $314, how much will you owe at the end of the year?

34. A's property is assessed at $6,750, and B's at $13,575, and A's tax is $52.65. How much is B's tax? *$105.89.*

35. A liquor dealer imported 45 casks of wine, invoiced 36 gal. each, at $1.50 per gal. He paid $1.75 per cask for transportation, a specific duty of $1 per gal., and 25% ad valorem duty. Deducting the customary allowance of 2% for leakage, what did the wine cost him? *$4,691.70.*

36. How many shares of bank-stock, at par, can a stock broker buy for $4,522.50, less his brokerage of ½%?

(37)

$1,650. Ithaca, N. Y., Nov. 19, 1867.

One year after date, I promise to pay Andrew D. White, or order, Sixteen Hundred Fifty Dollars, with interest, for value received.

Ezra Cornell

Indorsements: June 18, 1869, $125; Oct. 25, 1869, $475.

How much was due on settlement, Mar. 4, 1870?

38. When gold is worth 135, which is the better investment, U. S. 10-40's at 97½, or 5-20's at 107¾?

39. Find the average time for the payment of the following

STATEMENT OF %.

NEW YORK, *Feb. 1, 1869.*

O. & S. Ransom, Mobile, Miss.,

In % with **H. W. Perkins & Co.**, *Dr.*

1868.	*Oct.*	*19,*	*To Mdse.*	*@ 3 mo.,*		*$ 375*
	Nov.	*4,*	*,, do.*	*30 da.*		*757*
	Dec.	*28,*	*,, do.*	*Cash,*		*1,050*
1869.	*Jan.*	*15,*	*,, do.*	*@ 4 mo.*		*265*
			Am't due, Jan. 8, 1869,			*$2,447*

H. W. Perkins & Co.

40. I have a mortgage, bearing interest at 7%, payable annually. Will I gain or lose by selling it at ¼% discount, and investing the *proceeds in* U. S. 6's of '81, at 102½, when gold is quoted at 125?

CHAPTER 8.

RATIO AND PROPORTION.

SECTION I.

RATIO.

517. ***Ratio*** of numbers is the relative value of one number to another of the same kind.

518. The ***Terms*** of a ratio are the numbers whose values are compared.

519. The ***Antecedent*** is the first term; and

520. The ***Consequent*** is the second term.

EXAMPLE.—The relative value, or the ratio, of 15 to 5, is 3. In this example, 15 and 5 are the *terms* of the ratio, 15 is the *antecedent*, and 5 is the *consequent*.

NOTE.—The terms of a ratio are called a *Couplet*.

521. *Ratio is obtained by dividing the antecedent by the consequent.*

522. The ***Sign*** of ratio is the colon (:). It is written between the terms of a ratio, and is read, "the ratio of." Thus, 15 : 5 is read, "the ratio of 15 to 5."

Since the ratio is obtained by dividing the antecedent by the consequent, ratio may also be expressed by writing the antecedent for the numerator, and the consequent for the denominator of a fraction. Thus, $15 : 5 = \frac{15}{5}$.

523. A ***Simple Ratio*** is the ratio of any one number to another number of the same kind; and

524. A ***Compound Ratio*** is the ratio of the products of the corresponding terms of two or more simple ratios.

EXAMPLE.—The ratio of 15 to 5 is a simple ratio; and the ratio of the products of the corresponding terms of the ratios 15 : 5 and 8 : 2, or $15 \times 8 : 5 \times 2$, is a compound ratio.

The simple ratio of 12 : 4, or $\frac{12}{4}$, is 3;
" " " " 8 : 2, " $\frac{8}{2}$, " 4.
" compound " " $\left.\begin{matrix}12:4\\8:2\end{matrix}\right\}$, " $\frac{12}{4}\times\frac{8}{2}$ " 12, or 3 times 4, the product of the two simple ratios which form the compound ratio.

525. *Principles of Ratio.*

I. *The terms of a ratio must be of the same kind or denomination.*

II. *In an expressed ratio, the antecedent is a dividend and the consequent is a divisor.*

III. *The ratio is the quotient of the antecedent divided by the consequent.*

IV. *The product of any two or more ratios equals the ratio of their products;* and

V. *A compound ratio is reduced to a simple one, by multiplying all the antecedents together for a new antecendent, and all the consequents together for a new consequent.*

EXERCISES.

1. Express the ratio 18 to 3, and of 3 to 18; of 20 to 4, and of 4 to 20.

2. Express in both forms the ratio of 35 to 5; of 28 to 2; of 42 to 7; and of 9 to 15.

3. Find the ratio of 18 to 6; of 21 to 3; and of 14 to 2.

4. The antecedent is 16⅔, and the consequent is 3⅓. What is the ratio?

5. What is the ratio in each of the expressions 56 : 8; $51 : $3; 17½ yd. : 2½ yd.?

6. Find the ratio of 16⅝ to 13$\frac{4}{7}$; of 3.5 : .7; of .25 : 3.75; and of 5.7 : 8.4.

7. Express the compound ratio of $\left\{\begin{matrix}9:6\\5:10\end{matrix}\right\}$.

8. The antecedents of a compound ratio are 5, 72, and 30, and the consequents are 8, 3, and 5. Express the compound ratio.

10,800 : 120 = 90.

SECTION II.

SIMPLE PROPORTION.

526. ***Proportion*** is an equality of ratios.

527. ***Simple Proportion*** is an equality of two simple ratios.

528. The ***Sign*** of Proportion is the double colon (: :). It is written between the ratios, and is read "as," or "equals."

EXAMPLE.—The ratios 12 : 6 and 8 : 4 being equal, form the proportion 12 : 6 : : 8 : 4, which is read, "12 is to 6 as 8 is to 4," or "the ratio of 12 to 6 equals the ratio of 8 to 4."

Proportion may also be expressed by the sign of equality. Thus, 12 : 6 : : 8 : 4 may be written 12 : 6 = 8 : 4, or $\frac{12}{6}=\frac{8}{4}$.

529. The ***Extremes*** of a proportion are the first and fourth terms; and

530. The ***Means*** are the second and third terms.

531. We may write the proportion 12 : 6 : : 8 : 4 in the fractional form, and reduce the fractional expressions to similar fractions; $\frac{12}{6} :: \frac{8}{4} = \frac{48}{24} :: \frac{48}{24}$, or $\frac{12\times4}{6\times4} :: \frac{6\times8}{6\times4}$. The factors 12 and 4, of the first numerator, are the extremes of the proportion; and the factors 6 and 8, of the second numerator, are the means; and the products of these two sets of factors are equal. Hence,

532. *Principles of Proportion.*

I. *The product of the extremes equals the product of the means.*

II. *The first term is greater or less than the second, according as the third term is greater or less than the fourth.*

533. Ex. 1. What is the fourth term of the proportion 21 : 6 : : 14 : —?

SOLUTION.

$$\frac{6\times14}{21}=\frac{84}{21}=4$$

EXPLANATION. — 84, the product of the means, 6 and 14, is also the product of the extremes (**532**, I.). We now have 84, the product of two factors, and 21, one of the factors, to find the other factor, which we do by dividing 84 by 21. The result, 4, is the fourth term required.

Ex. 2. Find the second term of the proportion 15 : — : : 30 : 12.

SOLUTION.

$$\frac{15\times12}{30}=\frac{180}{30}=6$$

EXPLANATION.—Since the product of the extremes equals the product of the means, we multiply the extremes, 15 and 12, together, and divide the product, 180, by the known mean, 30. The result, 6, is the term required. Hence,

I. *The product of the means divided by either extreme gives the other extreme;* and

II. *The product of the extremes divided by either mean gives the other mean.*

PROBLEMS.

Find the unknown term in each of the following proportions:

1. 10 : 8 : : 15 : —.
2. 27 : — : : 12½ : 5.
3. 7½ : 6¼ : : — : 5.
4. — : 21 : : 15 : 35.
5. 12 : 18 : : 18 : —.
6. 6 : — : : 12 : 8.
7. 4 : 7 : : — : 6.
8. — : ⅘ : : ⅝ : ⅜.
9. .25 : — : : .3 : 2.4.
10. 15 : 9 : : 10 : —.

534. Ex. If 16 tons of hay cost $212, how much will 11 tons cost?

EXPLANATION.—*1st. Finding the ratios.*—The ratio of 16 T. to 11 T. must be the same as that of $212, the cost of 16 T., to $—, the cost of 11 T. The 16 T. and 11 T. must therefore form one couplet of the proportion, and the $212 and the $— the other couplet.

SOLUTION.

The ratios. $\begin{cases} 16\text{ T.} : 11\text{ T.} \\ \$212 : \$\text{—} \end{cases}$

Statements. $\begin{cases} 16\text{ T.} : 11\text{ T.} :: \$212 : \$\text{—} \\ 11\text{ T.} : 16\text{ T.} :: \$\text{—} : \$212 \end{cases}$

Finding unknown term. $$\frac{11\times\overset{\$53}{\cancel{\$212}}}{\underset{4}{\cancel{16}}}=\frac{\$583}{4}=\$145\tfrac{3}{4}$$

2d. Making the statement.—Since 16 T. are more than 11 T., $212, the cost of 16 T., must be more than $—, the cost of

11 T. Therefore, if we write 16 T. : 11 T. for the first ratio, we must write $212 : $— for the second (see **532**, II.) ; or, if we write 11 T. : 16 T. for the first ratio, we must write $— : $212 for the second.

3d. We find the unknown term by **533**, I. or II.

For convenience in making the statement, we take the two given numbers of the same kind, for the terms of the first ratio, and the other given number for the first term of the second ratio, or the third term of the proportion. Now, writing this third term, and applying (**532**, II.), we readily determine which term of the first couplet to write for the antecedent, and which for the consequent. Hence,

535. *Rule for Computations in Simple Proportion.*

I. *For the third term, write the number that is of the same kind as the unknown term.*

II. *Write the other two numbers as the first ratio, putting the greater for the first term, when the unknown term is to be less than the third term ; and for the second, when it is to be greater.*

III. *For the fourth or unknown term, multiply the second and third terms together, and divide the product by the first term.*

NOTES.—1. When the terms of the first ratio are of different denominations, they must be reduced to the same denomination before multiplying and dividing.

2. Cancel all like factors from the given extreme and either of the means.

See MANUAL.

PROBLEMS.

11. If 53 bu. of lime cost $13.25, how much will 29 bu. cost ?

12. If my store rent is $315 for 7 mo., how much is it for 2 yr. 5 mo. ? *$1,305.*

13. If 12¾ lb. of beef cost $1.78½, how much will 5½ lb. cost. ?

14. How much is a cental of wheat worth, at $1.40 per bushel ?

15. How much will 9 rm. 2 quires of note paper cost, if 3 rm. 14 quires cost $4.62½ ? *$11.37½.*

16. The driving-wheel of a locomotive makes 401 revolutions in running 1 mi. 20 rd. 4 ft. How many revolutions will it make in running 23 mi. 16 rd. 12 ft.? *8,694.*

17. If 31 trees bear 271 bu. 1 pk. of apples, how many apples will 56 trees bear, at the same rate? *490 bu.*

18. After cleansing 475 pounds of wool for manufacture, it weighed only 266 pounds. At the same rate, how much would 645 pounds weigh, after cleansing?

19. If a Mississippi River steam-boat runs 779 mi. in 3 da. 10 h., how far will she run in 4 da. 3 h.? *940½ mi.*

20. If 13⅝ cd. of wood cost $81.75, how much will 21.25 cd. cost?

21. When 8.7 tons of hay cost $73.95, how much must be paid for 9.75 tons? *$82.87½.*

22. If a cheese weighing 53 lb. 3 oz. costs $6.28, how much will a cheese cost that weighs 44½ lb.?

23. The interest on a certain sum of money for 1 yr. 5 mo. 22 da. is $46.55. What is the interest on the same sum for 11 mo. 12 da.? *$29.92½.*

24. If a pole 6 ft. high casts a shadow 7½ ft. long, how high is a tree whose shadow is 85 ft. long? *68 ft.*

SECTION III.

COMPOUND PROPORTION.

536. *Compound Proportion* is an equality of a compound and a simple ratio, or of two compound ratios; as,

$$\left.\begin{array}{r}5:9\\12:4\end{array}\right\}::10:6,\text{ and }\left.\begin{array}{r}5:9\\12:4\end{array}\right\}::\left\{\begin{array}{l}5:2\\2:3\end{array}\right.$$

537. Since in a simple proportion the product of the extremes equals the product of the means (see **532**, I.); and since a compound ratio is reduced to a simple one, by multiplying all the antecedents together for a new antecedent, and all the consequents together for a new consequent (see **525**, V.), it follows that

In a compound proportion the product of the extremes equals the product of the means.

538. Ex. If 4 men in 8 days cut 48 cords of wood, how many cords will 5 men cut in 6 days?

SOLUTION.

$$\left.\begin{array}{l}4\text{ men} : 5\text{ men}\\ 8\text{ days} : 6\text{ days}\end{array}\right\} :: 48\text{ cd.} : \text{— cd.}$$

$$\frac{5\times\overset{3}{\cancel{6}}\times\overset{\overset{3}{\cancel{6}}}{\cancel{48}}\text{ cd.}}{\underset{\cancel{2}}{\cancel{4}}\times\cancel{8}}=45\text{ cd.}$$

EXPLANATION. — Since the unknown term is cords, we write the 48 cords for the first term of the second ratio, or the third term of the proportion, as in **535**. Since the unknown term depends upon both the number of men and the number of days, we arrange the other given numbers—4 men and 5 men, 8 days and 6 days—as the terms of a compound ratio, observing the same directions in writing the terms of each couplet, as in writing the first ratio of a simple proportion. Thus, 4 men cut 48 cords, 5 men will cut more; hence, 4 men : 5 men; and in 8 days they cut 48 cords, in 6 days they will cut less; hence, 8 days : 6 days. Having made the statement, we multiply the second and third terms together, and divide the product by the first term, as in simple proportion. The quotient, 45 cords, is the fourth or unknown term. Hence,

539. *Rule for Computations in Compound Proportion.*

I. *For the third term, write the number that is of the same kind as the unknown term.*

II. *Write each two of the other numbers that are of the same kind, as a couplet of the compound ratio, observing the same directions as in writing the terms of the first ratio in a simple proportion.*

III. *For the fourth or unknown term, multiply the second and third terms together, and divide the product by the first term.*

See Manual.

PROBLEMS.

Find the unknown term in each of the following four proportions:

1. $\left.\begin{matrix} 7 : 21 \\ 4 : 8 \end{matrix}\right\} : : 3 : —.$ 2. $\left.\begin{matrix} 2.5 \text{ yd.} & : — \text{ yd.} \\ .25 \text{ yd. wide} & : \frac{1}{2} \text{ yd. wide} \end{matrix}\right\} : : \$7\frac{1}{2} : \$45.$

3. $\left.\begin{matrix} 25 : & 5 \\ 9 : & 135 \end{matrix}\right\} : : — : 7\frac{1}{2}.$ 4. $\left.\begin{matrix} — & : 14 \\ 3.2 & : 1\frac{3}{5} \\ 4.5 & : .6 \end{matrix}\right\} : : 13\frac{1}{3} : 5\frac{1}{4}.$

5. If the carpet for a room 15 ft. × 16 ft. costs $40, how much will a carpet of the same kind cost for a room 14 ft. × 18 ft.? *$42.*

6. A flouring mill running 10 hours per day, makes 1,365 barrels of flour in 13 days. How many barrels will the same mill make in 39 days, running 16 hours per day? 6552

7. I sold a village lot $5\frac{1}{2}$ rods front by $7\frac{1}{4}$ rods deep, for $580; and another lot $4\frac{2}{3}$ rods front by $9\frac{1}{6}$ rods deep, at the same rate. How much did I receive for the second lot? *$622.22.*

8. A pile of 4-foot wood 2,450 ft. long and 6 ft. high, was drawn to a R.R. station by 15 teams in 14 days. At the same rate, how many days will it take 24 teams to draw a pile 2,016 ft. long and 5 ft. high? *6.*

9. If 5,280 rails will build 360 rods of 8-rail fence, how many rails will be required to build 276 rods of 10-rail fence?

SECTION IV.

PARTNERSHIP.

540. A *Partnership*, or a *Company*, is an association of two or more persons for the transaction of business as an individual.

541. A *Firm* is the name under which a company transacts business.

NOTES.—1. Each member of a company or firm is a *Partner*.

2. A firm is sometimes called a *House*; as the House of Jay Cooke & Co.

542. *Profits* are the gains to be shared among the members of a firm; and

543. *Assessments* are sums to be paid by members of a firm, to meet expenses or cover losses.

544. *Capital* is the time, or the money, or both, invested in business.

545. A *Simple Partnership* is one in which each partner's share in one of the elements of capital—time or money—is the same; and

546. A *Compound Partnership* is one in which the partners' shares in one or both elements of capital vary.

CASE I.

Simple Partnership.

547. If one partner furnishes 3 times as much money as another, or if he furnishes the same amount of money, and it remains in business 3 times as long, his share of the profits and losses is 3 times as great as that of the other partner. Hence,

I. *When the time is equal, the profits and losses are shared by the partners in proportion to their respective shares of the money in business.*

II. *When the shares of money are equal, the profits and losses are shared by the partners in proportion to the respective times their money is in business.*

548. Ex. 1. A, B, and C enter into partnership, A furnishing $3,500, B $2,500, and C $2,000 of the capital. Their profits are $3,200. What is each man's share?

SOLUTION.

$3,500 + $2,500 + $2,000 = $8,000

$8,000 : $3,500 : : $3,200 : A's share.
$8,000 : $2,500 : : $3,200 : B's "
$8,000 : $2,000 : : $3,200 : C's "

Hence, *A's share is $1,400; B's share, $1,000; and C's share, $800.*

EXPLANATION. — Since the partners furnish different amounts of money for the same length of time, they will share in the profits in proportion to their respective shares of the money in business (**547**, I.). Adding their shares of money furnished, we have $8,000, the entire capital. Then,

Whole capital : each man's capital : : whole gain : each man's share of gain.

Finding the unknown term in each of the three proportions, we have each man's share of the gain.

Ex. 2. D and E formed a partnership for 1 year, each furnishing the same capital. At the end of 8 months, D drew out his capital, his interest continuing to the close of the year, when the profits were $4,500. What was each one's share?

SOLUTION.

8 mo. + 12 mo. = 20 mo.

20 mo. : 8 mo. : : $4,500 : D's share.
20 mo. : 12 mo. : : $4,500 : E's "

Hence, *D's share, $1,800 ;*
E's share, $2,700.

EXPLANATION.—Since the partners furnish the same amount of money, for different lengths of time, they will share in the profits in proportion to the respective times their money is in business (**547**, II.). Adding their shares of time, we have 8 mo. (D's share of time) + 12 mo. (E's share of time) = 20 mo., the sum of their shares of time. Then,

Whole time : each man's time : : whole gain : each man's share of gain.

PROBLEMS.

1. A and B form a partnership. A furnishes $2,500 of the capital, and B $4,000, and they gain $1,950. What is each man's share? *A's, $750; B's, $1,200.*

2. Three men, having 1 cow each, hire a pasture for the season, for $22.50. A's cow is in the pasture 4 months, B's cow 6 months, and C's cow 5 months. How much does each man pay? 6, 9, 7.50,

3. A factory, insured in the Ætna Ins. Co. for $5,000, in the Home Ins. Co. for $7,500, in the Continental Ins. Co. for $6,000, and in the North American Ins. Co. for $4,500, was damaged by fire to the amount of $10,120. How much of the loss fell upon each Co.?
Ætna, $2,200; Home, $3,300;
Continental, $2,640; N. A., $1,980.

4. A, B, and C bought some city lots in company, A furnishing $1,800 of the purchase money, B $1,500, and C $1,200. They sold the lots for $8,420. How much did each man gain?

CASE II.

Compound Partnership.

549. If one partner furnishes 3 times as much capital as another, either in money or in the time his money remains in the business, or in the product of money and time, his share of the profits and losses is 3 times as great as that of the other partner. Hence,

The product of each man's capital multiplied by the time it is in business, represents his proportionate share of the capital.

550. Ex. A, B, and C form a partnership for 8 months, A putting in $2,000, B $3,000, and C $1,500. If C draws out his capital in 4 months, and B his in 6 months, and the profits are $5,600, what is each man's share?

SOLUTION.

$2,000 for 8 mo. = $16,000 for 1 mo.
3,000 " 6 " = 18,000 " 1 "
1,500 " 4 " = 6,000 " 1 "
$40,000

$40,000 : $16,000 :: $5,600 : A's share.
$40,000 : $18,000 :: $5,600 : B's "
$40,000 : $6,000 :: $5,600 : C's "

Hence, *A's share is $2,240;*
B's, $2,520; C's, $840.

Explanation.—Since the times and shares of money are both unequal, each man's share of the profits depends both upon the amount of money furnished by him and the time it is used in the business. (See **549.**) A's money, $2,000, for 8 months, is the same as 8 times $2,000, or $16,000, for 1 month; B's money, $3,000, for 6 months, is the same as $18,000 for 1 month; and C's money, $1,500, for 4 months, is the same as $6,000 for 1 month. The whole proportionate capital is $40,000, of which A's share is $16,000; B's, $18,000; and C's, $6,000. Then,

Proportionate Capital : each man's share : : whole gain : each man's gain.

551. Upon the principles deduced in **547**, **549**, are based the

Rules for Computations in Partnership.

I. For Simple Partnership.

1. *For the whole capital, add all the partners' shares of money or time.*

2. *For each partner's share of the gain or loss, state a proportion, thus:—Whole capital : each partner's capital : : whole gain or loss : each partner's gain or loss.*

II. For Compound Partnership.

1. *For each partner's proportionate share of the capital, multiply his money by the time it is in business.*

2. *For the whole capital, add all the proportionate shares.*

3. *For each partner's share of the gain or loss, state a proportion, thus:—Whole proportionate capital : each partner's proportionate share : : whole gain or loss : each partner's gain or loss.*

PROBLEMS.

5. A and B engage in trade together, A putting in $1,500 for 9 months, and B $2,500 for 6 months. They gain $2,394. What is each one's share? *A's, $1,134; B's, $1,260.*

6. The firm of Sanford, Wright, & Thomas manufacture agricultural implements, and their capital is $12,000, of which Mr. S. furnished $5,500, Mr. W. $4,500, and Mr. T. $2,000. Last year they made $9,360. What were each partner's profits? 3510 1560

7. Three men harvested and thrashed a field of grain on shares, A furnishing 4 hands 5 days, B 6 hands 4 days, and C 5 hands 8 days. The whole crop was 630 bushels, of which they had $\frac{1}{5}$. How much did each receive? *A, 30 bu.; B, 36 bu.; C, 60 bu.*

8. A, B, and C are the partners in a store. A furnishes $2,300 for 1 yr., B $1,750 for 10 mo., and C $1,450 for 1 yr. 3 mo., and they *lose* $472.50. What is each man's loss?

SECTION V.

REVIEW PROBLEMS IN PROPORTION.

1. The property of an insolvent debtor amounts to $6,342, and his liabilities to $17,550. How much will a creditor receive on a debt of $1,250?

2. If .1875 bu. of sweet-potatoes cost $.30, what will be the cost of .875 bu.? *$1.40.*

3. A man whose estate is worth $19,250, directs, by his will, that his property shall be so divided among his four children that his daughter shall receive $4, his youngest son $5, and his second son $6, as often as his eldest son receives $7. How much will each child receive? *Daughter, $3,500; youngest son, $4,375; second son, $5,250; eldest son, $6,125.*

4. If 400 ft. of flooring 1¼ in. thick are required for a room, how much flooring 1½ in. thick will be required for a room 3 times as long and 2 times as wide? *2,880 ft.*

5. Four men paid $13 for a carriage to convey them from a R.R. station to their homes, which were distant 16 mi., 24 mi., 28 mi., and 36 mi. respectively. They paid in proportion to the distances they rode. How much did each man pay?

6. The interest of $286.25 for a certain time is $27.48. What is the interest of $59.50 for the same time?

7. A bin 12 ft. × 7 ft. × 6 ft. will hold 405 bushels of grain. How many bushels will a bin hold that is 8 ft. × 7 ft. × 4 ft.? *180.*

8. If 17¾ cords of wood will produce 745½ bushels of charcoal, how many cords of wood will be required to produce 1,677⅜ bushels?

9. A and B bought a mill, A paying $2,400, and B $3,200. 8 months afterward C bought ⅖ of A's share, and D bought ⅝ of B's share. The profits of the mill for the year were $5,040. How much was each man's share? *A, $1,872; B, $2,280; C, $288; D, $600.*

10. Three men shipped a cargo of 1,500 barrels of flour to England, A furnishing 700 barrels, B 200 barrels, and C the balance. In a storm 195 barrels were thrown overboard. How should the loss be shared among the owners?

CHAPTER 9

EVOLUTION.

SECTION I.

DEFINITIONS AND NOTATION.

552. A *Root* of a number is one of its equal factors.

553. The *Square Root* of a number is one of its two equal factors.

554. The *Cube Root* of a number is one of its three equal factors.

EXAMPLE.—3 is a *root* of 9, of 27, and of 81, because $9 = 3 \times 3$ or 3^2, $27 = 3 \times 3 \times 3$ or 3^3, and $81 = 3 \times 3 \times 3 \times 3$ or 3^4. 3 is the *square root* of 9, and the *cube root* of 27.

NOTES.—1. One of the four equal factors of a number is its *Fourth Root*, one of the five equal factors is its *Fifth Root*, and so on.

2. A number whose square root can be obtained, is a *Perfect Square;* and one whose cube root can be obtained, is a *Perfect Cube.*

555. *Involution* is the process of finding any required power of a number. (See 82–87).

556. *Evolution, or Extraction of Roots,* is the process of finding any required root of a number.

NOTE.—Involution and evolution are converse operations.

557. *Extraction of Square Root* is the process of finding one of the two equal factors of a number; and

558. *Extraction of Cube Root* is the process of finding one of the three equal factors of a number.

559. The *Sign of Square Root* is $\sqrt{}$, called the *Radical Sign;* and

560. The *Sign of Cube Root* is $\sqrt[3]{}$. Thus, $\sqrt{64}$ is read "The square root of 64;" and $\sqrt[3]{125}$ is read "The cube root of 125." See Manual.

NOTE.—A *Surd* is an indicated root which can not be obtained; as $\sqrt{5}$, $\sqrt[3]{7}$.

SECTION II.

EXTRACTION OF SQUARE ROOT.

561. The least and the greatest number that can be expressed by one figure are 1 and 9; by two figures, 10 and 99; by three figures, 100 and 999; and so on.

The squares of these numbers are

$1^2= 1$ $10^2= 100$ $100^2= 10{,}000$
$9^2=81$ $99^2=9{,}801$ $999^2=998{,}001$ and so on.

By examining these numbers and their squares, we see that

The square of a number expressed by	consists of
One figure	one or two figures;
Two figures	three " four "
Three "	five " six "
Four "	seven " eight "

and so on. That is,

I. *The square of any number consists of twice as many figures as the number, or one less.*

II. *If a number be separated into periods of two figures each, beginning with ones, its square root will consist of as many figures as there are full and partial periods in the number.*

562. If we write any digits, as 2 and 9, successively as ones, tens, hundreds, and so on, and square them, we shall have

$2^2= 4$ $20^2= 400$ $200^2= 40{,}000$
$9^2=81$ $90^2=8{,}100$ $900^2=810{,}000$

By examining these numbers, we see that

The square of the ones	is in the first period;
" " " tens	" second "
" " " hundreds	" third "

and so on. That is,

The square of the left-hand figure of a root is wholly in the left-hand period of the number or power.

563. If we square any numbers expressed by two figures, as 20 and 25, 60 and 63, 90 and 99, we shall have

$$\begin{array}{lll} 20^2=400 & 60^2=3{,}600 & 90^2=8{,}100 \\ 25^2=625 & 63^2=3{,}969 & 99^2=9{,}801 \end{array}$$

By comparing these roots and their squares, we see that

4 is the greatest square in 6, the hundreds of 625;
36 " " " 39, " " 3,969;
81 " " " 98, " " 9,801. That is,

The greatest square in the left-hand period of a number is the square of the left-hand figure of the root.

564. We will now square the number 37, for the purpose of learning of what parts the square is composed.

Ex. 37=30+7, and 37^2=30+7 multiplied by 30 + 7.

EXPLANATION. — The square of the ones=49; the product of the tens by the ones (7 × 30) + the product of the ones by the tens (30 × 7), or two times the product of the tens and ones=420; the square of the tens = 900; and the sum of these three partial products= 1,369. Hence,

SOLUTION.

$$\begin{array}{rcr} 30+\ 7 & = & 37 \\ 30+\ 7 & = & \underline{37} \\ \overline{210+49} & = & 259 \\ 900+210 & & \underline{111} \\ \overline{900+420+49} & = & 1369 \end{array}$$

The square of a number consisting of two figures, is equal to the square of the tens, plus two times the product of the tens and the ones, plus the square of the ones.

565. Ex. 1. What is the square root of 1,369?

EXPLANATION. — Separating the number into periods of two figures each, we find that the square root will consist of two figures. (**561**, II.)

FIRST SOLUTION.

$$\begin{array}{rr|rl} & 13\cdot69 & \underline{37} & \textit{Root.} \\ & \underline{9} & 60 & \textit{Trial divisor.} \\ \textit{Dividend.} & 469 & \underline{7} & \\ & \underline{469} & \underline{67} & \textit{Complete divisor.} \end{array}$$

SECOND SOLUTION.

13·69	37
9	
469	67
469	

Since 9 is the greatest square in 13, the first period, we write 3, its square root, for the first figure of the root (**563**). Taking 9, the greatest square, from the left-hand period, and annexing 69, the next period, to the remainder, we have 469. This number is made up of two times the product of the tens and ones of the root, plus the square of the ones (**564**); *i. e.*, of 30 (=3 tens) × 2 × the ones of the root, + the square of the ones. Dividing 469 by the trial divisor, 60 (=2 times 3 tens, or 30), we obtain 7, which we write for the second figure, or ones, of the root. Since 60=2 times 3 tens, and 469=2 times 3 tens × the ones + the square of the ones, we add 7 to the trial divisor, 60, making 67. Then multiplying 67, the complete divisor, by 7, the last figure of the root, we obtain, *1st*, 7 times 7 = the square of the ones; and *2d*, 7 times 60 = 2 times 3 tens × 7 ones = 2 times the product of the tens and the ones. The product, 469, is the same as the dividend, and 37 is the square root required.

In the Second Solution we have placed the quotient figure, 7, in the place of the 0 in the trial divisor, thus completing the divisor at once.

SOLUTION.

	55·50·25	745	*Root.*
	49		
1st dividend.	650	144	*1st divisor.*
	576		
2d dividend.	7425	1485	*2d divisor.*
	7425		

Ex. 2. What number is the square root of 555,025?

EXPLANATION. — In extracting the square root of a number, only two periods of figures are considered at once. Therefore, in obtaining any figure of the root, after the first, we regard the figure or figures of the root already found as tens, and the figure sought as ones, and find each succeeding figure in the same manner as we find the second figure of a root consisting of two figures, as will be seen in the Solution.

Ex. 3. Find the square root of 748.0225.

SOLUTION.

$$\begin{array}{r|l} 7\cdot48.02\cdot25 & 27.35 \\ 4 & \\ \hline 348 & 47 \\ 329 & \\ \hline 1902 & 543 \\ 1629 & \\ \hline 27325 & 5465 \\ 27325 & \\ \hline \end{array}$$

EXPLANATION.—Separating the number into integral and decimal periods, by counting left and right from ones, we proceed as in Ex. 1 and 2, putting a decimal point before the figure of the root obtained from using the first decimal period.

Ex. 4. Extract the square root of $\frac{49}{576}$.

EXPLANATION.—Since a fraction is squared by squaring each term separately (**335**), and since evolution is the converse of involution (**556**, Note), we extract the square root of each term separately.

SOLUTION.

$$\sqrt{\tfrac{49}{576}} = \sqrt{49} \div \sqrt{576} = \tfrac{7}{24}$$

566. Upon the principles deduced in **561–564**, is based the

Rule for Extraction of Square Root.

I. To determine the number of figures in the root.

Separate the number into periods of two figures each, counting left and right from ones.

II. For the first figure of the root.

1. *Find the root of the greatest square in the left-hand period, for the first figure of the root.*

2. *Subtract this square from the first period; and to the remainder annex the next period, for the first dividend.*

III. For the second figure of the root.

1. *Double the root already found, considered as tens, for the first trial divisor, by which divide the first dividend; and write the result for the second figure of the root, and also in the place of ones in the trial divisor, thus forming the complete divisor.*

2. *Multiply the complete divisor by the second figure of the*

root; subtract the product from the first dividend; and to the remainder annex the next period for a new dividend.

IV. For the succeeding figures of the root.

Proceed with the second, and with each succeeding dividend, in the same manner as with the first, until all the periods are used.

NOTES.—1. If any dividend is less than the divisor, annex a cipher to the root, and also to the divisor, and annex the next period to the dividend, for a new dividend.

2. If there is a remainder after all the periods have been used, *i. e.*, in extracting the square root of a surd, periods of decimal ciphers may be annexed, and the work extended to any required degree of exactness.

3. If the right-hand decimal period contains but one figure, annex a decimal cipher.

4. To extract the square root of a mixed fractional number, first reduce it to a mixed decimal number, or to an improper fraction.

PROBLEMS.

1. Extract the square root of 5,476. *74.*

2. Find the value of $\sqrt{75.69}$. *8.7.*

3. What is the square root of .0289?

4. A square plat of ground contains 87,616 square feet. What is the length of one side? *296 ft.*

5. $\sqrt{881,721}$ = what number?

6. Extract the square root of .455625.

7. What is the square root of 50,808,384? *7,128.*

8. The area of a square platform is $1,387\frac{9}{16}$ sq. ft. What is the length of one side? *37.25 ft.*

9. Find the value of $\sqrt{.000169}$. *.013.*

10. What is the square root of the fraction $\frac{361}{576}$?

11. $\sqrt{\frac{324}{841}}$ = what number? $\frac{18}{29}$.

12. The entire area of the six faces of a cubic block is $130\frac{2}{3}$ sq. in. What is one dimension of the block? $4\frac{2}{3}$ *in.*

13. Find the square root of $91\frac{121}{144}$. $9\frac{7}{12}$.

14. What is the value of $\sqrt{15}$? *3.872+.*

15. Extract the square root of 99. *9.9498+.*

16. $\sqrt{1127.750724}$ = what number? *33.582.*

SECTION III.

EXTRACTION OF CUBE ROOT.

567. If we cube an integral unit of each of the first four orders, we have

$1^3=1$ $10^3=1{,}000$ $100^3=1{,}000{,}000$ $1{,}000^3=1{,}000{,}000{,}000$

Since the cube of 1 is 1, and the cube of 10 is 1,000, the cube of any number between 1 and 10 must be a number between 1 and 1,000;

Since the cube of 10 is 1,000, and the cube of 100 is 1,000,000, the cube of any number between 10 and 100 must be a number between 1,000, and 1,000,000;

Since the cube of 100 is 1,000,000, and the cube of 1,000 is 1,000,000,000, the cube of any number between 100 and 1,000 must be a number between 1,000,000 and 1,000,000,000; and so on. That is,

The cube of a number expressed by	consists of
One figure	one, two, or three figures;
Two figures	four, five, or six "
Three "	seven, eight or nine "

and so on. Hence

I. *The cube of any number consists of three times as many figures as the number, or one or two less.*

II. *If a number be separated into periods of three figures each, beginning with ones, its cube root will consist of as many figures as there are full and partial periods in the number.*

568. If we write any digits, as 2 and 9, successively as ones, tens, hundreds, and so on, and cube them, we have

$2^3=8$ $20^3=8{,}000$ $200^3=8{,}000{,}000$

$9^3=729$ $90^3=729{,}000$ $900^3=729{,}000{,}000$

Examining these numbers, we see that

The cube of the ones	is in the first	period;
" " tens	" second	"
" " hundreds	" third	"

and so on. Hence,

The cube of the left-hand figure of a root is wholly in the left-hand period of the power.

569. If we cube any numbers expressed by two figures, as 20 and 25, 60 and 63, 90 and 99, we shall have

$20^3 = 8,000$ $60^3 = 216,000$ $90^3 = 729,000$
$25^3 = 15,625$ $63^3 = 313,047$ $99^3 = 970,299$

Comparing these roots and their cubes, we see that

8 is the greatest cube in 15, the thousands of 15,625;
216 " " 313, " " " 313,047;
729 " " 970, " " " 970,299. Hence,

The greatest cube in the left-hand period of a number is the cube of the left-hand figure of the root.

570. We will now cube the number 45, for the purpose of seeing of what parts the cube is composed. $45 = 40 + 5$, and $45^3 = 40 + 5$ multiplied by $40 + 5$ multiplied by $40 + 5$.

FIRST SOLUTION. SECOND SOLUTION.

$$\begin{array}{rcr}
40 + 5 & = & 45 \\
40 + 5 & = & 45 \\
\hline
(40 \times 5) + 5^2 & = & 225 \\
40^2 + (40 \times 5) & = & 180 \\
\hline
40^2 + 2 \times (40 \times 5) + 5^2 & = & 2025 \\
40 + 5 & = & 45 \\
\hline
(40^2 \times 5) + 2 \times (40 \times 5^2) + 5^3 & = & 10125 \\
40^3 + 2 \times (40^2 \times 5) + (40 \times 5^2) & = & 8100 \\
\hline
40^3 + 3 \times (40^2 \times 5) + 3 \times (40 \times 5^2) + 5^3 & = & 91125
\end{array}$$

The several parts of the final product, reading from the left, are

1st. The cube of the tens,	*64,000*
2d. Three times the square of the tens × the ones,	*24,000*
3d. Three times the tens × the square of the ones,	*3,000*
4th. The cube of the ones,	*125*
That is, 45^3 =	*91,125*

The cube of a number consisting of two figures, is equal to the cube of the tens, plus three times the square of the tens multiplied *by the ones, plus three times the tens multiplied by the square of the ones, plus the cube of the ones.*

571. Ex. 1. What is the cube root of 91,125?

SOLUTION.

	91·125	*45*	*Root.*
	64	*4800*	*Trial divisor.*
Dividend.	*27125*	*600*	
	27125	*25*	
		5425	*Complete divisor.*

EXPLANATION.— Separating the number into periods of three figures each, we find that the cube root will consist of two figures (**567**, II.).

Since 64 is the greatest cube in the left-hand period, 91, we write 4, its cube root, for the first figure of the root (**569**).

Taking 64, the greatest cube, from the left-hand period, and annexing 125, the next period, to the remainder, we have 27,125. This number is made up of 3 times the square of the tens × the ones, plus 3 times the tens × the square of the ones, plus the cube of the ones (**570**); *i. e.*, of $3 \times 40^2 \times$ the ones $+ 3 \times 40 \times$ the square of the ones + the cube of the ones.

Calling the first figure of the root tens, and multiplying its square by 3, we have 4,800 for a trial divisor. Dividing the dividend, 27,125, by the trial divisor, we obtain 5 for the second figure, or ones, of the root.

Since 4,800 = 3 times the square of 4 tens, and 27,125 = 3 times the square of 4 tens × the ones, plus 3 times 4 tens × the square of the ones, plus the cube of the ones, we add to 4,800, the trial divisor, 600 (= 3 × 4 tens or 40 × the ones), and also 25, the square of the ones, making 5,425, the complete divisor. Then, multiplying this complete divisor by 5, the second figure of the root, we obtain 27,125, which is made up of, 1*st*, 5 × 5 × 5, or the cube of the ones; 2*d*, 3 × 40 × 5 × 5, or 3 times the tens × the square of the ones; and 3*d*, 3 × 40 × 40 × 5, or 3 times the square of the tens × the ones. We have now used all of the given number, and 45 is the cube root required.

Ex. 2. Extract the cube root of 9,663,597.

EXPLANATION. — Since only two periods of figures are considered at once, in obtaining any figure of the root, after the first, we regard the figure or figures of the root already found as tens, and the figure sought as ones; and proceed in the same manner as in obtaining the second figure.

SOLUTION.

9·663·597	213
8	
1663	1200 + 60 + 1
1261	1261
402597	132300 + 1890 + 9
402597	134199

572. Upon the principles in **567–570** is based the

Rule for Extraction of Cube Root.

I. To determine the number of figures in the root.

Separate the number into periods of three figures each, counting left and right from ones.

II. For the first figure of the root.

1. *Find the root of the greatest cube in the left-hand period.*

2. *Subtract its cube from the period, and to the remainder annex the next period, for a dividend.*

III. For the second figure of the root.

1. *Considering the root already found as tens, multiply its square by 3, for a trial divisor, by which divide the dividend, and write the result for the second figure of the root.*

2. *Add to the trial divisor 3 times the product of the tens and ones of the root already found, and also the square of the ones, for a complete divisor.*

3. *Multiply the complete divisor by the last figure of the root; subtract the product from the dividend; and to the remainder annex the next period for a new dividend.*

IV. For each succeeding figure of the root.

Consider that part of the root already found as tens, and proceed in the same manner as in finding the second figure.

NOTES.—1. If any dividend is less than the divisor, annex a cipher to the root; two ciphers to the trial divisor, for a new divisor; and the next period to the dividend, for a new dividend.

2. In extracting the cube root of a surd, periods of decimal ciphers may be annexed, and the work extended to any required degree of exactness.

3. If a right-hand decimal period contains less than three figures, supply the deficiency by annexing a decimal cipher or ciphers.

4. If the given number is a fraction, take the cube root of the numerator and denominator separately; and if it is a mixed fractional number, first reduce it to an improper fraction, or to a mixed decimal number.

5. Since the trial divisor is less than the true divisor, in obtaining the root figure we must make allowance for this difference. See Manual.

PROBLEMS.

1. What is the cube root of 103,823; and of 24,389? *47; 29.*

2. $\sqrt[3]{274.625}$ = what number? *6.5.*

3. .000729 is the cube of what number?

4. What is the length of one side of a cubical block that contains 2 cu. ft. 1,457 cu. in.? *1 ft. 5 in.*

5. Find one of the three equal factors of 10,218,313. *217.*

6. $\sqrt[3]{131,096,512}$ = what number?

7. The length of a square stick of timber, which contains $13\frac{1}{2}$ cubic feet, is 32 times its width or thickness. What are its dimensions?

NOTE 6.—If the stick were cut, crosswise, into 32 equal parts, each part would be a cube. *24 ft. long, and 9 in. square.*

8. In digging a cellar, the length of which was 4 times, and the width 6 times its depth, 192 cubic yards of earth were removed. What were the dimensions of the cellar? *6, 24, and 36 ft.*

9. Extract the cube root of 187,149.248.

10. Of what number is 118,805,247,296 the cube? *4,916.*

11. In a granary is a bin that holds 270 bushels. Its length is 3 times, and its width $1\frac{3}{4}$ times its depth. What are its dimensions?

12. Extract the cube root of $\frac{8}{343}$, and $\frac{125}{2197}$.

13. $\sqrt[3]{16\frac{656}{1331}}$ and $\sqrt[3]{4\frac{12}{125}}$ = what numbers? *$2\frac{6}{11}$, and 1.6.*

14. What must be the interior measurement of a side of each of two boxes, one of which will hold a bushel of grain, and the other a gallon of oil? *12.907+ in.*, and *6.135+ in.*

CHAPTER 10.

PROGRESSIONS.

SECTION I.

DEFINITIONS.

573. A *Series* is a succession of numbers increasing or decreasing, either by a common difference or by a common ratio; as 3, 7, 11, 15; and 2, 6, 18, 54.

NOTE.—The numbers that form a Series are the *Terms.* The first and last terms are the *Extremes;* and the other terms are the *Means.* (See **529, 530.**)

574. An *Ascending Series* is one in which the terms increase in regular order, from the first.

575. A *Descending Series* is one in which the terms decrease in regular order, from the first.

576. An *Arithmetical Progression* is a series whose terms increase or decrease by a common difference; as 2, 7, 12, 17; and 24, 21, 18, 15.

577. A *Geometrical Progression* is a series whose terms increase or decrease by a common ratio; as 2, 10, 50, 250; and 48, 24, 12, 6.

SECTION II.

ARITHMETICAL PROGRESSION.

578. In the ascending arithmetical series 2, 5, 8, 11, 14, the common difference is 3, and the terms are formed as follows:

1st term, 2;
2d “ 5=2+3, or 1st term+common difference;
3d “ 8=2+3+3, “ “ +2 times com. diff.;
4th “ 11=2+3+3+3, “ “ +3 “ “
5th “ 14=2+3+3+3+3, “ “ +4 “ “

and the sum of the series is 2+5+8+11+14=40.

From this illustration we see, that in any arithmetical series there are five things to be considered: viz., the *First Term*, the *Last Term*, the *Common Difference*, the *Number of Terms*, and the *Sum of the Series*.

Dividing 40, the sum of the series 2, 5, 8, 11, 14, by 5, the number of terms, we have 8, which is the average of all the terms, or the *Average Term;* and adding 2 and 14, the extremes, we have 16, which is two times the average term.

579. From these illustrations we deduce the following

Principles of Arithmetical Progression.

I. *Any term in an ascending series is equal to the first term, plus the product of the common difference multiplied by the number of the term less 1.*

II. *The difference of the extremes is equal to the product of the common difference multiplied by the number of terms less 1.*

III. *The sum of the extremes is equal to two times the average term of the series.*

PROBLEMS.

1. The first term of an ascending arithmetical series is 6, the common difference is 3, and the number of terms is 57. What is the last term?

SOLUTION.

$57-1=56$, *times com. diff. is added.*
$56\times 3=168$, *sum of additions to 1st term.*
$6+168=174$, *last term.* (See I.)

Or,

$6+(56\times 3)=174$, *last term.*

2. The first term of a descending arithmetical series is 206, the common difference is 10, and the number of terms is 21. What is the last term?

SOLUTION.

$21-1=20$, *times com. diff. is subtracted.*
$20\times 10=200$, *sum of subtractions from 1st term.*
$206-200=6$, *last term.* (See II.)

Or,

$206-(20\times 10)=6$, *last term.*

3. The first term is 5, the last term is 117, and the number of terms is 15. What is the common difference?

SOLUTION.

117−5=112, sum of additions to 1st term.
15−1=14, number of additions.
112÷14=8, common difference. (See II.)

Or

$$\frac{117-5}{15-1} = 8, \textit{ common difference.}$$

4. The extremes are 7 and 95, and the common difference is 4. Find the number of terms in the series.

SOLUTION.

95−7=88, sum of additions to 1st term.
88÷4=22, number " "
22+1=23, number of terms.

Or

$$\frac{95-7}{4}+1=23, \textit{ number of terms.}$$

5. The extremes are 3 and 25, and the number of terms is 12. What is the sum of the series?

SOLUTION.

3+25=28, 2 times the average term. (See III.)
28÷2=14, the average term.
14×12=168, sum of the series.

Or

$$\frac{3+25}{2}\times 12=168, \textit{ sum of the series.}$$

580. Upon the principles and examples in **578, 579,** are based the

Rules for Computations in Arithmetical Progression.

I. To find either extreme.

Multiply the common difference by the number of terms less 1; and add the product to the less extreme, or subtract it from the greater.

II. To find the common difference.

Divide the difference of the extremes by the number of terms less 1.

III. To find the number of terms.

Divide the difference of the extremes by the common difference, and add 1 to the quotient.

IV. To find the sum of the series.

Multiply one half the sum of the extremes by the number of terms. See Manual.

PROBLEMS.

6. The less extreme of an arithmetical series is 5, the common difference is 7, and the number of terms is 13. What is the greater extreme? *89.*

7. Find the greater extreme of the progression of which 19 is the less extreme, 3 is the common difference, and 57 is the number of terms. 187

8. A boy 14 years old was apprenticed to a trade, and was to receive $50 the first year, and an increase of $75 yearly, till he was of age. How much did he receive the last year? *$500.*

9. The greater extreme of an arithmetical series is 215, the common difference is 13, and the number of terms is 15. What is the less extreme? *33.*

10. A man who owns a plot of 18 building lots, asks $1,000 for the one nearest the city, and $20 less for each succeeding lot. What is his price for the lot farthest from the city? 1340

11. The extremes of a series of 60 terms are 13 and 249. What is the common difference? *4.*

#3 12. If a laborer has $16 deposited in a savings-bank on Jan. 1, and $484 on Dec. 30 following, what are his average weekly deposits? 9

13. The extremes are $4\frac{3}{4}$ and $67\frac{3}{4}$, and the common difference is $1\frac{2}{5}$. What is the number of terms? *46.*

14. In how many years will the value of a piece of property be doubled, if it increases in value 16% the first year, and 7% each succeeding year?

15. What is the sum of the natural series of numbers 1, 2, 3, 4, and so on to 1,000 inclusive? *500,500.*

16. What is the number of strokes made by a clock in 12 hours? 74

17. The less extreme is $\frac{1}{2}$, the common difference is $\frac{1}{3}$, and the number of terms is 50. What is the greater extreme? $16\frac{5}{9}$

18. If you deposit $25 in a savings-bank the first week of the year, and $5 each succeeding week, how much will you deposit in the year? *$280.*

19. What is the 84th term of the series $90\frac{2}{3}$, 90, $89\frac{1}{3}$, etc.?

20. Insert 32 arithmetical means between the extremes 13 and 244. *1st mean, 20; 32d mean, 237.*

21. If the water in a lake is $16\frac{1}{2}$ feet deep 1 rod from a pier, and the bottom has a uniform slope of $3\frac{1}{2}$ feet to the rod, at what distance from the pier is the water 300 feet deep? *82 rods.*

22. Find the sum of 100 terms of the series $19\frac{5}{12}$, $19\frac{3}{4}$, $20\frac{1}{12}$, etc.

◆

SECTION III.

GEOMETRICAL PROGRESSION.

581. In the ascending geometrical series 2, 6, 18, 54, 162, the ratio is 3, and the terms are formed as follows:

1st	term,	2;			
2d	"	$6=2\times3$,	or 1st term	× ratio,	2×3^1;
3d	"	$18=2\times3\times3$,	" "	× square of ratio,	2×3^2;
4th	"	$54=2\times3\times3\times3$,	" "	× cube "	2×3^3;
5th	"	$162=2\times3\times3\times3\times3$,	" "	× 4th power, "	2×3^4;

and the sum of the series is $2+6+18+54+162=242$.

From this illustration we see that, in any geometrical series there are five things to be considered: viz., the *First Term*, the *Last Term*, the *Ratio*, the *Number of Terms*, and the *Sum of the Series*.

If we take the above series, 2, 6, 18, 54, 162, multiply it by the ratio, 3, placing the terms of the products over the corresponding numbers of the series, and then subtract the series from the product (or 3 times the series), the terms consisting of like numbers will disappear, and we shall have

3 times the series,		6	18	54	162	486
Series,	2	6	18	54	162	

(3 times—1 time=)2 times the series=486—2=484

Since 484 is 2 times the series, $484\div2=242$ is 1 time the series, or the sum of the series.

NOTE.—The pupil will notice that 2 is the first term of the series, 486 is 3 times the last term, and the divisor, 2, is the ratio less 1.

582. From these illustrations we deduce the following

Principles of Geometrical Progression.

I. *The first term and the ratio are the only factors used in forming a series.*

II. *In any term of a series, the first term is a factor once.*

III. *In any term of an ascending series, the ratio is a factor as many times as the number of terms less 1.*

IV. *The number of factors used in forming any term, is equal to the number of the term.*

V. *The product of the ratio and the greater extreme of a series, minus the less extreme, is as many times the sum of the series, as is expressed by the ratio less 1.*

PROBLEMS.

1. The first term of an ascending geometrical series is 4, the ratio is 2, and the number of terms is 7. What is the last term?

SOLUTION.

$7-1=6$, *times the ratio is a factor.* (See III.)
$2^6=64$, *product of ratio used as a factor.*
$4\times 64=256$, *last term.* (See I.)

Or

$4\times 2^{7-1}=4\times 2^6=256$, *last term.*

2. The first term of a descending geometrical series is 96, the ratio is 2, and the number of terms is 6. What is the last term?

SOLUTION.

$6-1=5$, *times the ratio is a divisor.* (See III.)
$2^5=32$, *product of ratio used as a divisor.*
$96\div 32=3$, *last term.*

Or

$96\div 2^{6-1}=96\div 2^5=3$, *last term.*

3. The first term of a geometrical progression is 7, the last term is 567, and the number of terms is 5. What is the ratio?

SOLUTION.

$567\div 7=81$, *product of ratio used as a factor.*
$5-1=4$, *times ratio is a factor.* (See III.)
$\sqrt[4]{81}=3$, *ratio.* **See Manual, Reference 310**

Or

$$\sqrt[5-1]{\frac{567}{7}}=\sqrt[4]{\frac{567}{7}}=3, \textit{ ratio.}$$

4. The extremes of a geometrical progression are 6 and 1,536, and the ratio is 4. What is the number of terms?

SOLUTION.

$1536 \div 6 = 256$, *prod. of ratio used as a factor.*

$256 \div 4 = 64$, $64 \div 4 = 16$, $16 \div 4 = 4$, $4 \div 4 = 1$.

4 (*the times the ratio is a factor*) $+ 1$ (*the time the less extreme is a factor*) $= 5$, *number of terms.* (See IV.)

5. The first term is 3, the last term is 375, and the ratio is 5. What is the sum of the series?

SOLUTION.

$375 \times 5 = 1875$, *5 times the last term.*

$1875 - 3 = 1872$, *4 times the series.* (See V.)

$1872 \div 4 = 468$, *sum of the series.*

Or

$\frac{375 \times 5 - 3}{5 - 1} = 468$, *sum of the series.*

583. Upon the principles and examples in **581**, **582**, are based the

Rules for Computations in Geometrical Progression.

I. To find the greater extreme.

Raise the ratio to a power 1 less than the number of terms, and multiply the result by the less extreme.

II. To find the less extreme.

Raise the ratio to a power 1 less than the number of terms, and divide the greater extreme by this result.

III. To find the ratio.

Divide the greater extreme by the less, and extract that root of the quotient whose index is 1 less than the number of terms.

IV. To find the number of terms.

Divide the greater extreme by the less, and this and each succeeding result by the ratio, till the quotient is 1.

The number of divisions will be the number of terms.

V. To find the sum of the series.

Multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio less 1.

See Manual.

PROBLEMS.

6. The first term of an ascending geometrical series is 7, and the ratio is 3. What is the 6th term? *1,701.*

7. The less extreme is 13, the ratio is 4, and the number of terms is 7. What is the greater extreme?

8. The 5th term of an ascending series is 5,625, and the ratio is 5. What is the first term? *9.*

9. The greater extreme is 845,824, the ratio is 2, and the number of terms is 12. What is the less extreme?

10. The extremes of a progression of 5 terms are 4 and 64. What is the ratio? *2.*

11. The four terms of a proportion are in geometrical progression, and the extremes are 8 and 2,744. What is the proportion?

NOTE 1.—First find the ratio. *8 : 56 : : 392 : 2744.*

12. The extremes of a series are 3 and 234,375, and the ratio is 5. How many terms are there in the series? *8.*

13. The extremes are 1 and $\frac{1}{729}$, and the ratio is $\frac{1}{3}$. Find the number of terms.

14. The extremes of a series are 2 and 4,374, and the ratio is 3. What is the sum of the series? *6,560.*

15. What is the sum of 9 terms of the series 2, 10, 50, etc.?

NOTE 2.—The greater extreme must first be found. *976,562.*

16. What debt can be discharged in a year, by paying 1 cent the first month, 3 cents the second, 9 cents the third, and so on, in that ratio, for the 12 months? *$2,657.20.*

17. If it were possible for a person having only 1 cent, to double his money every month for 4 years, how much money would he have? *$2,814,749,767,106.56.*

18. What is the sum of the series $18+6+2+\frac{2}{3}+\frac{2}{9}$, and so on, to infinity?

NOTE 3.—When the number of terms in a descending geometrical series is infinite, the series is called an *Infinite Series*, and the last term is 0. Hence, $\frac{(18\times3)-0}{3-1}$ = sum.

19. Find the sum of the infinite series $100+25+6\frac{1}{4}+1\frac{9}{16}$, etc. *$133\frac{1}{3}$.*

20. What is the sum of the series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, . . . to 0?

SECTION IV.

INTEREST BY PROGRESSIONS.

584. In *Simple Interest,* the amount of any sum is equal to the principal, plus the product of the interest for 1 year multiplied by the time in years (see **461**); the principal and the amount due at the close of the first year are both taken into account; and the amounts due at the end of the several years form an arithmetical series. Hence, in computations of

Simple Interest by Arithmetical Progression,

I. *The principal is the less extreme of an arithmetical series;*
II. *The interest for 1 year is the common difference;*
III. *The number of years plus 1 is the number of terms;* and
IV. *The amount is the greater extreme.*

585. In *Compound Interest,* the amount of any sum, for any year, is found by multiplying the amount due at the end of the preceding year by 1 plus the rate % (see **462**); and the amounts due at the end of the several successive years form a geometrical series. Hence, in computations of

Compound Interest by Geometrical Progression,

I. *The principal is the less extreme of a geometrical series;*
II. *1 plus the rate is the ratio;*
III. *The number of years plus 1 is the number of terms;* and
IV. *The amount is the greater extreme.*

PROBLEMS.

1. What is the amount of $500 for 8 years, at 7%? at 6%?

2. I have a lease of a building for 9 years, at $50 a year. If I allow 6% interest on the rents from the time they are due, and pay the whole amount at the expiration of the lease, how much must I then pay?

NOTE 1.—As there is no principal at the commencement of the first year, there are only 9 terms. *$558.*

3. If I save $150 each year, and put it at interest at 10%, how much will my savings amount to in 10 years? *$2,175.*

4. If a soldier's pension of $100 per annum remains unpaid for 9 years, how much will then be due him, allowing 6% simple interest?

NOTE 2.—Since the first year's pension has been due 9 years, and the last year's pension is now due, there are 10 terms in the series. *$1,270.*

5. What is the present worth of an annuity, or annual income, of $300 having 6 years to run, money being worth 8%?

NOTE 3.—The amount of the annuity due at the end of the 6 years, is the sum to be discounted, at 8%.

6. I hired a mill for 5 years, at an annual rent of $600, and paid the rent in advance, less the discount of 6%. How much did I pay?

7. After three years, I shall come into possession of property that pays $400 annually. How much ready money can I borrow, by hypothecating 5 years of this income, money being worth 10%?

NOTE 4.—The sum due in 5 years after the income commences, or 8 years from the present time, is the amount to be discounted for 8 years, at 10%.

8. What is the present worth of an annuity of $1000, 4 years in reversion (*i. e.* to commence in 4 years), and then having 7 years to run, allowing 8% for the use of money?

9. What is the present worth of a perpetual annuity of $1,500, to commence 7 years hence, if discounted at 6%?

NOTE 5.—The annuity is the interest of a principal that will earn $1,500, at 6%.

10. How much will $10,000 amount to in 5 years, at 6% compound interest? *$13,382.26.*

11. What is the compound interest of $425 for 5 years, at 7%?

12. If you deposit $500 in a savings-bank that pays 5% on deposits, compound interest payable quarterly, how much will your money amount to in 3 years?

13. What sum of money, at 6% compound interest, will amount to $89.54 in 4 years? *$70.92.*

14. The amount is $32,153.83, the time is 8 years, and the rate is 10% compound interest. What is the principal? *$15,000.*

15. Find the amount of an annuity of $185 for 4 years, at 7% compound interest. *$821.39.*

16. A clerk deposited $75 in a savings-bank every 6 months, upon which he received 6% interest compounded semi-annually. How much was standing to his credit at the end of 4 years?

17. How much is an annuity of $1,200 per annum worth in 10 years, at 5% compound interest? *$15,093.47.*

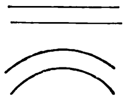
CHAPTER 11.
MENSURATION.

SECTION I.

DEFINITIONS.

586. *Mensuration* embraces the processes of measuring and computing the length of lines, the area of surfaces, and the capacity of solids and spaces. (See Chap. 2, Sec. VIII.)

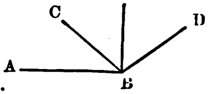
587. A *Curve Line* continually changes its direction, no part of it being a straight line.



588. *Parallel Lines* run in the same direction, at the same perpendicular distance apart.

589. An *Acute Angle* is less than a right angle; as *ABC*.

590. An *Obtuse Angle* is greater than a right angle; as *ABD*.



Note.—Acute and obtuse angles are *Oblique Angles*.

591. A *Plane Figure,* or a *Plane,* is a level surface bounded by lines.

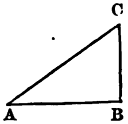
592. A *Polygon* is a plane bounded by straight lines.

593. A *Regular Polygon* has all its sides, and also all its angles equal.

594. A *Triangle* is a polygon of three sides.

595. A *Right-Angled Triangle* has one right angle; as *ABC*.

596. A *Hypothenuse* is the longest side of a right-angled triangle; as *AC*.

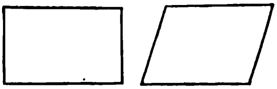


Note.—A triangle having three acute angles is an *Acute-Angled Triangle;* one having one obtuse angle is an *Obtuse-Angled Triangle ;* one having all its sides equal is an *Equilateral Triangle;* one having two sides equal is an *Isosceles Triangle;* and one having all its sides unequal is a *Scalene Triangle.*

See Manual.

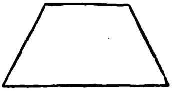
597. A *Quadrilateral* is a polygon of four sides.

598. A *Parallelogram* is a quadrilateral whose opposite sides are parallel, and consequently equal.



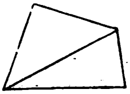
NOTE.—If all the sides of an oblique-angled parallelogram are equal, it is a *Rhombus*, or *Rhomb;* if only the opposite sides are equal, it is a *Rhomboid.*

599. A ***Trapezoid*** is a quadrilateral having only two sides parallel.



600. A ***Trapezium*** is a quadrilateral having neither two of its sides parallel.

601. A ***Diagonal*** is a straight line joining two opposite angles of a figure.



NOTES.—1. A regular polygon of five sides is a *Pentagon;* one of six sides is a *Hexagon;* one of seven sides is a *Heptagon;* one of eight sides is an *Octagon;* one of nine sides is a *Nonagon;* and one of ten sides is a *Decagon.*

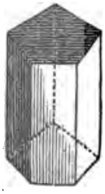
2. Any polygon of more than three sides may be divided, by diagonals all meeting at one angle, into as many triangles less 2 as the polygon has sides.

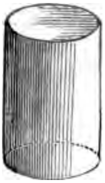
3. The total length of the sides of a polygon is its *Perimeter;* and the length of the circumference of a circle is its *Periphery.*

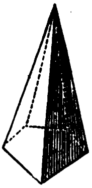
602. The ***Base*** of a figure is the side on which it is supposed to stand;

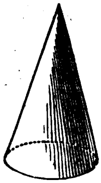
603. The ***Vertex*** is the point opposite, and furthest from the base; and

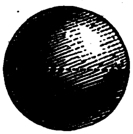
604. The ***Altitude*** is the perpendicular height of the vertex above the base.











605. A ***Prism*** is a solid whose bases or ends are equal, parallel polygons, and whose sides are parallelograms.

606. A ***Cylinder*** is a round solid of uniform diameter, whose bases or ends are equal, parallel circles.

607. A ***Pyramid*** is a solid whose base is a polygon, and whose sides are triangles, terminating in a point or vertex.

608. A ***Cone*** is a round solid whose base is a circle, whose top terminates in a point or vertex, and whose slant height is a straight line lying wholly in the curved surface.

609. A *Sphere,* or a *Globe,* is a solid bounded by one surface, which, in every part, is equally distant from a point within, called its center.

NOTE.—One half of a sphere is a *Hemisphere.*

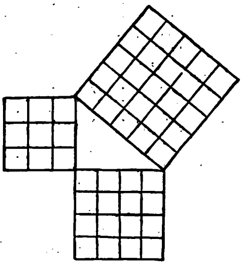
610. *Similar Surfaces* have their several angles equal each to each, and their sides about the equal angles proportional.

611. *Similar Solids* are contained by the same number of similar surfaces, similarly situated.

SECTION II.

MENSURATION OF LINES.

612. Some of the principles of mensuration can only be proved by a Geometrical analysis. Thus, this diagram illustrates the first of the following Geometrical Principles, but the illustration is not an analysis of the principle.



Geometrical Principles.

I. *The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

II. *The diameter of a circle : the circumference : : 113 : 355.*

NOTE.—By II. we find that, if the diameter of a circle is 1, the circumference is 3.14159, nearly. For ordinary purposes it is sufficiently accurate to call the circumference of a circle, 3$\frac{1}{7}$ times the diameter.

PROBLEMS.

1. Robert lives 117 rods north, and the school-house is 156 rods east, from the corners. What is the distance across the fields from Robert's house to the school-house? *195 rd.*

2. What is the length of a hand-rail to a flight of 16 stairs, each 12 inches wide and 9 inches high? *20 ft.*

3. My house is 24 ft. wide, the ridge is 9 ft. higher than the side walls, and the eaves project 1 ft. 6 in. beyond the sides of the house. How wide is each side of the roof? *16 ft. 6 in.*

4. A ladder 39 ft. long reaches to the top of a building, when its foot stands 15 ft. from the building. How high is the building?

5. From the top of a certain building, 36 feet high, to the opposite side of the street, is 164 feet. How wide is the street? *160 ft.*

6. Two streets, one 48 and the other 64 feet wide, cross at right angles. What is the distance between the diagonal corners?

7. What is the side of a square whose diagonal is 50 feet?

8. Round a cylinder 5 ft. 10 in. high and 1 ft. in circumference, a string is wound spirally from bottom to top, passing 14 times round. How long is the string?

9. The slant height of a cone is 21.8 inches, and the diameter of the base is 2.64 inches. How high is the cone? *21.76 in.*

10. A pole 45 ft. high is supported by three guys attached to the top, and reaching the ground at the distances of 60 ft., 108 ft., and 200 ft. from the foot of the pole. What are the lengths of the rods?
75 ft.; 117 ft.; 205 ft.

11. What is the circumference of a circle 8 feet in diameter?

12. What is the length of the tire on a carriage wheel 5 feet in diameter?

13. What is the circumference of a lake 721 rods in diameter?

14. What is the girth of an oak log which is 32 inches through?

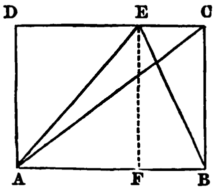
15. Find the diameter of a circle which is 33 rods in circumference.

16. In a park is a fountain whose basin is 8 ch. 20 l. in circumference. What is the diameter of the basin?

17. The extreme end of the minute-hand of a town clock moves forward 19 inches in 12 minutes. How long is the minute-hand?
$15\frac{17}{142}$ in.

SECTION III.

MENSURATION OF SURFACES.

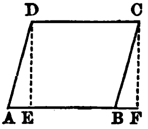


613. By examining this diagram, we see *1st.* The diagonal *AC* divides the parallelogram into two equal parts; and consequently, the area of the triangle *ABC* is equal to one half the area of the parallelogram *ABCD*, or to $\frac{1}{2}$ of $AB \times BC$. (See **188**).

2d. The areas of the triangles *AFE* and *BFE* are equal to one half the areas of the parallelograms *AFED* and *BFEC*, respectively; consequently, the area of the whole triangle ABE is equal to one half the area of the parallelogram $ABCD$, or to the area of the triangle ABC. Hence

I. *The area of a triangle is equal to one half the area of a parallelogram having the same base and altitude;* or

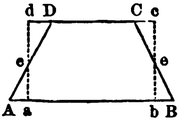
II. *To one half the product of its base multiplied by its altitude.*



614. If in the rhombus $ABCD$, the line DE be drawn perpendicular to the base, and the part AED be placed on the opposite side, the line AD on BC, the figure $EFCD$ will be a square.

By the same process, the rhomboid will be reduced to a rectangle. Hence

The area of any parallelogram is equal to the product of the base multiplied by the altitude.



615. If the right-angled triangles Aae and Bbe be applied to the spaces Dde and Cce, respectively, the figure $abcd$ will be a rectangle, equal in area to the trapezoid $ABCD$, because the side DC will be increased as much as the side AB is diminished; $AB+CD$ will equal $ab+cd$; and ab or cd will equal $\frac{AB+CD}{2}$. Hence

616. *The area of a trapezoid is equal to the product of one half the sum of its parallel sides multiplied by its altitude.*

617. From what has been said in **613**, **614**, it is evident that

I. *The area of any polygon is equal to the sum of the areas of any set of triangles into which it may be divided;* and

II. *The area of a regular polygon is equal to one half the product of the periphery multiplied by the altitude of one of its equal triangles.*

618. *Geometrical Principles.*

I. *The area of a circle is equal to the product of one half the circumference and one half the diameter;* or

II. *To one fourth the product of the circumference and diameter;* or

III. *To the product of the square of the diameter multiplied by .7854.*

IV. *The surface of a sphere or globe is equal to 4 times the area of a circle of the same diameter.*

V. *The areas of similar figures are to each other as are the squares of any one of their similar dimensions.* See Manual.

PROBLEMS.

1. The base of a right-angled triangle is 12 inches, and the perpendicular is 8 inches. What is the area? *48 sq. in.*

2. How many feet of boards will it take to cover the gable of a barn 32 feet wide, the ridge being 8 feet above the plates?

3. The base of a triangle is 8 ft. 1 in., and its area is 2,861½ sq. in. What is its altitude? *4 ft. 11 in.*

4. The base of a rhomboid is 223 feet, and its altitude is 96 feet. What is its area?

5. Find the area of a trapezoid whose sides are 9 and 17 inches long, and 13 inches apart. *1 sq. ft. 25 sq. in.*

6. How much lumber in an inch board 12 ft. long, 16 in. wide at each end, and 8 in. wide in the middle? *12 sq. ft.*

7. What is the area of a circle 20 feet in diameter?

8. My horse is tied to a stake in the pasture, by a rope 11 ft. long. On how much land can he graze? *380.1336 sq. ft.*

9. The area of the bottom of a tin pan is 196 sq. in. What is its diameter? *15.79 in.*

10. How many square inches of map surface on a 15-inch school globe? *706.86.*

11. The slant height of a pyramid is 11 inches, and the base is 4 inches square. How many square inches on the entire surface?

12. The periphery of the base of a cone is 40 inches, and the slant height is 38 inches. How many square inches are there on the lateral or convex surface?

NOTE.—The cone may be regarded as a pyramid of an infinite number of sides, and the periphery of its base as the sum of the bases of all the triangles which form its convex surface. *5 sq. ft. 40 sq. in.*

13. What is the surface of a prism 18 ft. long and 21 in. square?

14. What is the surface of a round pillar 14 inches in diameter and 30 feet long?

15. How many feet of inch lumber in a box 6 ft. 6 in. long, 4 ft. 2 in. wide, and 3 ft. 2 in. deep, inside measurement?

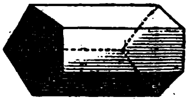
16. Two men start from the same place, at the same time. One of them travels south, at the rate of 3 miles an hour, and the other west at the rate of 4 miles an hour, for 7 hours. They then travel directly towards each other, at the rate of 3½ miles an hour, till they meet. How many hours do they travel, and how many square miles do they travel round?

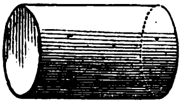
SECTION IV.

MENSURATION OF CAPACITIES.

619. The capacity of a prism or a cylinder 4, 5, or 6 feet in length, is 4, 5, or 6 times as much as 1 foot in length of the same prism or cylinder. (See **191.**) Hence,

The capacity of a prism or cylinder is equal to the product of the area of its base multiplied by its length.





NOTE.—Lumber 1 inch thick or less is sold by surface measure. If more than 1 inch thick, it is computed at this thickness; *i. e.*, the product of the surface measure in square feet multiplied by the thickness in inches, is the number of feet of lumber of standard thickness.

620. *Geometrical Principles.*

I. *The solidity of a pyramid is ⅓ that of a prism, and the solidity of a cone ⅓ that of a cylinder, having the same base and altitude.*

II. *The solidity of a sphere is ⅔ that of a cylinder whose diameter and altitude are, each, equal to the diameter of the sphere.*

III. *The capacities of similar solids are to each other as are the cubes of any one of their similar dimensions.* See Manual.

PROBLEMS.

1. The ends of a prism 20 feet long are right-angled triangles, the two shorter sides of each of which measure 16 and 20 inches. Find the cubic contents of the prism.

2. How many feet of timber in a log 31 feet long and 17¼ inches in diameter? *51.78+.*

3. I have a cylindrical cistern 6 feet in diameter and 8 feet deep. What is its capacity in hogsheads? *26 hhd. 54.05+gal.*

4. The area of the base of an octagonal pyramid is 78 sq. ft., and its altitude is 19 ft. 6 in. What are its cubic contents?

5. What are the cubic contents of a cone 7 ft. in diameter at the base, and 16 ft. 9 in. high?

6. Find the solidity of a 13-inch school globe.

7. The capacity of a hollow globe of glass is 65.45 cubic inches. What is its diameter?

8. A leaden ball 1 inch in diameter weighs $\frac{3}{14}$ lb. How much does a leaden ball 5 inches in diameter weigh? *$26\frac{11}{14}$ lb.*

9. A cast-iron ball 4 inches in diameter weighs 9 lb. What is the weight of a cast-iron ball 7 inches in diameter? *$48\frac{15}{64}$ lb.*

10. A marble monument consists of a pedestal 18 inches square and 3 feet high, on which stands a pyramid 16 inches square and 7 feet high. What did it cost, at $16.25 per cubic foot? *$177.09.*

11. A log chain and $3\frac{3}{4}$ quarts of water fill a cubical box whose inside edge measures 8 inches. How many cubic inches are in the chain?

12. In a stick of timber 50 feet long, and 7 × 10 inches, there are how many feet, timber measure? How many feet, board measure?

13. Find the contents, in timber measure and in board measure, of a stick of timber 18 ft. long, 12 in. wide, 15 in. thick at one end, and 10 in. thick at the other.

14. Find the side of the largest square stick of timber that can be cut from a log 2 feet in diameter. *17 inches, nearly.*

15. How much will the flooring for a two-story house 24 × 32 feet cost, at $40 per M., the flooring being $1\frac{1}{4}$ inches thick? *$76.80.*

16. How much lumber in a stock of 9 boards 13 ft. long, 9 in. wide, and $1\frac{1}{2}$ in. thick? *$131\frac{5}{8}$ ft.*

17. I wish to have built in my cellar, a cistern that shall hold 20 hhd., and to have it a cylinder 5 ft. 10 in. in diameter. How deep will it be? *6 ft. 3.63+in.*

18. How much 2-inch plank must I buy for a 5-foot walk on the street sides of a corner lot 4 × 8 rods, the walk to be placed 2 ft. 6 in. from the fence? And how much will it cost me, at $16 per M.?

19. In a granary is a bin $12\frac{2}{3}$ ft. long, 8 ft. 7 in. wide, and 5.4 ft. deep. How many bushels of grain will it hold?

20. What is the capacity, in hogsheads, of a rectangular cistern 12 ft. long, 8 ft. wide, and 6 ft. 4 in. deep?

21. A laborer built a wall 5 rd. long, 5 ft. thick at the bottom, 2 ft. thick at the top, and 5 ft. high, in 2 days, building 2 ft. in height the first day. On which day did he lay the most wall?

22. If a trough 5 feet long holds 12 pailfuls of water, how many pailfuls will a similar box hold that is 8 feet long.

23. If a pint of wine will fill 15 cone-shaped wine-glasses, how many times will a gallon of wine fill a similar glass of $1\frac{1}{2}$ times the diameter at the top? *$35\frac{5}{9}$.*

CHAPTER 12.

MISCELLANEOUS PROBLEMS.

1. The quotient is $436\frac{517}{735}$, and the divisor is 735. What is the dividend?

2. A cubic foot of water weighs 62 lb. 8 oz. What is the pressure on 1 acre at the bottom of the sea, where the water is 1,000 fathoms deep?

3. Find the prime factors of $.531\dot{3}$.

4. The expenses of an excursion party, consisting of 8 gentlemen and 9 ladies, were $2.40 a piece, which were paid by the gentlemen. How much did each pay?

5. $.725 is what fraction of $1?

6. How many feet in .735 of a mile?

7. If the cost of manufacturing kip boots is $4.60 a pair, and they are sold at 25% profit, what is the selling price?

8. If $33\frac{1}{3}$ lb. of tea cost $$29\frac{1}{6}$, how much will $12\frac{1}{2}$ lb. cost?

9. How much lumber in a stock of 12 boards 14 ft. long and 10 in. wide?

10. A grocer bought 7 doz. brooms @ $2.25, and retailed them at $$.31\frac{1}{4}$ apieco. How much did he gain on the lot?

11. One week, 2,230 barrels of flour, which cost $9.25 per barrel, were received at the port of Cleveland, and it was sold at the rate of $3.15 per sack of 49 pounds. What was the gain?

12. How many pickets each 3 inches wide, placed 3 inches apart, will it take for a fence round a lot 4 × 10 rods?

13. What is the shortest distance that is an exact number of times a 1-ft. rule, a 2-ft. rule, a yard-stick, and a 10-ft. pole?

14. At $3.25 per C., how many broom handles can I buy for $26.52?

15. After 4% of a flock of sheep had been killed by dogs, and 68 had been sold to a butcher, $\frac{4}{7}$ of the original flock were left. How many sheep were in the flock at first?

16. If $21\frac{3}{4}$ bushels of oats are required to seed $9\frac{2}{3}$ acres, how many bushels will be required to seed a field of 17.2 acres?

17. How many sheets of tin, each 14 × 22 in., will it take to cover a roof, each side of which is 30 ft. long and 18 ft. 4 in. wide?

18. A printer's price for business cards is $2.75 for the first hundred, and $1.25 per hundred for any number after the first hundred. How much will 1,500 cards cost?

19. A man, dying, leaves an estate of $53,166, but it is incumbered to the amount of $17,496. His widow receives $\frac{1}{3}$ of what remains after paying the incumbrances, and the balance is divided equally among 5 children. What is the widow's share? How much does each child receive?

20. A fruit dealer pays $4.25 per bushel for 3 bushels of chestnuts, and sells them at $.10 a pint, tin measure. What are his profits?

21. What fraction equals .00096?

22. A grain buyer paid $1.40 per bushel for 2,380 bushels of wheat, and $.50 per bushel for transportation to New York, where he sold it at a loss of 20%. How much did he lose?

23. If 25 cows average 9 quarts of milk each, per day, through the year, and it is sold at an average of 7 cents per quart, and the expenses of keeping and labor are $78 per head, what are the annual profits?

24. What will be the face of a sight draft on New York, that costs $664 in Louisville, Ky., exchange on New York being at $3\frac{3}{4}$% premium?

25. I invested one half of my capital in bank-stock, and the balance in R.R. stock. I gained 11% on the bank-stock, and lost $7\frac{1}{2}$% on the R.R. stock, and my net gain was $175. How much was my capital?

26. If I buy 1,600 bushels of oats in Iowa, at a net cost of $.56 per bushel, transportation included, and sell them in New York @ $.54, how much do I gain?

27. A grocer paid $22.75 for a barrel of mess pork, and retailed $\frac{3}{8}$ of it at $.12½ per lb., and the balance @ $.14. What were his profits?

28. Portland, Me., is in latitude 43° 39′ north, and L. Titicaca, on the same meridian, is in latitude 16° 42′ south. How many miles, air line distance, from Portland to L. Titicaca?

29. A note at 8 mo. for $750, with interest, was discounted by a Boston bank, 3 months after date. What were the proceeds?

30. If three men can build a sidewalk 240 ft. long and 6 ft. wide, in 15 days, in how many days can 5 men build a walk 180 ft. long and 4 ft. wide?

31. A mechanic contracted to work a year for $40 per month, his wages payable at the end of each month. Nothing was paid him till the close of the year, when he received the whole amount, with 8% interest. How much did he receive?

32. A tax of $3,156 is levied on a Union school-district, whose assessed valuation is $493,125. What is the rate?

33. In the above-mentioned district A's property is assessed at $750, B's at $3,850, C's at $1,600, and D's at $14,500. How much is each one of them taxed?

34. Memorandum:—Nov. 19, 1866, gave a note for $1,650, with 6% interest. June 18, 1868, paid $125; Oct. 25, 1868, paid $475. March 4, 1869, took up the note. How much was the last payment?

35. How many squares of flooring in a floor 44 × 75 ft.?

36. In the right wing of an army there were 18,675 men, in the center 23,518, and in the left wing 11,498. The left wing was re-inforced by 16,488 new troops, and the center by 3,486. The commanding general then ordered 9,894 men from the left wing to the right, and 5,145 from the right wing to the center. How many troops were then in each division of the army?

37. A pile of 4-foot wood 244 feet long and 5 feet high, was sold for $152.50. What was the price per cord?

38. What is the commercial weight of a nugget of gold that weighs 3 oz. 3 pwt. 19¼ gr.?

39. A merchant sold broadcloth at 5% less than the marked price, and yet made a profit of 25%. At what % advance on cost were the goods marked?

40. A miller pays $1.45 per bu. for 225 bu. of wheat. If 4.5 bu. make 1 bar. of flour, and he sells the ship-stuffs for $54.75, at what price per bar. must he sell the flour, to realize a net profit of $104.50 by the transaction?

41. A 100-acre farm is a trapezoid in shape, the shorter of the two parallel sides being 64.7 rods, and the longer 135.3 rods. How far apart are these two sides?

42. A box of glass contains 50 sq. ft. How many panes in each of three boxes of glass, marked, respectively, 8 × 10, 9 × 16, and 10 × 18?

43. For what amount must I draw my note, payable in 60 days, to obtain a discount of $250 from a Philadelphia bank?

44. A teacher receives a salary of $1,050, and 6% of his expenses equals 20% of his savings. How much does he save yearly?

45. If the transportation of 51.2 tons of freight costs $268.80, how much should be paid for the transportation of 32¾ tons?

46. I paid 1¼% for an insurance of $1,075 on a building worth $1,500. If the building should burn, what would be my loss?

47. A field, which is $3\frac{1}{2}$ times as long as it is wide, contains 22.4 acres. What are its dimensions?

48. A block of marble contains $54\frac{11}{16}$ cubic feet, and the length, breadth, and thickness are as 7, 4, and 1. What are the dimensions?

49. $\frac{3}{4}$ mi. + $7\frac{7}{8}$ rd. + $\frac{2}{3}$ yd. = what distance?

50. A note for $356, dated Mar. 10, at 10 mo., with interest at 7%, was discounted at the American Exchange Bank of New York City, Aug. 25. What were the proceeds?

51. A drover bought 135 head of cattle @ $23, and 147 head @ $19, and shipped them to New York, at a cost of $1,597. He sold 163 head @ $37, and the balance @ $31. Did he gain or lose, on the whole drove, and how much?

52. The latitude of Chicago is 41° 54′ N., and Mobile is $706\frac{337}{5000}$ miles S. of Chicago. In what latitude is Mobile?

53. I sold a horse for $\frac{1}{5}$ more than he cost me, receiving $216 for him. How much did he cost me?

54. A cannon-ball 15 inches in diameter weighs 456 pounds. What is the diameter of a 260-pound shot?

55. A mechanic having $852.75, paid $\frac{2}{7}$ of his money for a half-acre lot of land. How much would an acre cost, at the same rate?

56. A sixty-day note for $237.40, dated Poughkeepsie, N. Y., June 21, was discounted at the Second National Bank of Troy, June 28. How much money was received?

57. How far is it from one of the lower corners to the diagonal upper corner of a room 20 ft. long, 16 ft. wide, and 12 ft. high?

58. When it is 9 o'clock, A. M., at Cincinnati, 84° 24′ west, it is 9 o'clock 47 min. 12 sec., A.M., at Montpelier. What is the longitude of Montpelier?

59. A farmer exchanged 2 bu. of beans @ $$1.31\frac{1}{4}$, for sugars at $.10 and $.11 per lb., taking the same quantity of each kind. How many pounds of sugar did he receive?

60. What is the interest, in this State, on a mortgage for $490, for 1 yr. 5 mo. 24 da.?

61. A and B were partners in business, with a capital of $1,250, of which A furnished $750. At the end of the first year A's share of the profits was $340.65, when B sold his interest to C for $637.50. At the end of the second year C's share of the profits was $247.80, when he bought A's interest for $876.74. How much did A and B each make? And how much had C invested more than he had realized?

62. From the product of the sum and difference of 3.6 and 2.24, subtract the difference between the squares of 3.6 and 2.24.

63. A can build 50 rods of fence in 14 days, B can build it in 25 days, C in 8 days, and D in 20 days. In what time can they build it, if they all work together?

64. The product is $\frac{7}{9}$, and the multiplier is $\frac{3}{8}$ of $\frac{5}{9}$ of 2. What is the multiplicand?

65. One day a boy bought peaches at the rate of 3 for 4 cents, and sold them at the rate of 2 for 5 cents, and cleared $4.20. How many peaches did he buy and sell?

66. If each one of us breathes 30 cubic feet of air per hour, in how long a time will we breathe as much air as this school-room contains?

67. What is the equated time for the payment of $100 due in 6 mo., $120 due in 7 mo., and $160 due in 10 mo.?

68. Pekin is in 118° E. longitude, and San Francisco is in 122° W. longitude. When it is noon at Pekin, what is the hour at San Francisco?

69. The floor of a public hall 56 × 84 feet, is of boards 14 feet long and 6 inches wide, which are nailed with 10-penny nails, 8 to each board. Allowing 68 nails to a pound, how many pounds of nails are in the floor?

70. There are $22\frac{1}{2}$ bricks to a cubic foot of brick wall. What part of the wall consists of mortar?

71. How many bricks in the walls of a house 48 ft. long, 25 feet wide, and 18 ft. high, the walls being 1 foot thick, and allowing $2\frac{1}{2}$% for openings?

72. If $2\frac{1}{2}$ yd. of cassimere @ $$1\frac{3}{4}$ are worth as much as .7 of a ton of coal, how much is the coal worth per ton?

73. When 8 eggs sell for $.25, what are they worth per dozen?

74. What is the least common multiple of the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{7}$, and $\frac{7}{9}$?

NOTE.—Reduce the fractions to least similar fractions.

75. What is the least common multiple of 222, 104, 68, 54, and 34? What is the greatest common divisor of the same numbers?

76. The Oswego Starch Co. drew on a customer in Milwaukee for $1,275, at 60 days after sight. The bank charged $\frac{1}{2}$% for collecting, and required 2 days for transmission each way. Exchange on Milwaukee being at $1\frac{1}{4}$% discount, what were the proceeds of the draft?

77. At $.36 per sq. yd. for plastering, and $.75 per roll for paper-hanging, how much will it cost to plaster the walls and ceiling, and paper the walls of a room 18 × 16 × 9 ft., making allowance, in papering, for 2 windows, each 3 × 6 ft., and 3 doors, each 3 × 7 ft., the wall-paper being 1 ft. 6 in. wide and 7 yd. in a roll?

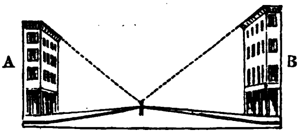
78. A 4-rod road extends along one end and one side of a farm which is 90.5 × 120 rd., the farm extending to the middle of the road. How much of the farm is in the road?

79. How many days will it take a ship to sail from St. John's, Newfoundland, to Valentia Bay, Ireland, a distance of 1,950 miles, if she sails at the rate of 9.5 knots per hour?

80. What length of a board 9 inches wide will make a square foot?

81. At $13.50 per M., what is the value of a stock of 13 boards each 14 ft. long, 16 in. wide at one end, and tapering to a point?

82. My agent in Toledo bought 5,000 barrels of apples @ $1.60, commission 2½%. I sent him a draft for the amount, which I purchased at ½% discount. I paid $.30 a barrel to transport the apples to New York, and sold them @ $2.10. What were my profits?



83. In a straight line between two buildings standing on opposite sides of a public square, is a post. The building, A, is 55 ft. high, and B 64 ft. From the foot of the post to the base of the building, B, is 76 ft.; from the top of the post to the top of the same building is 95 ft.; and from the top of the post to the top of the building, A, is 80 feet. What is the height of the post?

84. What is the horizontal distance between the buildings?

85. What is the distance from the top of one building to the top of the other?

86. The average diameter of the earth is 9,111 miles. How many square miles on its surface?

87. Find the number of cubic miles in the earth.

88. If 6 masons build a pier 35 ft. long, 18 ft. high, and 4 ft. wide, in 15 days of 8 hours each, how many masons will be required to build a pier 48 ft. long, 21 ft. high, and 5 ft. wide, in 12 days of 10 hours each?

89. Divide an estate of $7,500 among 3 children, 10, 12, and 15 years old, so that their respective shares, at 7% interest, shall amount to the same sum when they are 21 years old.

90. A mortgage for $13,275, dated St. Louis, Mo., Oct. 10, 1865, bears the following indorsements: May 7, 1866, $1,250; Dec. 11, 1866, $760; June 23, 1867, $500; Nov. 8, 1867, $850; July 20, 1868, $350. The mortgage was taken up Jan. 1, 1869. What amount was then paid?

91. What is the present value of a paid-up lease having 4 years to run, if the property will rent for $2,000 per annum, money being worth 6% compound interest?

92. A carpenter, a mason, and a painter built a house, by contract, for $3,000. The carpenter worked 108 days, the mason 72 days, and the painter 45 days, and the materials used cost $1,775. How much did each man receive for his labor?

93. Last year my expenses, which were 80% of my last year's income, equalled 96% of my expenses this year, and my income equalled 75% of this year's income. Last year I saved $480. How much do I save this year?

94. A broker bought 115 shares of Express stock at 79½. He exchanged 63 shares at 85 for U. S. 5–20's at 111, and the balance at par for R.R. stock at 78. He afterward sold the 5–20's at 116⅛, and the R.R. stock at 72. Did he gain or lose, and how much?

95. Find the balance of the following account, and the equated time for its payment:

Dr. GEO. H. THOMAS. *Cr.*

1869					1869				
Jan.	13	To Mdse. @ 4 mo.	23	30	Feb.	25	By Cash,	25	00
Feb.	12	" " 4 "	42	83	Apr.	7	" "	75	00
Mar.	23	" " 6 "	169	22	May	22	" "	20	00
Apr.	19	" " Cash,	73	19	July	7	" Note,	75	00
June	6	" " @ 30 da.	48	53	"	29	" Cash,	25	00
					Aug.	16	" "	50	00

96. A widow who is left with a daughter 16, and a son 8 years old, is to have the income of property that pays an annual rent of $1,500 above taxes and repairs, till the daughter is 21 years old. The daughter is then to have the income, till the son attains his majority, when the property is to be his. How much is each one's interest in the property worth to-day, money being worth 6% simple interest?

INDEX.

PAGE
Addition of Compound Numbers... 146
" " Decimals.............. 81
" " Fractions.............. 191
" " Integers.............. 17
Analysis.............. 223
Arithmetical Progression.............. 311
Average of Payments.............. 275

Banking.............. 269

Changes of Dividend and Divisor... 165
Commission.............. 241
Common Divisors.............. 172
" Multiples.............. 176
COMPOUND NUMBERS.............. 113
Compound Proportion.............. 292
CONVERSE OPERATIONS.............. 205
Converse Reductions.............. 210
Cube Root, Extraction of.............. 306

DECIMALS.............. 73
Definitions in Compound Numbers. 113
" " Decimals.............. 73
" " Evolution.............. 300
" " Factors and Multiples 164
" " Integers.............. 9
" " Mensuration.............. 321
" " Percentage.............. 229
" " Progressions.............. 311
Discount.............. 265
Division of Compound Numbers.... 156
" " Decimals.............. 89
" " Fractions.............. 199
" " Integers.............. 50
Divisors, Common.............. 172

EVOLUTION.............. 300
Exchange.............. 273
Extraction of Cube Root.............. 306
" " Square ".............. 301

FACTORS AND MULTIPLES.............. 164
FRACTIONS.............. 179

Geometrical Progression.............. 315
Government Securities.............. 267

Insurance.............. 238
INTEGERS.............. 9
Interest.............. 252
" by Progressions.............. 319

Longitude and Time.............. 224

MANUAL OF METHODS AND SUGGESTIONS.............. 5
Measurement of Right-Angled Surfaces and Solids.............. 103

PAGE
MENSURATION.............. 321
Mensuration of Capacities.............. 327
" " Lines.............. 323
" " Surfaces.............. 324
MISCELLANEOUS PROBLEMS.............. 329
Multiples, Common.............. 176
Multiplication of Compound Nos... 154
" " Decimals.............. 85
" " Fractions.............. 195
" " Integers.............. 35

Notation of Compound Numbers... 115
" " Decimals.............. 74
" " Evolution.............. 300
" " Fractions.............. 179
" " Integers.............. 10
" " Percentage.............. 229

Partnership.............. 294
PERCENTAGE.............. 229
Percentage, the Five General Cases of.............. 231
Preface.............. 3
Price, Quantity, and Cost.............. 216
Prime Numbers.............. 171
Profit and Loss.............. 248
Progression, Arithmetical.............. 311
" Geometrical.............. 315
PROGRESSIONS.............. 311
Properties of Composite Numbers.. 169
Proportion, Compound.............. 292
" Simple.............. 289

RATIO AND PROPORTION.............. 287
Reductions, Converse.............. 210
" of Compound Numbers. 115
" " Fractions.............. 183
Review Problems in Compound Numbers.. 161
" " Converse Operations 226
" " Decimals.............. 110
" " Fractions.............. 203
" " Integers.............. 69
" " Percentage.............. 283
" " Proportion.............. 299

Simple Proportion.............. 289
Square Root, Extraction of.............. 301
Stocks.............. 245
Subtraction of Compound Numbers. 149
" " Decimals.............. 83
" " Fractions.............. 191
" " Integers.............. 26

Taxes and Duties.............. 248

United States Money.............. 95

APPENDIX.

ANNUAL INTEREST.

A. In Vermont, a note or other written obligation containing the words "with interest annually," or "with annual interest," is a legal contract on the part of the maker of such obligation, to pay the interest on the principal at the end of each year, whether the principal or any part of it is due or not. If he fails to pay the interest when it becomes due,—*i. e.*, at the end of each year,—the law allows to the holder of the obligation, in the nature of damages, simple interest on the unpaid yearly interests until they are paid; but it does not allow interest on this interest.

B. *Annual Interest* is the interest allowed when, by the terms of a note or other written obligation, interest becomes due and payable at the end of each year.

C. The Vermont statute of 1866 provides that, when notes, bills, or other obligations draw annual interest, and any part or the whole of this interest remains unpaid—

First.—"*The annual interests that remain unpaid shall be subject to simple interest from the time they become due to the time of final settlement;*" and

Second.—"*If in any year, reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year, with interest thereon from the time of payment, shall be applied: first, to liquidate the simple interest that has accrued from the unpaid annual interests; secondly, to liquidate the annual interests that have become due; and thirdly, to the extinguishment of the principal.*"

The practical applications of the several provisions of this law are properly embraced under two cases, namely:

I. When no partial payments have been made; and

II. When partial payments have been made.

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CASE I.

No Partial Payments.

FIRST METHOD.

D. Ex. 1. A note for $850, with interest annually, was taken up at the end of 5 years. How much interest had accrued?

SOLUTION.

Interest on principal due at end of each year, $850 × .06 = $51.

“ “ “ *for 5 years,*	*$51 × 5 =*	*$255*
“ “ *1st yearly int. for 4 yr.,*	*$51 × .24 (= .06 × 4) =*	*12.24*
“ “ *2d* “ “ “ *3* “	*51 × .18 (= .06 × 3) =*	*9.18*
“ “ *3d* “ “ “ *2* “	*51 × .12 (= .06 × 2) =*	*6.12*
“ “ *4th* “ “ “ *1* “	*51 × .06 =*	*3.06*
	Total interest,	*$285.60*

Explanation.—The interest due on the principal at the end of each year is $51; and in 5 years it is $255. The $51 of interest due at the end of the first year draws simple interest for 4 years; the $51 due at the end of the second year draws simple interest for 3 years; the $51 due at the end of the third year draws simple interest for 2 years; the $51 due at the end of the fourth year draws simple interest for 1 year; and the $51 due at the end of the fifth year draws no interest. The sum of the interest on the principal for 5 years, and the simple interest on the $51 of yearly interest for 4 years, 3 years, 2 years, and 1 year, is $285.60, the total interest required.

Ex. 2. How much interest will accrue on a note for $350 in 4 yr. 8 mo. 12 da., interest payable annually?

SOLUTION.

4 yr. 8 mo. 12 da. = 28.2% = .282.

Interest due on principal at the end of each year, $350 × .06 = $21.

Interest on principal for 4 yr. 8 mo. 12 da.,	*$350 × .282 =*	*$98.70*
“ “ *1st yearly int. for 3 yr. 8 mo. 12 da.,*	*$21 × .222 =*	*4.662*
“ “ *2d* “ “ “ *2* “ *8* “ *12* “	*21 × .162 =*	*3.402*
“ “ *3d* “ “ “ *1* “ *8* “ *12* “	*21 × .102 =*	*2.142*
“ “ *4th* “ “ “ *8* “ *12* “	*21 × .042 =*	*.882*
	Total interest,	*$109.79*

E. From these two examples it will be seen that, when interest is payable annually, the total interest of any sum of money for any given time, is made up of *the interest on the principal for the whole time, and the simple interest on each year's interest of the principal from the time it is due until paid.*

PROBLEMS.

Interest being payable annually, what is the interest

1. Of $650 for 4 years? *$170.04.*
2. Of $1,375 for 3 yr. 9 mo.? 335.36
3. Of $287.50 for 5 yr. 4 mo. 15 da.? *$105.01.*
4. Of $3,125 for 2 yr. 11 mo. 20 da.? What is the amount?

F. Thus far, the simple interest on each year's interest of the principal has been found by a separate computation. The whole amount of this simple interest may readily be found by one computation, as is shown in the next two examples.

Ex. 1. A note for $850, with interest annually, was taken up at the end of 5 years. How much interest had accrued?

SOLUTION.

Interest on principal due yearly, $\$850 \times .06 = \$51.$

1 year's interest draws simple int. for 4 *yr.* $+ 3$ *yr.* $+ 2$ *yr.* $+ 1$ *yr.* $= 10$ *yr.*

Simple interest on 1 year's interest for 10 yr.,

$$\$51 \times .60\ (= .06 \times 10) = \$\ 30.60$$

Interest on principal for 5 yr., $\$850 \times .30\ (= .06 \times 5) = 255$

Total interest, $\$285.60$

Explanation.—$51, the interest due at the end of the first year, draws simple interest for the second, third, fourth, and fifth years; $51, the interest due at the end of the second year, draws simple interest for the third, fourth, and fifth years; $51, the interest due at the end of the third year, draws simple interest for the fourth and fifth years; $51, the interest due at the end of the fourth year, draws simple interest for the fifth year; and $51, the interest due at the end of the fifth year, draws no interest.

From this we see that the same sum, $51, is at simple interest for 4 yr. + 3 yr. + 2 yr. + 1 yr., or 10 yr. in all. $30.60, the interest on 1 year's interest for 10 years, + $255, the interest on the principal for 5 years, is $285.60, the total interest required.

Ex. 2. What is the interest of $350 for 4 yr. 8 mo. 12 da., interest payable annually?

SOLUTION.

Interest due on principal at the end of each year, $\$350 \times .06 = \$21.$

1st year's interest draws simple interest for	*3 yr.*	*8 mo.*	*12 da.*
2d " " " " " "	*2*	*8*	*12*
3d " " " " " "	*1*	*8*	*12*
4th " " " " " "		*8*	*12*
Or 1 " " " " " "	*8 yr.*	*9 mo.*	*18 da.*

Simple interest on 1 year's interest for 8 yr. 9 mo. 18 da.,

$\$21 \times .528 = \$11.09$

Interest on principal for 4 yr. 8 mo. 12 da., $\$350 \times .282 = 98.70$

Total interest, $109.79

G. When interest is payable annually, the total interest of any sum of money for any given time, computed by the First Method, is made up of

I. *The interest on the principal for the whole time;* and

II. *The simple interest on one year's interest for the sum of the periods of time the several yearly interests remain unpaid.*

PROBLEMS.

Interest being payable annually, find the interest of

5. $275 for 3 years.	*$52.47.*
6. $241 for 4 yr. 2 mo.	66.03
7. $3,675.87½ for 3 yr. 9 mo. 24 da.	*$913.90.*
8. $1,000.46 for 5 yr. 3 mo. 16 da. Find the amount.	344.725

SECOND METHOD.

a

H. Ex. 1. A note for $850, with interest annually, was taken up at the end of 5 years. How much interest had accrued?

SOLUTION.

$$\frac{5\ yr. - 1\ yr.}{2} = 2\ yr.$$

Interest on principal for 5 yr., $\$850 \times .30\ (= .06 \times 5) = \$255$

Simple interest on this interest for 2 yr.,

$\$255 \times .12\ (= .06 \times 2) = 30.60$

Total interest, $285.60

Explanation.—The interest due at the end of each year becomes a principal, which draws simple interest for the remainder of the time. In this example we have $51 of yearly interest on interest 4 years, $51 on interest 3 years, $51 on interest 2 years, and $51 on interest 1 year; or $51 on interest 10 years in all. Since the interest of $1 for 2 years = the interest of $2 for 1 year; the interest of $1 for 3 years = the interest of $3 for 1 year; and so on, we have

Int. of $51	for 4 yr.	= int. of	4 × $51,	or int. of	$204,	for 1 yr.
" 51	" 3 "	= "	3 × 51,	"	153,	" 1 "
" 51	" 2 "	= "	2 × 51,	"	102,	" 1 "
" 51	" 1 "	=		"	51,	" 1 "
" 51	" 10 "	= "	10 × $51,	"	$510,	" 1 "
Int. of 5 × $51 for $\frac{1}{5}$ of 10 yr. or $\left(\frac{5\text{ yr.} - 1\text{ yr.}}{2} =\right)$ 2 yr.		= "	2 × 255,	"	510,	" 1 "

The interest on the principal for 5 yr. = five times $51; $\frac{1}{5}$ of 10 yr. = 2 yr.; and the simple interest on $51 for 10 yr. = the simple interest on 5 times $51 for $\frac{1}{5}$ of 10 yr. or 2 yr. But $\frac{1}{2}$ of one year less than the given time $\left(\frac{5\text{ yr.} - 1\text{ yr.}}{2}\right)$ = 2 yr., the time for which all of the yearly interest draws interest. We therefore subtract 1 from the number of years, divide the remainder by 2, and find the interest on $255 for the time indicated by this quotient, 2 yr. Adding the two results, we have $285.60, the total interest required, the same as by the First Method.

Ex. 2. How much interest is due on a note for $350, at the end of 4 yr. 8 mo. 12 da., interest payable annually

SOLUTION.

$$\frac{4\ yr. - 1\ yr.}{2} + 8\ mo.\ 12\ da. = 2\ yr.\ 2\ mo.\ 12\ da. = 13.2\% = .132.$$

Interest on principal for 4 yr., $350 × .24 (= .06 × 4)	=	*$84*
Simple interest on this int. for 2 yr. 2 mo. 12 da., $84 × .132	=	*11.09*
Interest on principal for 8 mo. 12 da., $350 × .042	=	*14.70*
	Total interest,	*$109.79*

Explanation.—Interest is due at the end of each year for 4 yr., making 4 times 1 year's interest. At the end of 4 years, the simple interest of the yearly interests is equal to the interest on 1 year's interest of the principal for 3 yr., 2 yr., and 1 yr., or 6 yr. in all; *i. e.*, 1 year's interest is at interest for 6 yr.; or 4 times 1 year's interest is at interest for $\frac{1}{4}$ of 6 yr., which is $1\frac{1}{2}$ yr. But $\frac{1}{2}$ of $\overline{4 \text{ yr.} - 1 \text{ yr.}} = 1\frac{1}{2}$ yr. = 1 yr. 6 mo., the time that all of the yearly interest for 4 yr. draws simple interest. The whole time being 4 yr. 8 mo. 12 da., the interest due at the end of each year for the 4 yr. draws simple interest for 8 mo. 12 da. more than this time, making 2 yr. 2 mo. 12 da., the whole time for which we compute simple interest on the yearly interest accrued in 4 years. Proceeding as in the solution of the last example, we find the yearly interest on $350 for 4 yr. to be $84. We next find the simple interest on $84 for 2 yr. 2 mo. 12 da. to be $11.09. We have now obtained $84, the interest on the principal for 4 yr., and $11.09, the amount of the interests accruing on the several yearly interests in the 4 yr. 8 mo. 12 da. We then compute the simple interest on the principal for 8 mo. 12 da., and obtain $14.70. Adding these results, we have $109.79, the total interest required.

J. When interest is payable annually, the total interest of any sum of money for any given time, computed by the Second Method, is made up of

I. *The interest on the principal for the years;*

II. *The simple interest on this interest for the period of time expressed by the months and days added to the quotient of the number of years less 1 divided by 2;*

And, when there are months and days in the given time,

III. *The interest on the principal for the months and days.*

PROBLEMS.

Interest being payable annually, what is the interest of

9. $800 for 5 years?
10. $124 for 4 yr. 8 mo.? *$38.59.*
11. $96.84 from Nov. 27, 1870, to July 10, 1874?
12. $874.45 for 3 yr. 9 mo. 14 da.? What is the amount? *$1,090.13.*

Solve the following problems by each of the methods now given.

Interest being payable annually, what is the interest

13. Of $272 for 3 yr. 3 mo. 3 da.?
14. Of $731.29 for 4 yr. 15 da.? *$193.57.*

15. Of $356 for 2 yr. 7 mo. 23 da.?

16. Of $100 from April 1, 1868, to July 10, 1875? *$51.90.*

17. Of $64.25 from Oct. 10, 1869, to the present time?

At annual interest, what is the amount

18. Of $124 for 5 yr. 8 mo.?

16 19. Of $65.15 for 1 yr. 7 mo. 15 da.? *$71.65.*

20. Of a note for $520, dated Aug. 5, 1870?

CASE II.

Partial Payments.

K. By reference to the extract from the Vermont law (see page 337, C), we see that when partial payments have been made on any written obligation drawing annual interest,

First.—Balances are to be computed to the end of each year in which a payment or payments are made.

Second.—If the amount of the payment or payments at the end of the year in which they are made, does not exceed the simple interest due on the yearly interests accrued to the same time, it must be subtracted from this simple interest. The remainder does not draw interest, but at the date to which the next balance is computed, it is to be added to the simple interest due on the unpaid yearly interests.

Third.—If the amount of the payment or payments at the end of the year in which they are made, exceeds the simple interest on the yearly interests, but does not exceed the sum of this simple interest and the yearly interests accrued to the same time, it must be subtracted from such sum. The remainder is unpaid annual interest, and draws simple interest to the date to which the next balance is computed.

Fourth.—If the amount of the payments at the end of the year in which they are made, exceeds the total interest then due, it is to be subtracted from the whole amount of the obligation.

Ex. Face of mortgage, Oct. 1, 1866, $7,250.

Indorsements,	Feb. 1, 1868,	$2,452.50;
	Jan. 15, 1869,	187.50;
	Oct. 8, 1871,	25;
	Jan. 16, 1874,	250;
	June 1, 1874,	3,000.

Settlement, Jan. 20, 1875. What was the amount due?

SOLUTION.

	Interest on unpaid interests.	Unpaid yearly interests.	Principals and amounts.
First Balance.			
To Oct. 1, 1868, 2 *yr.* = 12% = .12.			
$\frac{2\ yr. - 1\ yr.}{2} = 6\ mo. = 3\% = .03.$			
Mortgage,			*$7,250*
Yearly interests on mortgage for 2 *yr.*, $7,250 × .12=		*$870*	
Int. on these interests, $870 × .03=	*$26.10*		
Am't of interests due,			*896.10*
Am't of mortgage at end of 2d yr.,			*$8,146.10*
From Feb. 1 *to Oct.* 1, 1868, 8 *mo.*=4%=.04.			
Am't of 1st payment for 8 *mo.*, $2,452.50 × 1.04=			*2,550.60*
New prin., or unpaid bal. on mortgage, Oct. 1, 1868,			*$5,595.50*
Second Balance.			
To Oct. 1, 1869, 1 *yr.*=6%=.06.			
Interest on new prin. for 1 *yr.*, $5,595.50 × .06=		*$335.73*	
From Jan. 15 *to Oct.* 1, 1869, 8 *mo.* 16 *da.*=$4.2\frac{2}{3}\%=.042\frac{2}{3}$.			
Am't of 2d payment for 8 *mo.* 16 *da.* (*less than yearly interest due*), $187.50 × $1.042\frac{2}{3}$=		*195.50*	
Bal. of unpaid ann. int. and prin., Oct. 1, 1868,		*$140.23*	*$5,595.50*
Third Balance.			
To Oct. 1, 1872, 3 *yr.*=18%=.18.			
$\frac{3\ yr. - 1\ yr.}{2} = 1\ yr. = 6\% = .06.$			
Yearly interests on prin. for 3 *yr.*, $5,595.50 × .18=		*1,007.19*	
Interest on unpaid yearly interests for 3 *yr.*, $140.23 × .18=	*$25.24*		
Interest on accrued yearly interests, $1,007.19 × .06=	*60.43*		
Simple interest due on yearly interests, Oct. 1, 1872,	*$85.67*		
From Oct. 8, 1871, *to Oct.* 1, 1872, 11 *mo.* 23 *da.*=$5.8\frac{5}{6}\%=.058\frac{5}{6}$.			
Am't of 3d payment for 11 *mo.* 23 *da.* (*less than simple int. due on unpaid yearly interests*), $25 × $1.058\frac{5}{6}$=	*26.47*		
Unpaid balances, Oct. 1, 1872,	*$59.20*	*$1,147.42*	*$5,595.50*

Fourth Balance.	Interest on unpaid interests.	Unpaid yearly interests.	Principals and amounts.
Unpaid balances brot. forward,	$59.20	$1,147.42	$5,595.50
To Oct. 1, 1874, 2 *yr.*=12%=.12.			
$\frac{2\ yr. - 1\ yr.}{2}$ = 6 *mo.*=3%=.03.			
Yearly interests on prin. for 2 *yr.*, $5,595.50 × .12=		671.46	
Interest on unpaid yearly interests for 2 *yr.*, $1,147.42 × .12=	137.69		
Interest on accrued yearly interests, $671.46 × .03=	20.14		
Am't of interests due,			2,035.91
Total of prin. and interests at end of 6th yr.,			$7,631.41
From Jan. 16 *to Oct.* 1, 1874, 8 *mo.* 15 *da.*=4¼%=.0425.			
From June 1 *to Oct.* 1, 1874, 4 *mo.*=2%=.02.			
Amount of 4th payment for 8 *mo.* 15 *da.*, $250 × 1.0425= $ 260.63			
Am't of 5th payment for 4 *mo.*, $3,000 × 1.02= 3,060			3,320.63
New prin., or unpaid bal. on mortgage, Oct. 1, 1874,			$4,310.78
Final Settlement.			
From Oct. 1, 1874, *to Jan.* 20, 1875, 3 *mo.* 19 *da.*=18⅙%=.018⅙.			
Int. on new prin. for 3 *mo.* 19 *da.*, $4,810.78 × .018⅙=			78.31
Amount due on settlement, Jan. 20, 1875,			$4,389.09

EXPLANATION.—At the end of the second year, the amount of the first payment exceeds the amount of the simple and yearly interests due (page 343, **K**, *Third*). We therefore subtract the amount of the payment computed from its date to the time to which the first balance is computed, from the amount of the mortgage computed to the same time; and we have left $5,595.50, the balance unpaid at the end of the second year.

At the time to which the second balance is computed, the amount of the second payment is less than the yearly interest due (page 343, **K**, *Second*). We therefore subtract the amount of the second payment, $195.50, from the yearly interest due, $335.78, and we have left $140.23 of unpaid yearly interests, and $5,595.50, the whole of the principal at the beginning of the year.

At the time to which the third balance is computed, the amount of the third payment is less than the simple interest due on the unpaid yearly interests (page 343, **K**, *First*). Therefore we find, *1st*, The yearly interests on the principal to the time to which the third balance is computed, *i.e.*, for 3 years; *2d*, The interest on the $140.23 of unpaid yearly interests for 3 years; *3d*, The interest on the yearly interests accrued since the date of the second balance; *4th*, The amount of unpaid simple interest due; and *5th*, The amount of the third payment to the time to which the third balance is computed. Next subtracting this amount, $26.47, from the amount of simple interest due, $85.67, and then adding the unpaid yearly interests, we have left unpaid at the beginning of the next year, $59.20 of simple interest, $1,147.42 of yearly interests, and $5,595.50 of principal.

At the time to which the fourth balance is computed, the sum of the amounts of the payments exceeds the amount of all the interests due (page 343, **K**, *Third*). We therefore subtract the sum of the amounts of the payments computed from their respective dates to the date of the fourth balance, from the total amount of the mortgage computed to the same date; and we have left $4,310.78, the unpaid balance at the time to which the fourth balance is computed.

As the final settlement occurs within the year following the date of the fourth balance, we add $78.31, the simple interest accrued at the time of final settlement, to $4,310.78, the principal, and we have $4,389.09, the required amount due.

L. Upon the preceding facts and explanations are based the following

Rules for Computing Annual Interest.

I. No Partial Payments.

First Method.—1. *Compute the interest on the principal for one year.*

2. *Compute the interest on this interest for the sum of the times the several years' interests remain unpaid.*

3. *Compute the interest on the principal for the whole time.*

4. *Add the last two results.*

Second Method.—1. *Subtract 1 from the number of years, divide the remainder by 2, and to this quotient add the given months and days.*

2. *Compute the simple interest on the principal for the years.*

3. *Compute the interest on this simple interest, for the whole time found by the first step in this Method.*

4. *Compute the interest on the principal for the months and days.*

5. *Add the last three results.*

II. Partial Payments.

1. Compute the balance or balances to the end of each year in which a payment is made.

2. When the amount of the payment, or the sum of the amounts of the payments at the end of the year in which they are made, equals or exceeds the total interest due,

Subtract this amount from the sum of the principal and accrued interests.

The remainder will be a new principal.

3. When the amount of the payment or payments at the end of the year exceeds the simple interest on the unpaid yearly interests, but is less than all the accrued interest,

Subtract such amount from the sum of the simple and yearly interests due.

The remainder will be unpaid yearly interest, and will draw simple interest to the time to which the next balance is computed, or till paid.

4. When the amount of the payment or payments at the end of the year is less than the interest on the unpaid yearly interests,

Subtract such amount from the simple interest accrued on the unpaid yearly interests.

The remainder will be unpaid simple interest on unpaid yearly interests; and it must be added to the simple interest accrued on the unpaid yearly interests at the date of the next balance.

Notes.—1. The last balance must be computed to the date of settlement.

2. The result obtained by the third step in the Second Method is the simple interest of the accrued yearly interests.

3. Annual interest at any greater rate than 6% is illegal.

4. For any time not exceeding 2 years, the amount of any sum is the same at annual interest as at compound interest. When the time exceeds 2 years, the amount is less at annual interest than at compound interest.

PROBLEMS.

21. What is the amount of a note for $600, which has run 4 yr. 3 mo. 15 da., with interest annually? *$769.98.*

22. A note for $956, bearing date, Burlington, Vt., Oct. 9, 1870, was paid June 15, 1874, with annual interest. What amount was paid?

23. To how much will $250 amount in 3 years, 1st. At simple interest? 2d. At annual interest? 3d. At compound interest?

24. Allowing annual interest for payment in advance, what debt due in 2 yr. 6 mo. will $500 cash pay to-day? *$578.60.*

25. What is the simple interest of the accrued annual interests of $2,000, for 3 yr. 7 mo. 20 da.? *$35.40.*

26. What is the difference between the amounts of $942 for 4 yr. 6 mo., at annual interest and at compound interest? *$1.46.*

27. Face of mortgage drawing annual interest, Jan. 15, 1868, $4,500. Payment of $2,000, June 1, 1871. Taken up, Jan. 1, 1873. What amount was due? *$3,810.28.*

28. Jan. 1, 1866, I gave my note for $5,000, with interest annually. June 1, 1868, I paid $500; May 1, 1871, I paid $1,000; and Oct. 1, 1874, I took up the note. How much was the last payment? *$6,369.87.*

29. Memorandum:—Sept. 5, 1868, gave a note for $2,500, "with interest annually." Paid $125, Apr. 1, 1869; $475, Aug. 25, 1869; and the balance, Jan. 1, 1874. How much was the last payment? *$2,630.29.*

30. A mechanic bought a house and lot in Montpelier for $2,750, paying $1,000 down, and giving a mortgage for the balance, due in 4 years, with interest annually. 1 yr. 6 mo. afterward he paid $842. What amount was due on the mortgage at maturity?

31. Nov. 7, 1867, a banker in Bennington loaned to a manufacturing company $10,000, at annual interest. June 28, 1868, $4,800 was paid; and July 14, 1871, $3,750. What sum was due Jan. 3, 1874?

32. Upon a note for $576, with annual interest, dated St. Johnsbury, Vt., March 14, 1870, there was paid Oct. 31, 1871, $50; and June 11, 1872, $285. How much was due Sept. 25, 1873? *$344.49.*

33. On a mortgage for $5,000, with interest annually, and dated Jan. 1, 1865, were indorsed payments as follows: July 1, 1868, $50; Nov. 1, 1869, $100; Dec. 1, 1870, $2,000; April 1, 1871, $25; Nov. 1, 1871, $2,000; and June 1, 1874, $20. The mortgage was taken up June 20, 1875. How much was the last payment? *$3,842.14.*

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